2) Waves and Particles

revision of movie:
Quantum physics is essentially all about “things that ought to be particles are also waves” and “things that ought to be waves are also particles”.

Thus, let’s make sure we are all on the same page regarding waves…..
2.1) Introduction to wave mechanics

What is a wave?

**Definition of wave:**

A perturbation of some *property* is transported through a *medium*, without transport of the medium itself

**Book:** A.P. French, “Vibrations and waves”
Examples:

- **transverse wave**
- **Rope waves**
- **Seismic wave**
- **Sound wave**
- **Elm wave**
Waves

Water wave

Traffic waves

Spin waves

Gravitational wave
2.1.1) Waves, frequencies, wavelengths

\[ y(x) = A \sin \left( \frac{2\pi}{\lambda} x \right) \]

- **y(x)**: amplitude
- **\( \frac{2\pi}{\lambda} \)**: wavelength

![Diagram of a wave with a child pulling on it](image)
Waves

\[ y(x, t) = A \sin \left( \frac{2\pi}{\lambda} (x - Vt) \right) \]

Form of progressive/travelling wave

wave velocity
amplitude
wavelength
Waves

\[ y(x, t) = A \sin \left( \frac{2\pi}{\lambda} (x - Vt) \right) \]

- Argument of \( \sin \) is called **phase**
- \( V \) here is the **phase velocity**

At \( t = 0 \):

\[ t = \frac{\lambda}{V} \]

\[ t = \frac{\lambda}{2V} \]
Waves

Rewrite wave form:

\[ y(x, t) = A \sin \left( \frac{2\pi}{\lambda} (x - Vt) \right) \]

\[ k = \frac{2\pi}{\lambda} \]  

\[ y(x, t) = A \sin (kx - \omega t) \]  \hspace{1cm} (6)

\[ \frac{\omega}{k} = V \]  \hspace{1cm} (8)

- \textbf{k} is the \textit{wave number} \hspace{1cm} (unit 1/m)
- \textbf{\omega} is the \textit{angular frequency}
- \textbf{\nu} is the \textit{frequency} \hspace{1cm} (unit Hz = 1/s)

\[ \omega = 2\pi \nu \]  \hspace{1cm} (9)
Waves

\[ y(x, t) = A \sin (kx - \omega t) \] (6)

\[ k = \frac{2\pi}{\lambda} \] (7)

**Relation between frequency, wave length or wave number and phase velocity of any wave** (unit m/s)

\[ \frac{\omega}{k} = V \] (8)

\[ \nu \lambda = V \] (10)

\( k \) is the wave number

\( \omega \) is the angular frequency

\( \nu \) is the frequency

\[ \omega = 2\pi\nu \] (9)
Wave velocities

\[
\frac{\omega}{k} = V \quad (8) \quad \nu \lambda = V \quad (10)
\]

Examples:

sound in solid

\[
V = \sqrt{\frac{Y}{\rho}} \approx 5000 \text{ m/s}
\]

\[
\nu = 440 \text{ Hz} \quad \lambda = 11.4 \text{ m}
\]

gravitational waves

\[
V = c = 299792458 \text{ m/s}
\]

\[
\nu = 440 \text{ Hz} \quad \lambda = 681 \text{ km}
\]
Wave velocities

\[ \frac{\omega}{k} = V \quad (8) \]
\[ \nu \lambda = V \quad (10) \]

**Examples II:**

- **water wave (tsunami)**
  \[ V = 500 \text{ km/h} \]
  \[ \nu = 3.3 \text{ /h} \]
  \[ \lambda = 151 \text{ km} \]

- **light wave (elm)**
  \[ V = c = 299792458 \text{ m/s} \]
  \[ \lambda = 700 \text{ nm} \]
  \[ \nu = 4.2 \times 10^{14} \text{ Hz} \]
2.1.2) The wave equation

Is there a general equation that governs wave behavior?

\[ y(x, t) = A \sin (kx - \omega t) \]

We see:

\[ \frac{\partial^2}{\partial x^2} y(x, t) = - k^2 A \sin (kx - \omega t) = - k^2 y(x, t) \quad (11) \]

\[ \frac{\partial^2}{\partial t^2} y(x, t) = - (-\omega)^2 A \sin (kx - \omega t) = - \omega^2 y(x, t) \quad (12) \]
Wave equation

Any function:

\[ y(x, t) = f(x - Vt) \]

Chain rule:

\[ \frac{\partial^2}{\partial x^2} y(x, t) = (1)^2 f''(x - Vt) \]
\[ \frac{\partial^2}{\partial t^2} y(x, t) = (-V)^2 f''(x - Vt) \]

Fulfills wave-equation: (13)

\[ \frac{\partial^2}{\partial x^2} y(x, t) = \frac{1}{V^2} \frac{\partial^2}{\partial t^2} y(x, t) \]
The wave equation

\[
\frac{\partial^2}{\partial x^2} y(x, t) = -k^2 y(x, t) \quad \text{(11)}
\]

\[
\frac{\partial^2}{\partial t^2} y(x, t) = -\omega^2 y(x, t) \quad \text{(12)}
\]

\[
y(x, t) = -\frac{1}{\omega^2} \frac{\partial^2}{\partial t^2} y(x, t)
\]

With:

\[
\frac{\omega}{k} = V \quad \text{(8)}
\]

\[
\frac{k}{\omega} = \frac{1}{V}
\]

General wave equation

\[
\frac{\partial^2}{\partial x^2} y(x, t) = \frac{1}{V^2} \frac{\partial^2}{\partial t^2} y(x, t) \quad \text{(13)}
\]

• Change in time causes change in space and vice versa.
Wave equation

\[ y(x, t) = f(x - Vt) \] moves to the right with velocity \( V \)!!
Wave equation

\[ y(x, t) = f(x - Vt) \] moves to the right with velocity \( V \!! \)

- \[ y(x, t) = f(x + vt) \] Moves to the left with velocity \( V \), also fulfills wave equation
- Can be generalized to 2D, 3D
- There are many wave-equations, one for each medium.
Superposition principle

The wave equation is linear. That means any combination of waves is also a solution.

let:
\[ \frac{\partial^2}{\partial x^2}y(x, t) = \frac{1}{V^2}\frac{\partial^2}{\partial t^2}y(x, t) \]
\[ \frac{\partial^2}{\partial x^2}w(x, t) = \frac{1}{V^2}\frac{\partial^2}{\partial t^2}w(x, t) \]

Then:
\[ \frac{\partial^2}{\partial x^2}[y(x, t) + w(x, t)] = \frac{1}{V^2}\frac{\partial^2}{\partial t^2}[y(x, t) + w(x, t)] \]
2.1.3) Standing waves

What happens if we combine two identical waves travelling in opposite directions?

https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html
2.1.3) Standing waves

What happens if we combine two identical waves travelling in opposite directions?

Using (6), we can write this as:

\[ y(x, t) = A \sin (kx - \omega t) + A \sin (-kx - \omega t) \]

Animation from: [https://www.youtube.com/watch?v=ic73oZoqr70](https://www.youtube.com/watch?v=ic73oZoqr70)
Standing waves

\[ y(x, t) = A \sin (kx - \omega t) + A \sin (-kx - \omega t) \]

Trigonometric identity

\[ \sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \]  \hspace{1cm} (14)

\[ y(x, t) = A \left[ \sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t) \right. \\
\hspace{1cm} + \sin(-kx)\cos(\omega t) - \cos(-kx)\sin(\omega t) \right] \\
\hspace{1cm} - \sin(kx) \cos(kx) \sin(\omega t) \]

\[ y(x, t) = -2A \cos(kx) \sin(\omega t) \]
Standing waves

Formula for some standing wave

\[ y(x, t) = \tilde{A} \cos(kx) \sin(\omega t) \]  

\[ t = \frac{\pi}{2\omega} \quad \text{just before} \quad t = \frac{\pi}{\omega} \quad t = \frac{3\pi}{2\omega} \]
Standing waves

Boundary condition: $y(0,t) = y(L,t) = 0$ (16b)

Resonance condition for standing wave

$L = n \frac{\lambda}{2}$  \quad $\lambda = \frac{2L}{n}$  \quad $n = 1, 2, 3, \ldots$ (16)

* Q: Eq. (15) is an example that does not fulfill Eq. (16b). Find another example that does.
Standing waves

Examples:
Backreflected string wave

Musical instruments

Antenna current charge

Micro-wave oven

http://whatmusicreallyis.com/research/physics/
2.1.4) Phenomena characteristic for waves

Interference

Superposition principle: Waves taking different paths get added.

Standing wave: example where superimposed waves always cancel at anti-node:
Interference

Usually (2D, 3D) more options:

Circular waves on a water surface
Interference

Usually (2D, 3D) more options:

Two circular waves: strengthen/cancel

https://www.youtube.com/watch?v=ovZkFMuxZNc

Two circular waves: — strengthen  — cancel
Interference

Waves can show interference

- strengthening in certain directions/ at certain times: **constructive** interference

- weakening in certain directions/ at certain times: **destructive** interference
Diffraction

Waves can turn around corners:

https://www.youtube.com/watch?v=BH0NfVUTWG4
Diffraction

Decompose wave into lots of spherical waves:

Could see this from 2D wave equation
Slit smaller than wavelength: emits circular waves going in **ALL** directions

Slit larger than wavelength: waves destructively interfere if direction not almost forward *(tutorial, waves and optics course)*
Double slit interference
Diffraction and Interference

Double slit interference

equal phase fronts
Diffraction and Interference

Double slit interference

d
screen
Diffraction and Interference

Double slit interference

\[ y_1(r_1, t) = \frac{A}{r_1} \sin(kr_1 - \omega t) \]
\[ y_2(r_2, t) = \frac{A}{r_2} \sin(kr_2 - \omega t) \]

Don’t worry about 1/r prefactors (energy conservation, see optics course later)
Double slit interference

\[ y_1(r_1, t) = \frac{A}{r_1} \sin(kr_1 - \omega t) \]

\[ y_2(r_2, t) = \frac{A}{r_2} \sin(kr_2 - \omega t) \]
Diffraction and Interference

Double slit interference

wave amplitude at position $z$

$$y(z, t) = y_1(r_1(z), t) + y_2(r_2(z), t)$$
Diffraction and Interference

Double slit interference

The mean wave intensity at position $z$ is given by:

$$I(z) = \left| y(z, t) \right|^2 = y_1^2 + y_2^2 + 2y_1y_2$$

(\text{\textit{...is time average}})
Diffraction and Interference

\[ I(\theta) \approx I_0 \cos(\pi d \frac{\sin \theta}{\lambda}) \]  (17)
Diffraction and Interference

Path difference: $r_2 - r_1$
Diffraction and Interference

Path difference

$r_2 - r_1 = \lambda$
Path difference \( r_2 - r_1 = \lambda/2 \)
Diffraction and Interference

\[ I(\theta) \approx I_0 \cos^2 \left( \pi d \frac{\sin \theta}{\lambda} \right) \] (17)

double slit interference pattern
Examples:

- Colors reflected from CD
- VLA Radio Astronomy
- Water in bay
2.1.5) Electromagnetic waves

You will learn in Electro-magnetism lecture:

Changing **magnetic** field causes **electric** field (induction)  
Changing **electric** field causes **magnetic** field
Electromagnetic waves

Electric field

Magnetic field

travelling direction of wave (transverse wave)
**Electromagnetic waves**

**Electric field**

**Magnetic field**

**Electromagnetic wave equation:**

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \mathbf{E}(\mathbf{r}, t) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E}(\mathbf{r}, t) \tag{18}
\]

**Speed of light (vacuum)**

\[
c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \tag{19}
\]

\[
c = 29,979,245,8 m/s
\]
Electromagnetic waves

\( \nu \lambda = c \quad (10) \)

**Color** | **Wavelength**
---|---
**violet** | 380–450 nm
**blue** | 450–495 nm
**green** | 495–570 nm
**yellow** | 570–590 nm
**orange** | 590–620 nm
**red** | 620–750 nm