

## COMPLEX TOPOLOGICAL K-THEORY : PROJECT DESCRIPTION

The aim of algebraic topology is to understand a topological space by associating to it groups whose algebraic properties reflect the topological properties of the space. The goal of this project is to understand one such family of groups, collectively called topological K-theory.

K-theory can be defined in two ways, using vector bundles over the space, or using a more algebraic formulation through continuous functions on the space. We will begin the project by learning about both these approaches, prove their equivalence, and use them to define the  $K^0$  group associated to a space.

We will then learn various properties of the  $K^0$  group, such as half-exactness and excision. We then define the  $K^{-1}$  group using invertible continuous functions, and discuss the boundary map connecting  $K^0$  to  $K^{-1}$ . This allows us to define the higher K-groups,  $K^{-n}$ .

Our goal then is to understand the Bott periodicity theorem, which states that there is a natural isomorphism between  $K^{-n}$  and  $K^{-n-2}$ . We will then use this theorem to compute certain K-groups, and also understand vector bundles over spheres, the projective plane, etc.

Finally, we will also discuss the ring structure on the  $K^0$  group and the fundamental product theorem, which gives us another equivalent description of the Bott isomorphism.

We will follow [Park] and [Hatcher] for most of the material, occasionally using [Husemoller] and [Notes] as well.

### REFERENCES

- [Atiyah] M. Atiyah, *K-Theory*, Westview Press (2009)
- [Hatcher] A. Hatcher, *Vector Bundles and K-theory*, <https://www.math.cornell.edu/~hatcher/VBKT/VB.pdf>
- [Husemoller] D. Husemoller, *Fibre Bundles (3rd Ed)*, Springer-Verlag (1994)
- [Karoubi] M. Karoubi, *K-theory: An Introduction*, Springer (1978)
- [Notes] Notes from *School on K-theory and its Applications*, ISI Bangalore (2016)
- [Park] E. Park, *Complex Topological K-Theory*, Cambridge University Press (2008)

### IMPORTANT COMMENTS

- (1) We will have weekly meetings in which you are expected to give a progress report and discuss questions. You must treat these meetings with utmost seriousness, and must cancel them only in case of unavoidable emergencies. (Taking the GATE/GRE do not count as emergencies)
- (2) Every other week or so, you will be expected to give a short board presentation on what you have learnt.
- (3) You should start writing your project report immediately after completing the first topic. I expect to see the first draft of Chapter I by the end of Semester IX.
- (4) Read the DUGC guidelines for the project, and pay attention to the deadlines given therein. If you have any questions, make sure that you ask them *now*.
- (5) Please make sure that you complete all the paperwork on time, and ensure that you do not rush your PEC members by doing things last-minute.