CHM 421/621 Assignment 1

August 28, 2023

Due on 29^{th} Aug., 2023.

1. For a system with the fundamental equation

$$u = \left(\frac{\theta}{R}\right) s^2 - \left(\frac{R\theta}{v_0^2}\right) v^2$$

   (a) Find the three equations of state.

   (b) Verify that the equations of state are homogeneous zero order, i.e. that T, P and μ are intensive parameters.

   (c) Show that $\mu = -u$ here.

   (d) Express μ as a function of T and P.

2. A particular system obeys the relation

$$u = Av^{-2}\exp(s/R)$$

   N moles of this substance, initially at temperature T_0 and pressure P_0, are expanded isentropically ($s =$ constant) until the pressure is halved. What is the final temperature?

3. Show that if a single-component system is such that PV^k is constant in an adiabatic process (k is a positive constant) the energy is

$$U = \frac{1}{k-1} PV + Nf(PV^k/N^k)$$

   where f is an arbitrary function.

4. Two particular systems have the following equations of state:

$$\frac{1}{T^{(1)}} = \frac{3}{2} R \frac{N^{(1)}}{U^{(1)}}$$
$$\frac{1}{T^{(2)}} = \frac{5}{2} R \frac{N^{(2)}}{U^{(2)}}$$

where R is the gas constant. The mole number of the first system is $N^{(1)} = 2$ and that of the second is $N^{(2)} = 3$. The two systems are separated by a diathermal wall, and the total energy in the composite system is 2.5×10^3 J. What is the internal energy of each system in equilibrium?

5. The fundamental equation of a particular type of two-component system is

$$S = NA + NR\ln\frac{U^{\frac{3}{2}}V}{N^{\frac{5}{2}}} - N_1 R\ln\frac{N_1}{N} - N_2 R\ln\frac{N_2}{N}$$
$$N = N_1 + N_2$$

where A is an unspecified constant. A closed rigid cylinder of total volume 10 L is divided into two chambers of equal volume by a diathermal rigid membrane, permeable to the first component but impermeable to the second. In one chamber is placed a sample of the system with original parameters $N_1^{(1)} = 0.5$, $N_2^{(1)} = 0.75$, $N_1^{(2)} = 1$, $N_2^{(2)} = 0.5$, $V^{(2)} = 5$ L, $T^{(1)} = 300$ K and $T^{(2)} = 250$ K. After equilibrium is established, what are the values of $N_1^{(1)}$, $N_1^{(2)}$, T, $P^{(1)}$, and $P^{(2)}$?