CHM 325 Assignment 5

November 7, 2022

Due on 14^{th} November, 2022.

1. Check whether the functions given below satisfy the differential equations given on their right.

$$(i) \quad y = x^2 + cx \qquad\qquad x\frac{dx}{dy} = x^2 + y$$

$$(ii) \quad c_1e^{2x} + c_2e^{-3x} \qquad \frac{d^2y}{dx^2} + \frac{dy}{dx} - 6yx^2 = 0$$

$$(iii) \quad y = cx + c^2 \qquad\qquad y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$$

2. Solve the following differential equations by a suitable approach.

$$(i) \quad \frac{dy}{dx} = -\frac{y}{x-1}$$

$$(ii) \quad a\left(x\frac{dy}{dx} + 2y\right) = xy\frac{dy}{dx}$$

$$(iii) \quad x^2\frac{dy}{dx} = y^2\frac{dx}{dy}$$

3. Newton's law of cooling states the rate at which the temperature of a hot body having temperature $T > T_0$ at time t drops to reach the surrounding temperature T_0 is given by

$$\frac{dT}{T} = k(T - T_0)$$

where k is a constant and T is in degrees Celcius. A copper pellet initially at 200°C is dropped into a large bucket of water at 20°C. After 6 minutes, the temperature of the pellet is 100°C. How much longer will it take for the pellet to cool down to 25°C. Assume that the temperature of water does not change during the cooling.

4. Determine which of the following two differential equations is exact and solve it.

$$(i) \quad xdy + ydx = xy^3dx$$

$$(ii) \quad \frac{dy}{dx} = \frac{x-y}{x+y}$$

5. A curve is described such that the intercept of its tangent at any point (x, y) with the y axis is equal to $2xy^2$. Determine the curve $y(x)$.

6. Solve the differential equation

$$\frac{dy}{dx} = \frac{2x+y-4}{x-y+1}$$

using the substitution $x = u + x_0$ and $y = v + y_0$, where both the numerator and denominator go to zero at $x = x_0, y = y_0$.

7. Find the general solutions of

$$(i) \quad (x^2 + y^2)\frac{dy}{dx} = 1$$

$$(ii) \quad \frac{dx}{dt} = 3x - 5\sin(t)$$

$$(iii) \quad x\frac{dy}{dx} + y = 3x^3y^2$$

8. Consider the chemical reaction $A \rightleftharpoons B$. The rate equation of the reaction can be written as

$$\frac{dA}{dt} = -k_1A + k_2B$$

where k_1 and k_2 are rate constants. If $A(0) = A_0$ and $B(0) = B_0$, solve the equation for $A(t)$ and $B(t)$.

9. Check whether e^x, $\sinh x$ and $\cosh x$ are linearly independent.

10. Find the general solutions of

$$
\begin{aligned}
(i) &\quad y''(x) - y'(x) - 2y(x) = 0 \\
(ii) &\quad y''(x) + 2y'(x) + 4y(x) = 0 \\
(iii) &\quad y'''(x) - 2y''(x) - y'(x) + 2y(x) = 0
\end{aligned}
$$

11. The motion of a 1-dimensional simple harmonic oscillator subjected to frictional damping is given by

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega^2 x(t) = 0$$

where ω is the fundamental angular frequency and γ is the frictional coefficient. Given that at $t = 0$ the oscillator is stretched by x_0 and starts from rest, solve the equation to obtain $x(t)$. Sketch the behaviour of x/x_0 as a function of $\omega_0 t$ for (a) $\gamma = 0$, (b) $0 < \gamma < 2\omega_0$, (c) $\gamma = 2\omega$, and (d) $\gamma > 2\omega_0$.