CHM 325 Assignment 3

September 15, 2022

<u>Due on 29th September, 2022.</u>

1. Use the comparison test to show that $\int\limits_1^\infty \frac{\sqrt{x}}{1+x}\ dx$ converges.

2. Show that $\int\limits_0^\infty t^n \mathrm{e}^{-xt}\ dt = \frac{n!}{x^{n+1}}$.

3. Show that the integral $\int\limits_0^\infty \frac{\sin\ x}{x}\ dx$ converges and compute it. Hint: Use the result for $\int\limits_0^\infty \mathrm{e}^{-xt}\ dt$, for the second part of the question.

4. Compute the integral $\int\limits_{-\infty}^\infty \frac{\sin^2(\mathrm{ax})}{x^2}\ dx$, where $a > 0$.

5. Compute the integral $\int\limits_0^\infty \mathrm{e}^{-x^2}\cos(ax)\ dx$.

6. Show that $\int\limits_0^\infty \mathrm{e}^{ax}\cos(x)\ dx$ is a continuous function of a for $a > 0$.

7. Show that $I(a,b) = \int\limits_0^\infty \mathrm{e}^{-a^2x^2-b^2x^2}\ dx = \frac{\pi^{\frac{1}{2}}}{2a}\mathrm{e}^{-2ab}$.

8. Evaluate

   (a) $\int\limits_0^\infty \mathrm{e}^{-au}u^{\frac{3}{2}}\ du$

(b) $3\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{1}{4}\right)$

(c) $\int\limits_0^1 (\ln x)^n \, dx$

9. Evaluate $\int\limits_0^\infty x^m e^{-x^n} \, dx$, where m and n are positive integers, in terms of a gamma function.

10. Show that $\int\limits_{-\infty}^{\infty} f(x)\delta(g(x)) \, dx = \sum\limits_i \frac{f(x_i)}{|g'(x_i)|}$, where f, g are continuous functions and x_i are the roots of $g(x)$.