# CHM 325 Assignment 2

September 1, 2022

Due on $12^{th}$ September, 2022.

1. Differentiate the following functions of $x$

$$
\begin{aligned}
&a)(2+x)\mathrm{e}^{-x^2}\\
&b)\frac{\sin\ x}{x}\\
&c)x^2\ \tan\ 2x\\
&d)\sqrt{x^2-3x+1}\\
&e)x^{\sin\ x}\\
&f)\tan^{-1}\left(\mathrm{e}^{-x}\right)\\
&g)\ln\left(\sec\ x+\tan\ x\right)
\end{aligned}
$$

2. The tangent line to a curve at some point $(a,b)$ has the slope $m = \left(\frac{dy}{dx}\right)_{x=a} = f'(a)$. Show that the slope of the line perpendicular to the curve at $(a,b)$ is equal to $-\frac{1}{m}$.

3. Find the local extrema and the inflection points of $f(x) = 3x^4 - 4x^3 - 24x^2 + 48x - 20$ over the entire $x$ axis.

4. The free energy difference between two phases of some solid at any temperature is given by the expression $\Delta F(x,T) = F_1(x,T) - F_2(x,t) = ax^2 + bx^4$, with $a = a_0(T-c)$, where $a_0, b, c > 0$ are parameters. $x$ is called an order parameter describing the two phases, such that $x = 0$ in phase 2 and $0 < x \leq 1$ in phase 1. At any temperature, determine the critical points in $\Delta F$ and classify them in to maximum or minimum.

Also determine the inflection points, if any. If the most stable phase at any temperature is identified by the one with lower free energy, determine the temperature at which the system will undergo a phase transition.

5. The height of a body shot vertically upward is given as $h(t) = 40t - 32t^2$. How high will it go before it returns to ground?

6. Show that the rectangle of largest possible area for a given perimeter is a square.

7. The blackbody radiation law is given by

$$\rho(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{\mathrm{e}^{hc/\lambda k_B T} - 1}$$

where $\rho(\lambda, T)d\lambda$ is the energy between $\lambda$ and $\lambda + d\lambda$, $\lambda$ is the wavelength of the radiation, $h$ is the Plancl's constant, $k_B$ is the Boltzmann constant, $c$ is the speed of light, and $T$ is the temperature in Kelvin. The Wien displacement law says that $\lambda_{max} T$ = constant where $\lambda_{max}$ is the value of $\lambda$ at which $\rho(\lambda, T)$ is maximum. Derive the Wien displacement law from the blackbody radiation law. Show that "constant" $= hc/4.965k_B$.

8. Using l'Hopital's rule determing the following limits

$$a) \lim_{x \to 0} \frac{e^x - 1}{x}$$
$$b) \lim_{x \to 0} \frac{1 - \cos\, x}{x^2}$$
$$c) \lim_{x \to 0} \frac{\ln(1 + x)}{x}$$
$$d) \lim_{x \to 0} \frac{1 - \cos^2\, x}{x^2}$$