CHM 325 Assignment 1

August 21, 2022

Due on 30^{th} Aug, 2022.

1. If y is expected to a real-valued function of x, then determine the maximum domains of $y(x)$ for the following definitions of the function:
   (a) $y = \sqrt{16 - x^2}$
   (b) $y = \frac{1}{x^2+8}$
   (c) $y = \ln\, x$
   (d) $y = \frac{1}{x-1}$
2. Which of the following functions is periodic? What are their periods?
   (a) $\tan\, 2x$
   (b) $|\cos\, x|$
   (c) $(\sin\, x)/x$
3. Find the following limits
   (a) $\lim\limits_{x\to 0} \frac{\sin 3x}{x}$
   (b) $\lim\limits_{x\to 0} \frac{\sin 2x}{\sin x}$
   (c) $\lim\limits_{x\to 1} \frac{1+\cos \pi x}{\tan^2 \pi x}$
   (d) $\lim\limits_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$
4. Check whether the function $f(x) = (x^4 + x^3 - 3x + 2x - 1)/(x - 1)$ is continuous at $x = 1$.

5. Show that the following equations have at least one solution :

   (a) $2x^4 + 2x + 1 = 0$ between -1 and 1.

   (b) $\cos\, x = x$ betweem 0 and $\pi/2$.

6. Using ϵ-δ notation prove that $f(x) = x^2$ is continuous at $x = 2$.

7. The function $f(x) = (x^3 - 1)/(x^2 - 1)$ is not defined at $x = 1$. Is the point $x = 1$ a removable discontinuity? What value must $f(x)$ be assigned at $x = 1$ to make it continuous there?

8. Find α and β such that $f(x)$ given below is continuous at $0 < x < 2\pi$.

$$f(x) = \begin{cases} -\sin x & 0 < x < \pi/2 \\ \alpha \sin\, x + \beta & \pi/2 < x < 3\pi/2 \\ \left(x - \frac{3\pi}{2}\right)^2 & 3\pi/2 < x < 2\pi \end{cases} \tag{1}$$