CHM 322/642 Assignment 2

February 13, 2017

Due on 20^{th} Feb., 2017

1. Consider the wave packet $\psi(x) = Ce^{ip_0x/\hbar}e^{-|x|/(2\Delta x)}$ where C is a normalisation constant.
   (a) Normalize $\psi(x)$ to unity.
   (b) Obtain the corresponding momentum space wave function $\phi(p_x)$ and verify that it is normalised to unity according.
   (c) Suggest a reasonable definition of the width of Δp_x of the momentum distribution and show that $\Delta x \Delta p_x \gtrsim \hbar$.

2. Prove the following relations of the Dirac delta function :
   (a) $\int \delta(a-x)\delta(x-b)dx = \delta(a-b)$
   (b) $f(x)\delta(x-a) = f(a)\delta(x-a)$
   (c) $f(x)\delta''(x) = -f'(0)\delta(x)$

3. (a) Consider a particle moving along x direction described by the wave function $\psi(x,t)$. What is the probability that it will have a momentum p at time t ?
   (b) Prove that the certainty of finding a particle somewhere in space implies that the certainty that it can be found with some value of momentum. Consider particle in 1-dimension only. (**Hint**: Use Parseval's theorem).

4. Compute the Fourier transform of (i) $f'(x)$, (ii) $f''(x)$ and (iii) $\nabla^2 g(\vec{r})$, in terms of Fourier transforms of the original functions.

5. Calculate the Fourier transform of the Dirac delta function and show that if we know the position of a particle exactly at any instant then the momentum is completely uncertain.

6. Consider a particle of mass m moving in a potential given by $V(x) = 0, -a < x < a$ and $V(x) = \infty$, elsewhere.

   (a) Derive the stationary quantum states for this particle along with their energies.

   (b) Show that the eigenstates corresponding to different quantum numbers are orthogonal in the sense that $\int dx\ \psi_m^*(x)\psi_n(x) = \delta_{m,n}$.

   (c) Assuming that any function $f(x)$ that satisfies the boundary conditions of this problem can be written as a linear superposition of the eigenstates of the problem, i.e. $f(x) = \sum_{n=1}^{\infty} C_n\psi_n(x)$, show that the coefficients in the expansion are given by $C_n = \int dx\ \psi_n^*(x)f(x)$.

   (d) What can you say about the parity of the eigenfunctions?

7. When an electron in a certain excited energy level in a 1-dimensional box of length 2.00Å makes a transition to the ground state, a photon of wavelength 8.79 nm is emitted. Find the quantum number of the initial state.

8. An electron in a stationary state of a 1-dimensional box of length 0.300 nm emits a photon of frequency $5.05 \times 10^{15} s^{-1}$. Find the initial and final quantum numbers for this transition.

9. An electron confined in a 1-dimensional region has an optical spectrum with the longest wavelength of absorption at 140 nm. What is the confinement length of the electron?

10. Calculate the current density of a particle in the ground-state of 1-dimensional box problem. Show that the equation of continuity holds for this state.