General Relativity - 2017

Indian Institute of Science Education and Research Bhopal

Assignment 5

1. Lightlike geodesic equations in Schwarzschild spacetime is given by,

$$\begin{aligned}
\frac{d}{d\lambda}\left[\left(1-\frac{2GM}{c^2r}\right)\frac{dt}{d\lambda}\right] &= 0,\\
\frac{d}{d\lambda}\left[r^2\frac{d\phi}{d\lambda}\right] &= 0,\\
\left(1-\frac{2GM}{c^2r}\right)c^2\left(\frac{dt}{d\lambda}\right)^2-\left(1-\frac{2GM}{c^2r}\right)^{-1}\left(\frac{dr}{d\lambda}\right)^2-r^2\left(\frac{d\phi}{d\lambda}\right)^2 &= 0.
\end{aligned}$$

(a) Replacing $r = 1/u$ show that the a photon trajectory can be described by

$$\frac{d^2u}{d\phi^2}+u=\frac{3GM}{c^2}u^2.$$

(b) Show that for $M = 0$, the solution to this equation is given by $b = r\sin(\phi)$ for some constant b.

(c) Treating $3GM/c^2$ to be very small, show that the perturbed solution is given by

$$u=\frac{1}{b}\sin\phi+\frac{3GM}{2b^2c^2}\left(1+\frac{1}{3}\cos 2\phi\right)$$

(d) Hence show that the total bending of light around a massive object of mass M is given by $\frac{4GM}{bc^2}$. What is the minimum value of b? The Sun being the massive object, the deflection is around $1.75''$.

2. (a) Define $\bar{t} = t+2m\ln(r-2m)$. Here $m = GM/c^2$. Find out the metric in $\bar{t}, r$ coordinate system.

(b) Is the metric regular at $r = 2m$?

(c) Define $v = \bar{t} + r/c$, and find the metric in (v, r) coordinate system.

(d)Find out the light like geodesics in this coordinate system, and draw space time diagram in $(\bar{t}, r)$ plane. Draw light cones in this plane.