General Relativity - 2017

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Assignment 4

1. For non relativistic system $|T_{ij}| \ll |T_{00}|$. (a) In this case, show $R_{ij} \simeq \frac{1}{2} g_{ij} R$, and $R = 2R_{00}$. It leads us to $G_{00} \simeq 2R_{00}$. (b) Now, in the weak and static field limit, show that $R_{00} = \frac{1}{2}\nabla^2 g_{00}$. With all these information, write down the weak-field limit of the Einstein's equations.

2. Consider the three dimensional space with the line element

$$ds^2 = (1 - 2M/r)^{-1} dr^2 + r^2 d\Omega^2.$$

Calculate the radial distance between the sphere at $r = 2M$ and the sphere at $r = 3M$. (b) Calculate the spatial volume between the two spheres in part (a).

3. A static, spherically symmetric spacetime must have a metric of the form

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2 d\Omega^2.$$

(a) Show that R_{tt} has the following form

$$R_{tt} = \frac{f''}{2h} - \frac{f'}{4h}\left(\frac{f'}{f} + \frac{h'}{h}\right) + \frac{f'}{rh}$$

(b) Let $h = f^{-1}$. Show that $R_{rttr} = \frac{1}{2} f''(r) = -\frac{2M}{r}$ for $f = (1 - \frac{2M}{r})$. How should you interpret your result? Do give special attention to $r \to \infty, r = 2M$, and $r = 0$. There are also other non-zero components of the the Riemann tensor.

4. Let the spacetime metric has the following form (this describes metric of our universe):

$$ds^2 = -dt^2 + a(t)^2(d\chi^2 + sin^2\chi d\Omega^2) \ , 0 \leq \chi \leq \pi$$

(a) Find the form of the 4-velocity for a photon propagating in the χ-direction. This should involve one unknown function. (b) Let $k^\alpha = \nu_0(1/a, 1/a^2, 0, 0)$. Check that the geodesic equation $k^\beta D_\beta k^\alpha = 0$ is satisfied for $\alpha = t$.