

General Relativity - 2017

Indian Institute of Science Education and Research Bhopal

Assignment 2

1. A straight line in Cartesian coordinate system is given by

$$\frac{d^2x}{ds^2} = 0, \quad \frac{d^2y}{ds^2} = 0 .$$

Write down these equations in polar coordinate systems, and find out the Christoffel symbols.

2. Solve the above equations, and show that the solution is a straight line.
3. Write down the metric on a two-dimensional sphere and calculate all the Christoffel symbols. Show that a great circle is a solution of the geodesic equation.
4. An affine parameter λ is one for which the equation of geodesics motion has the form

$$\frac{dx^\alpha}{d\lambda^2} + \Gamma_{\beta\gamma}^\alpha \frac{dx^\beta}{d\lambda} \frac{dx^\gamma}{d\lambda} = 0.$$

Show that all affine parameters are related by linear transformations with constant coefficients.

5. Show by direct calculation from the geodesic equation that the norm of the four-velocity is a constant along a geodesic.