Assignment 5

Lattice vibration and Magnetization

1. In a linear chain of monatomic lattice with spacing a, particles/ions of mass m are connected by nearest-neighbor springs of spring constant K. In addition to the elastic forces, each particle is subjected to a damping force $F_D = -\gamma \dot{u}(na)$, where $\dot{u}(na)$ is the displacement of the n^{th} particle from the equilibrium position. How does the damping change the frequencies ω(k), and what is the relaxation time of the modes? Assume $(\gamma/m)^2 \ll K/m$ and discuss the limiting cases $k \approx 0$ and $k \approx \pi/a$.

2. Cosider an ideal gas (with density $n = N/V$) of stable vector mesons (massive particles with spin 1) with magnetic moment μ.
a) Compute Magnetization per unit volume.
b) Compute paramagnetic susceptibility.
c) Verify Curie law for $k_B T \gg \mu H$.

3. As we discussed in the class that classical analysis of lattice vibration can not explain low temperature behaviour of specific heat. Einstein, once again, was the first to apply quantum treatment to calculate specific heat due to lattice vibration. He assumed that all N atoms in three dimensions are vibrating with same frequency ω in all three directions.

(i) Calculate the partition function of the system at temperature T.

(ii) Show that the specific heat at constant volume is given by,

$$C_V = 3 N k_B \left(\frac{\theta_E}{T}\right)^2 \frac{e^{\theta_E/T}}{(e^{\theta_E/T} - 1)^2}$$

where, θ_E is Einstein temperature and given by,

$$\theta_E = \frac{\hbar \omega}{k_B}.$$

(iii) Find high and low temperature behaviour of specific heat.

5. Consider a system of N magnetic dipoles, each containg a dipole moment μ. The dipoles are fixed in space at their centers but are free to rotate about any axis passing through their centers and perpendicular to the axis of the dipole. The moment of inertia of each of these dipoles about any such axis is I. The system is kept in thermal equilibrium with a heat bath of temperature T. There is also an external uniform magnetic field B which we take to be in z direction. Calculate magnetization and susceptibility. Calculate susceptibility at high temperature and veryfy Curie's law.