PHY102: Mid-semester examination
Full marks : 100 Time 120 mins.

Instructions: (a) Attempt all the questions. (b) Each question carries 20 marks. (c) No credit for answers without any explanation or justification even if your answer is correct. (d) Keep your answer book neat and clean. (e) Your handwriting should be clear and readable.

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1. Two vectors are given by,

$$\vec{V}_1 = 3\hat{i} - \hat{j} + 2\hat{k}, \qquad \vec{V}_2 = -2\hat{i} + 5\hat{j} - 4\hat{k}.$$

Find
(i) $\vec{V}_1 \cdot \vec{V}_2$, (ii) $\vec{V}_1 \times \vec{V}_2$, (iii) the angle between these two vectors. (iv) the projection of $\vec{V}_1$ along $\hat{i} + \hat{j} - \hat{k}$.

2. Find divergence and curl of the following vector

$$\vec{V} = 2x\ \hat{i} + 5y^2\ \hat{j} + 3e^z\ \hat{k}.$$

3. A vector $\vec{P}$ is given by $\vec{P} = x^2 y\ \hat{i} - 2xy\ \hat{j} + 3(x^2 + y^2)\hat{k}$. Check the following relation

$$\int_{\mathcal{A}} (\vec{\nabla} \times \vec{P}) \cdot \vec{da} = \oint_{\mathcal{L}} \vec{P} \cdot \vec{dl}$$

for the shaded area $\mathcal{A}$ which is bounded by the path $\mathcal{L}$ as shown in the Fig. 1. The closed path $\mathcal{L}$ consists of lines joining $(0,0)$ and $(1,0)$, $(1,0)$ and $(1,1)$, $(1,1)$ and $(0,1)$ and finally $(0,1)$ and $(0,0)$.

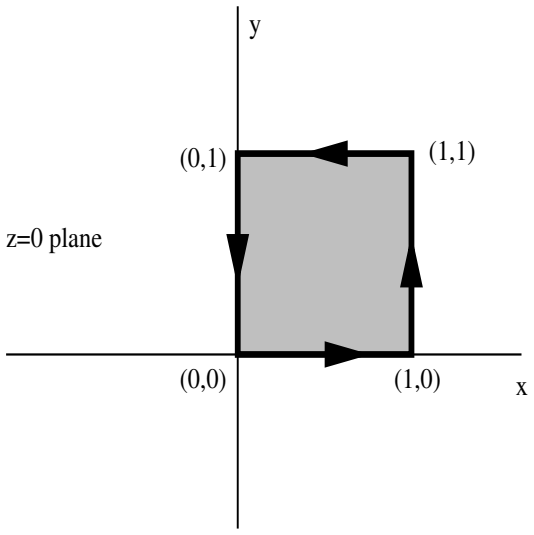

Fig. 1: for question no. 3.

4. There is a line charge distribution from $x = -L$ to $x = L$ with a line charge density $\lambda(x)$. Find out the electric potential and electric field at a point P on x axis and at a distance d from the origin ($x = 0$ line) as shown in Fig. 2 when,
(a) $\lambda(x) = c$ (c is a constant) and
(b) $\lambda(x) = x^2$.

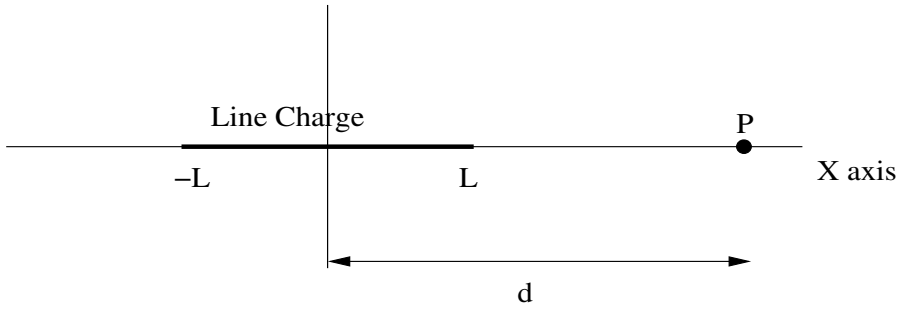

Fig. 2: for question no. 4.

5. (a) Find the capacitance per unit length of two (infinitely long) coaxial metal cylindrical tubes of radii a and b ($a > b$) (as shown in Fig. 3).
(b) Also find the electrostatic energy per unit length of the system if the outer and inner cylinders carry a constant surface charge density σ_a and σ_b respectively.

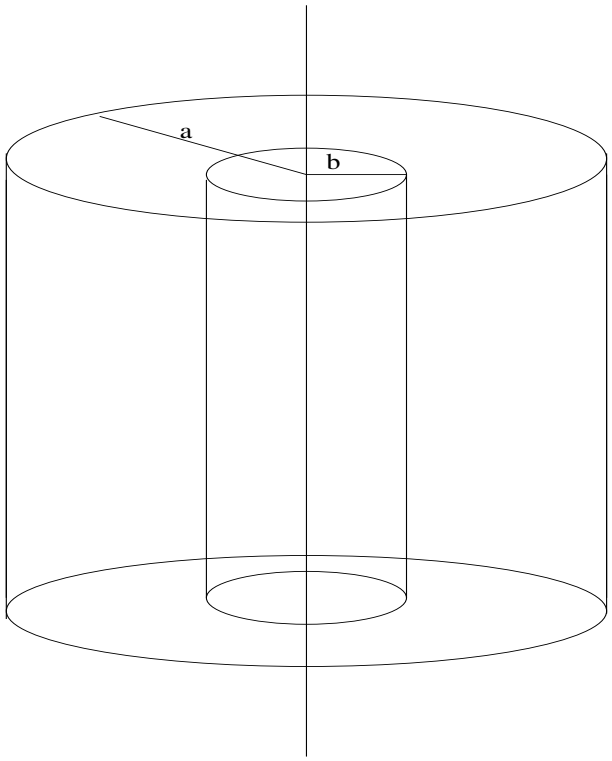

Fig. 3: for question no. 5.