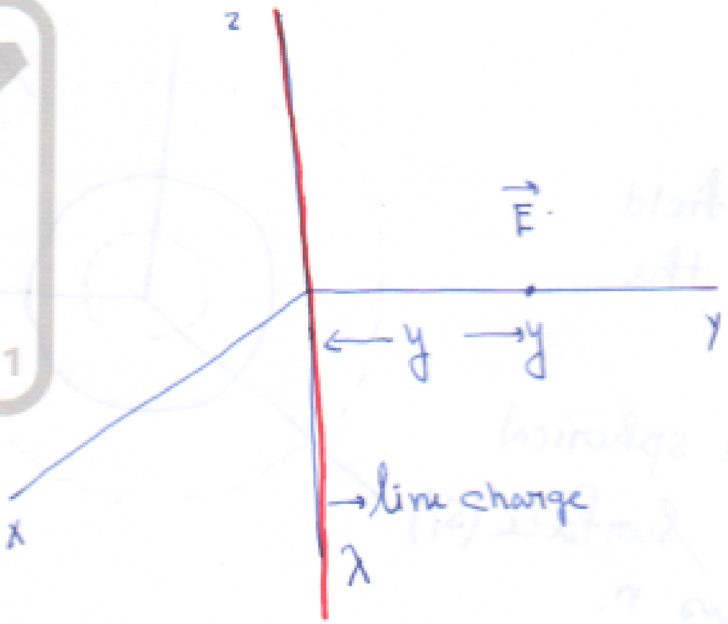
Prob. 1



Electric field at a distance y on y axis is

$$\vec{E}(y) = \frac{\lambda}{2\pi\epsilon_0} \cdot \frac{1}{y}\hat{j} \quad \left[\text{See the last problem set}\right]$$

~~Work done~~ Force on a charge Q at y

$$\vec{F} = \frac{\lambda Q}{2\pi\epsilon_0} \cdot \frac{1}{y}\hat{j}$$

Work done to displace the charge from $y = y_2$ to $y = y_1$

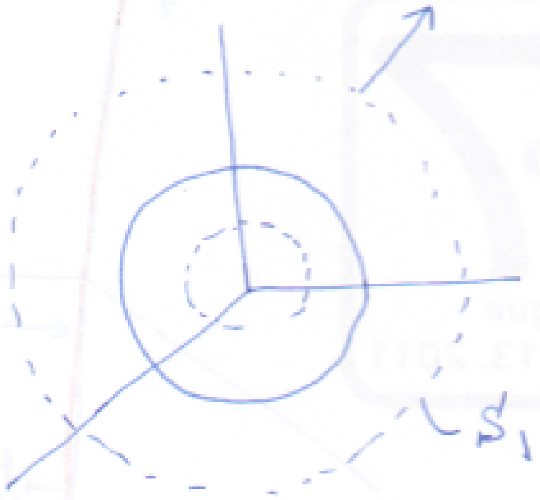
$$W = -\int_{y_2}^{y_1} \vec{F} \cdot \vec{dl}$$

$\vec{dl} = dy\hat{j}$ Hence $\vec{F} \cdot \vec{dl} = \frac{Q\lambda}{2\pi\epsilon_0} \cdot \frac{dy}{y}$

$$\therefore W = -\int_{y_2}^{y_1} \frac{Q\lambda}{2\pi\epsilon_0} \cdot \frac{dy}{y} = -\frac{Q\lambda}{2\pi\epsilon_0} \ln y \Big|_{y_2}^{y_1}$$

$$= \frac{Q\lambda}{2\pi\epsilon_0} \ln \frac{y_2}{y_1}$$

Prob 2



Electric field out side the sphere:

Consider a spherical Gaussian surface (S_1) of radius r.

$$\oint_{S_1} \vec{E}\cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

$\vec{E}$ is radia (from symmetry).

$$d\vec{s} = r^2 \sin\theta \, d\theta \, d\varphi \, \hat{r}$$

$$\therefore \oint_{S_1} \vec{E}\cdot d\vec{s} = \int_0^{\pi}\int_0^{2\pi} E(r)\, r^2 \sin\theta \, d\theta \, d\varphi = \frac{q}{\epsilon_0}$$

$$\therefore \vec{E}(r) = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$

$\vec{E}$ inside the sphere:

Consider a spherical Gaussian surface S_2.

$\vec{E}$ is radial (from symmetry)

$$\oint_{S_2} \vec{E}\cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

$$\therefore 4\pi r^2 E(r) = \frac{1}{\epsilon_0}\cdot\frac{q}{\frac{4}{3}\pi R^3}\cdot\frac{4}{3}\pi r^3 = \frac{q}{\epsilon_0}\frac{r^3}{R^3}$$

$$\therefore \boxed{\vec{E}(r) = \frac{q}{4\pi\epsilon_0}\cdot\frac{r}{R^3}\,\hat{r}}$$

Prob 2 Continue

$$W = \frac{\epsilon_0}{2}\int d\tau\, \vec{E}^2$$

$$= \frac{\epsilon_0}{2}\left[\underbrace{\int d\tau\, E_{in}^2}_{\text{Inside the sphere}} + \underbrace{\int d\tau\, E_{out}^2}_{\text{outside the sphere}}\right]$$

$$= \frac{\epsilon_0}{2}\left[\underbrace{\int r^2 dr \sin\theta\, d\theta\, d\phi}_{\text{Inside}} \frac{q^2}{16\pi^2\epsilon_0^2}\cdot\frac{r^2}{R^6}\right.$$

$$\left. + \underbrace{\int r^2 dr \sin\theta\, d\theta\, d\phi}_{\text{outside}}\cdot\frac{q^2}{16\pi^2\epsilon_0^2}\cdot\frac{1}{r^4}\right]$$

$$= \frac{q^2}{8\pi\epsilon_0}\left[\frac{1}{R^6}\int_0^R r^4 dr + \int_R^\infty \frac{dr}{R^2}\right]$$

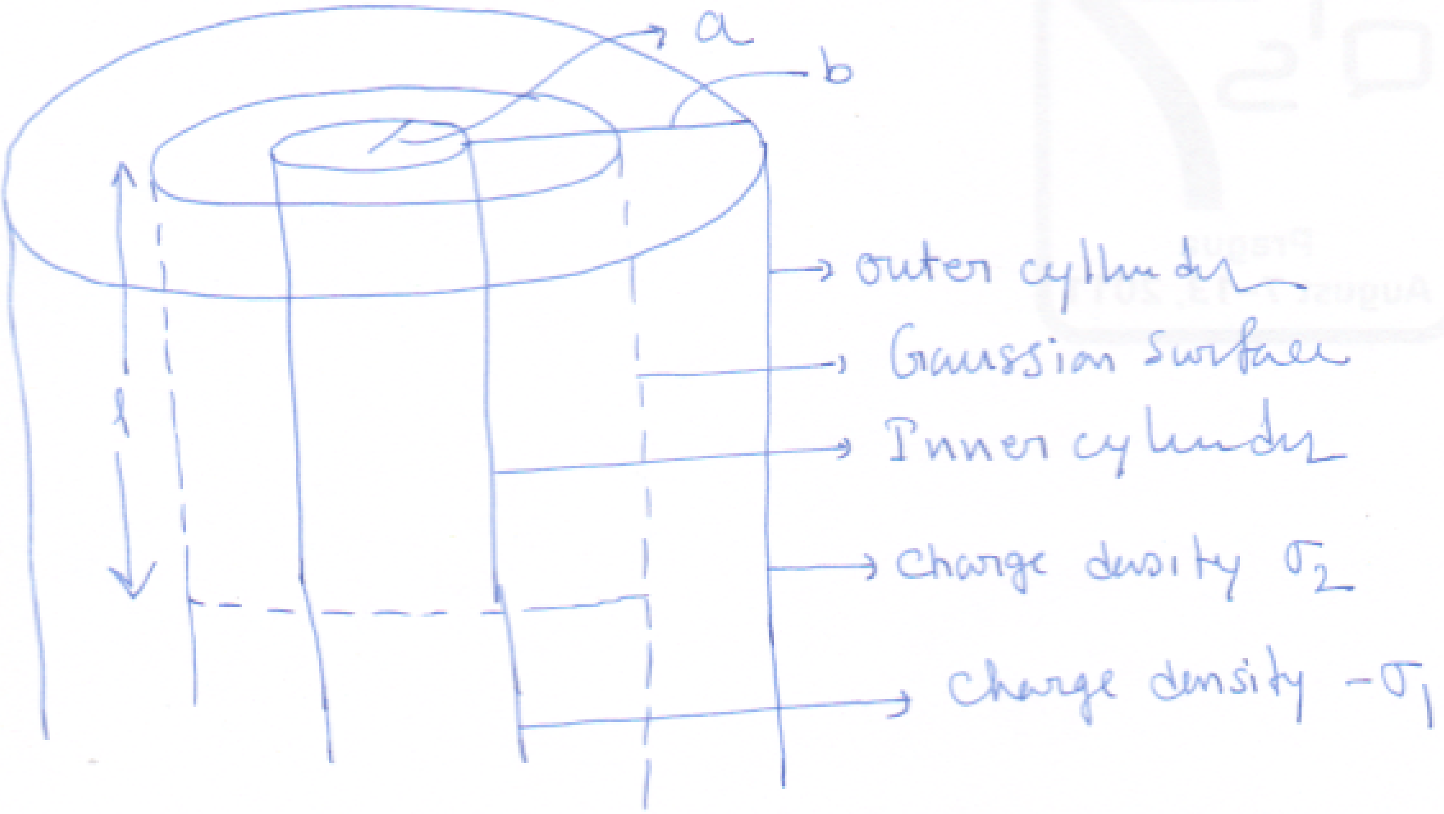
$$= \frac{q^2}{8\pi\epsilon_0}\left[\frac{1}{5R} + \frac{1}{R}\right] = \frac{3q^2}{20\pi\epsilon_0 R}$$

3. Check Griffiths Example 2.8

$$W = \frac{1}{8\pi\epsilon_0}\cdot\frac{q^2}{R}$$

Thus $W_{shell} \ll W_{solid\ sphere}$.

Q.4 See Griffiths Example 2.11.



Q.5 Electric field between two cylinders.

$\vec{E} \rightarrow$ radially outward (from symmetry)

Consider the Gaussian surface (cylindrical surface) of radius r and length l.

$$E(r)\, 2\pi r \cancel{l} = -\frac{2\pi a \sigma_1 \cancel{l}}{\epsilon_0}$$

Now $2\pi a \sigma_1 L = Q$

$$\therefore E(r) = \frac{Q}{2\pi\epsilon_0} \cdot \frac{1}{L} \cdot \frac{1}{r}$$

$$\phi_{out} - \phi_{in} = -\int_b^a \underline{E} \cdot dr = \frac{Q}{2\pi\epsilon_0} \cdot \frac{1}{L} \ln\frac{b}{a} = \phi$$

$$\therefore C = \frac{Q}{\phi} = \frac{2\pi\epsilon_0 L}{\ln b/a} \Rightarrow \boxed{\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln b/a}}$$