Tutorial 3

Vector Analysis

PHY 102

January 23, 2015

Problem no. 2

- Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along
(a) the straight line from $(0, 0, 0)$ to $(2, 1, 3)$.
- Equation for a line passing through two points (x_0, y_0, z_0) and (x_1, y_1, z_1) :

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} = \frac{z - z_0}{z_1 - z_0}$$

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} = \frac{z - z_0}{z_1 - z_0} = t$$

$$\begin{aligned} x &= x_0 + (x_1 - x_0)t \\ y &= y_0 + (y_1 - y_0)t \\ z &= z_0 + (z_1 - z_0)t \end{aligned}$$

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- The equations are

$$\begin{aligned} x &= x_0 + (x_1 - x_0)t \\ y &= y_0 + (y_1 - y_0)t \\ z &= z_0 + (z_1 - z_0)t \end{aligned}$$

- Here $x_0 = y_0 = z_0 = 0$ and $x_1 = 2, y_1 = 1, z_1 = 3$. Hence,

$$x = 2t, \qquad y = t, \quad and \quad z = 3t$$

- Hence

$$dx = 2dt, \quad dy = dt, \quad and \quad dz = 3dt.$$

Problem no. 2

- Therefore,

$$\begin{aligned} \vec{F} &= 12t^2\hat{i}+(12t^2-t)\hat{j}+3t\hat{k} \\ d\vec{l} &= dx\hat{i}+dy\hat{j}+dz\hat{k} = (2\hat{i}+\hat{j}+3\hat{k})dt \end{aligned}$$

- Hence

$$\vec{F}\cdot\vec{dl} = (24t^2+(12t^2-t)+9t)dt$$

- The initial point $(0,0,0)$ corresponds to $t=0$ and the final point $(2,1,3)$ corresponds to $t=1$.
- Finally

$$\int_a^b \int_P \vec{F}\cdot\vec{dl} = \int_0^1 (24t^2+(12t^2-t)+9t)dt = \cdots$$

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- Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along
  (b) the space curve $x = 2t^2, y = t, z = 4t^2 - t$ from $t = 0$ to $t = 1$.
- Here $x = 2t^2, y = t, z = 4t^2 - t$ hence
  $dx = 4tdt, dy = dt, dz = 8tdt - dt$
- Therefore, $\vec{dl} = (4t\hat{i} + \hat{j} + (8t - 1)\hat{j})dt$.
  and $\vec{F} = 12t^4\hat{i} + (16t^4 - 4t^2 - t)\hat{j} + (4t^2 - t)\hat{k}$
- Calculate $\vec{F} \cdot \vec{dl}$ and integrate from $t = 0$ to $t = 1$.

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- Here $x = 2t^2, y = t, z = 4t^2 - t$ hence $dx = 4tdt, dy = dt, dz = 8tdt - dt$
- Therefore, $\vec{dl} = (4t\hat{i} + \hat{j} + (8t - 1)\hat{j})dt$. and $\vec{F} = 12t^4\hat{i} + (16t^4 - 4t^2 - t)\hat{j} + (4t^2 - t)\hat{k}$
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- Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along
  (c) the curve defined by $x^2 = 4y, 3x^3 = 8z$ from $= 0$ to $x = 2$.
- Replace y and z in terms of x also dy and dz in terms of dx.
- Calculate $\vec{F} \cdot \vec{dl}$ and integrate over x from $x = 0$ to $x = 2$.

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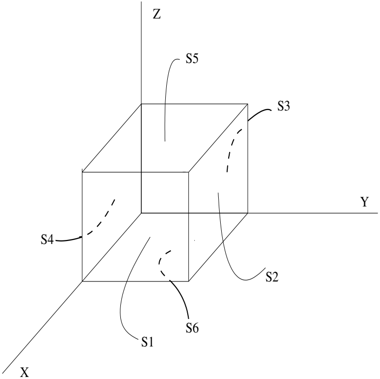
- Replace y and z in terms of x also dy and dz in terms of dx.
- Calculate $\vec{F} \cdot \vec{dl}$ and integrate over x from $x = 0$ to $x = 2$.

Problem no. 3

- Consider a vector field $\vec{V} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$. Compute

$$\oint_S \vec{V} \cdot d\vec{s}$$

over the surface of a cube of side 1 as shown in figure.



Problem no. 3

- We can break the integral in the following way

$$\oint_S \vec{V}\cdot d\vec{s} = \int_{S_1}\vec{V}\cdot d\vec{s}_1 + \int_{S_2}\vec{V}\cdot d\vec{s}_2 + \int_{S_3}\vec{V}\cdot d\vec{s}_3 + \int_{S_4}\vec{V}\cdot d\vec{s}_4 + \int_{S_5}\vec{V}\cdot d\vec{s}_5 + \int_{S_6}\vec{V}\cdot d\vec{s}_6$$

$$d\vec{s}_1 = dydz\hat{i},\ d\vec{s}_2 = dxdz\hat{j},\ d\vec{s}_3 = -dydz\hat{i},\ d\vec{s}_4 = -dxdz\hat{j}$$
$$d\vec{s}_5 = dydx\hat{j},\ d\vec{s}_6 = -dydx\hat{j}$$

$$\int_{S_1}\vec{V}\cdot d\vec{s}_1 = \int_0^1\int_0^1 x^2 dydz = \int_0^1\int_0^1 dydz = 1$$
$$\int_{S_2}\vec{V}\cdot d\vec{s}_2 = \int_0^1\int_0^1 y^2 dxdz = \int_0^1\int_0^1 dxdz = 1$$

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$$\oint_S \vec{V}\cdot d\vec{s} = \int_{S_1}\vec{V}\cdot d\vec{s_1} + \int_{S_2}\vec{V}\cdot d\vec{s_2} + \int_{S_3}\vec{V}\cdot d\vec{s_3} + \int_{S_4}\vec{V}\cdot d\vec{s_4} + \int_{S_5}\vec{V}\cdot d\vec{s_5} + \int_{S_6}\vec{V}\cdot d\vec{s_6}$$

$$d\vec{s_1} = dydz\hat{i},\ d\vec{s_2} = dxdz\hat{j},\ d\vec{s_3} = -dydz\hat{i},\ d\vec{s_4} = -dxdz\hat{j}$$

$$d\vec{s_5} = dydx\hat{j},\ d\vec{s_6} = -dydx\hat{j}$$

$$\int_{S_1}\vec{V}\cdot d\vec{s_1} = \int_0^1\int_0^1 x^2 dydz = \int_0^1\int_0^1 dydz = 1$$

$$\int_{S_2}\vec{V}\cdot d\vec{s_2} = \int_0^1\int_0^1 y^2 dxdz = \int_0^1\int_0^1 dxdz = 1$$

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- We can break the integral in the following way

$$\oint_S \vec{V} \cdot d\vec{s} = \int_{S_1} \vec{V} \cdot d\vec{s_1} + \int_{S_2} \vec{V} \cdot d\vec{s_2} + \int_{S_3} \vec{V} \cdot d\vec{s_3} + \int_{S_4} \vec{V} \cdot d\vec{s_4} + \int_{S_5} \vec{V} \cdot d\vec{s_5} + \int_{S_6} \vec{V} \cdot d\vec{s_6}$$

$$d\vec{s_1} = dydz\hat{i}, \; d\vec{s_2} = dxdz\hat{j}, \; d\vec{s_3} = -dydz\hat{i}, \; d\vec{s_4} = -dxdz\hat{j}$$
$$d\vec{s_5} = dydx\hat{j}, \; d\vec{s_6} = -dydx\hat{j}$$

$$\int_{S_1} \vec{V} \cdot d\vec{s_1} = \int_0^1 \int_0^1 x^2 dydz = \int_0^1 \int_0^1 dydz = 1$$
$$\int_{S_2} \vec{V} \cdot d\vec{s_2} = \int_0^1 \int_0^1 y^2 dxdz = \int_0^1 \int_0^1 dxdz = 1$$

Problem no. 2

$$\int_{S_3} \vec{V} \cdot d\vec{s_3} = -\int_0^1 \int_0^1 x^2 dydz = 0 \cdot \int_0^1 \int_0^1 dydz = 0$$

$$\int_{S_2} \vec{V} \cdot d\vec{s_2} = -\int_0^1 \int_0^1 y^2 dxdz = 0 \cdot \int_0^1 \int_0^1 dxdz = 0$$

and

$$\int_{S_5} \vec{V} \cdot d\vec{s_5} = \int_0^1 \int_0^1 z^2 dydx = \int_0^1 \int_0^1 dydx = 1$$

$$\int_{S_6} \vec{V} \cdot d\vec{s_6} = -\int_0^1 \int_0^1 z^2 dxdy = 0 \cdot \int_0^1 \int_0^1 dxdy = 0$$

Hence,

$$\oint_S \vec{V} \cdot d\vec{s} = 3.$$