Tutorial 1

Vector Analysis

PHY 102

January 9, 2015

Problem no. 2

- $\vec{A} = 3\hat{i} - \hat{j} + 5\hat{k}$, $\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{C} = 5\hat{i} - 2\hat{j} - 1\hat{k}$. Find the unit vector along $\vec{A} + 2\vec{B} + 3\vec{C}$.

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- Recall the definition of unit vector along a vector $\vec{V}$

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- Simplify the expression.

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- Join P and Q. How can you write $\vec{PQ}$ in terms of $\vec{OP}$ and $\vec{OQ}$.?

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$$\vec{PQ} = \vec{OQ} - \vec{OP}.$$

Problem no. 4

- Show that the necessary and sufficient condition that the vectors $\vec{A} = A_1\hat{i}+A_2\hat{j}+A_3\hat{k}$, $\vec{B} = B_1\hat{i}+B_2\hat{j}+B_3\hat{k}$ and $\vec{C} = C_1\hat{i}+C_2\hat{j}+C_3\hat{k}$ be linearly independent is that the determinant

$$\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

is non-zero.

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- Recall the condition for linear independence: Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are linearly independent if the equation

$$m_1\vec{A} + m_2\vec{B} + m_3\vec{C} = 0$$

has no other solution than $m_1 = 0, m_2 = 0$ and $m_3 = 0$.

- Write down the equation in cartesian coordinate.
Consider

$$\vec{A} = A_1\hat{i}+A_2\hat{j}+A_3\hat{k},\ \ \vec{B} = B_1\hat{i}+B_2\hat{j}+B_3\hat{k},\ \ \vec{C} = C_1\hat{i}+C_2\hat{j}+C_3\hat{k}$$

- What are the equations for m_1, m_2 and m_3 ?

Problem no. 4

- Can you write these equations in the following form?

$$\begin{pmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = 0$$

- This equation has solution $m_1 = 0, m_2 = 0$ and $m_3 = 0$ only when the determinant of the matrix is **non-zero**

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$$\vec{A} \cdot \frac{\vec{B}}{|\vec{B}|}$$

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- Calculate $\vec{A} \cdot \vec{B}$ and $|\vec{B}|$.
- What about projection of $\vec{B}$ along $\vec{A}$.