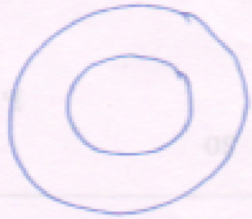
Quiz-2

Ans: (1)



Electric field inside $r<a$

$\vec{E} = 0$

As there is no charge.

Electric field between $a < r < b$

Bound charges

Surface bound charges

$\sigma_b^1 = \vec{P}\cdot\hat{n} = \vec{P}\cdot(-\hat{r}) = -\frac{k}{a}$ (bound charge at inner surface)

$\sigma_b^{(2)} = \vec{P}\cdot\hat{n} = \frac{k}{r}\hat{r}\cdot\hat{r} = \frac{k}{r}$ (bound charge at outer surface)

Volume bound charge

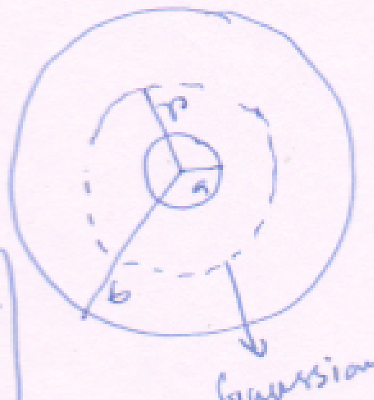
$\rho_b = -\vec{\nabla}\cdot\vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{k}{r}\right) = -\frac{k}{r^2}$

$$E(r)\,4\pi r^2 = \frac{1}{\epsilon_0}\left[\sigma_b^1\cdot 4\pi a^2 + \int_a^r \rho_b\, dv\right]$$

$$= \frac{1}{\epsilon_0}\left[-4\pi k a + 4\pi\int_a^r\left(-\frac{k}{r^2}\right)r^2 dr\right]$$

$$= \frac{1}{\epsilon_0}\left[-4\pi k a - 4\pi k r + 4\pi k a\right]$$

$$= -\frac{4\pi k r}{\epsilon_0}$$



$$\boxed{\vec{E}(r) = -\frac{1}{\epsilon_0}\frac{k}{r}\hat{r}} \qquad \underline{a < r < b}$$

Electric field outside $r > b$

$$E(r)\,4\pi r^2 = \frac{Q_{total}}{\epsilon_0} \qquad \underline{r > b}$$

$$Q_{total} = \sigma^{(1)}_{b}\,4\pi a^2 + \sigma^{(2)}_{b}\,4\pi b^2 + \int_a^b \rho_b\, dV$$

$$= -4\pi k a + 4\pi k b - k\int_a^b \frac{4\pi r^2}{r^2}\,dr$$

$$= -4\pi k a + 4\pi k b - 4\pi k b + 4\pi k a$$

$$= 0$$

$$\therefore \boxed{\vec{E}(r) = 0 \qquad r > b}$$

To find electric field inside we can do the following also.

Since there is no free charge $\vec{D} = 0$ everywhere

For $r < a$,

$$\vec{P} = 0 \Rightarrow \vec{E} = 0$$

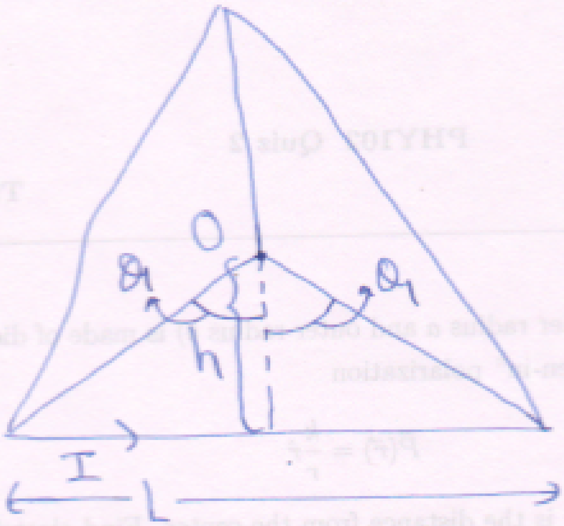
For $a < r < b$

$$\vec{P} = \frac{k}{r}\hat{r} \Rightarrow \vec{E} = -\frac{k}{r}\hat{r}$$

For $r > b$

$$\vec{P} = 0 \Rightarrow \vec{E} = 0.$$

Ans-2



Magnetic field at O due to one arm

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi h} \cdot \left[\sin\theta_1 - \sin(-\theta_1) \right]$$

$$= \frac{\mu_0 I}{4\pi h} \, 2\sin\theta_1$$

here, $h = \frac{L}{2\sqrt{3}}$, $\theta_1 = 60^\circ$

$$\therefore \vec{B}_1 = \frac{\mu_0 I \, 2\sqrt{3}}{4\pi L} \cdot 2 \cdot \frac{\sqrt{3}}{2} \odot$$

$$= \frac{\mu_0 I}{2\pi L} \cdot 3 \odot$$

For three arms

$$\boxed{\vec{B} = 3\vec{B}_1 = \frac{9\mu_0 I}{2\pi L} \odot}$$