PHY102: Assignment 3

1. Calculate divergence and curl of the following vector

$$\vec{V} = e^{3x} \sin y \ \hat{i} + \frac{\cos^2 y}{1 + 5y^2} \ \hat{j} + \tan y \log z \ \hat{k}.$$

2. Suppose we are in two dimensions. We have x axis and y axis. The unit vectors along positive x and positive y directions are $\hat{i}$ and $\hat{j}$ respectively. Now we make a coordinate transformation and go to (r, θ) coordinate system. The coordinates of these two coordinate systems are related by,

$$x = r \cos \theta \quad \text{and} \quad y = r \sin \theta.$$

The unit vectors in (r, θ) coordinate system are given by $\hat{r}$ and $\hat{\theta}$. $\hat{r}$ is the direction along which r increases keeping θ fixed and $\hat{\theta}$ is the direction along which θ increases keeping r fixed. Look at the figure on the last page. Find the relation between $(\hat{r}, \hat{\theta})$ and $(\hat{i}, \hat{j})$.

Ans: $\hat{r} = \hat{i} \cos \theta + \hat{j} \sin \theta$, $\hat{\theta} = -\hat{i} \sin \theta + \hat{j} \cos \theta$.