Assmnt-8 Solution

1)

a) $\mathcal{E} = -\frac{d\phi}{dt} = -Bl\frac{dx}{dt} = -Blv$

$|\mathcal{E}| = IR$

$I = \frac{|\mathcal{E}|}{R} = \frac{Blv}{R}$

b) Force on the bar

$F = IBl = \frac{B^2 l^2}{R} v$ (acting on left)

c) $m\frac{dv}{dt} = -F = -\frac{B^2 l^2}{R} v$

$\Rightarrow \frac{dv}{v} = -\frac{B^2 l^2}{mR} dt$

$mv = -\frac{B^2 l^2}{mR} t + c$

at $t = 0$, $v = v_0$

$\therefore c = mv_0$

$\therefore \boxed{v = v_0 e^{-\frac{B^2 l^2}{mR} t}}$

$\boxed{I = \frac{Bl}{R} v_0 e^{-\frac{B^2 l^2}{mR} t}}$

d) Power delivered to resistor

$$\frac{dW}{dt} = I^2 R = \frac{B^2 l^2}{R^{\cancel{2}}} v_0^2 \, e^{-\frac{2B^2 l^2}{mR} t} \cancel{R}$$

$$dW = \frac{B^2 l^2 v_0^2}{R} e^{-\frac{2B^2 l^2}{mR} t} \, dt$$

$$W = \frac{B^2 l^2 v_0^2}{R} \int_0^\infty e^{-\frac{2B^2 l^2}{mR} t} \, dt$$

$$= \frac{\cancel{B^2 l^2} v_0^2}{\cancel{R}} \cdot \left(-\frac{1}{2}\right) \frac{m\cancel{R}}{\cancel{B^2 l^2}} e^{-\frac{2B^2 l^2}{mR} t} \Big|_0^\infty$$

$$= -\frac{1}{2} m v_0^2 \left[e^{-\infty} - e^{-0} \right]$$

$$= \frac{1}{2} m v_0^2$$

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(2) $B = \mu_0 n I$ (magnetic field inside)

flux through a single turn $F_1 = \pi R^2 \mu_0 n I$

Total flux through nl turns $F = \pi R^2 \mu_0 n^2 I l$

$\therefore F = LI \Rightarrow L = \pi R^2 \mu_0 n^2 l$

$\therefore \boxed{L/l = \pi R^2 \mu_0 n^2}$

(3) $\varepsilon_0 - L\frac{dI}{dt} = IR \Rightarrow I(t) = \frac{\varepsilon_0}{R} + k e^{-R/L\, t}$

at $t=0$, $I(t) = 0 \Rightarrow k = -\frac{\varepsilon_0}{R}$

$$I(t) = \frac{\varepsilon_0}{R}\left(1 - e^{-\frac{R}{L} t}\right)$$