Assignment - 6

Solution

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Q. 1

$$\vec{P} = k\vec{r}$$

a)

$$\rho_b = -\vec{\nabla}\cdot\vec{P} = -\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 P_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta P_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}P_\phi\right]$$

$$= -\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 kr\right]$$

$$= -3k$$

Hence volume bound charge density $\boxed{\rho_b = -3k}$

Surface bound charge density

$$\sigma_b = \vec{P}\cdot\hat{n} \quad \text{for spherical surface } \hat{n} = \hat{r}$$

Hence $\sigma_b = \left(\vec{P}\cdot\hat{r}\right)\Big|_{r=R} = kR$ $\quad\boxed{\sigma_b = kR}$

b) Electric field inside $\vec{E}_{in}$

$\vec{E}_{in}$ is due to ρ_b.

We consider a spherical Gaussian surface ~~and~~ of radius r $(r<R)$ and apply Gauss law

$$4\pi r^2 E(r) = \frac{1}{\epsilon_0}\cdot\frac{4}{3}\pi r^3(-3k)$$

$$\Rightarrow \boxed{\vec{E}_{in}(r) = -\frac{k}{\epsilon_0}\vec{r}}$$

Electric field outside : $\vec{E}_{out}$

Consider a Gaussian surface of radius r $(r>R)$
Apply Gauss law

$$4\pi r^2 E(r) = \frac{1}{\epsilon} Q_{in}$$

Now Q_{in} is total bound charges (due to both ρ_b and σ_b)

$$Q_{in} = \int \rho_b \, d\tau + \int \sigma_b \, ds$$

$$= -\cancel{3}K \cdot \frac{4}{\cancel{3}}\pi R^3 + 4\pi R^2 KR$$

$$= 0$$

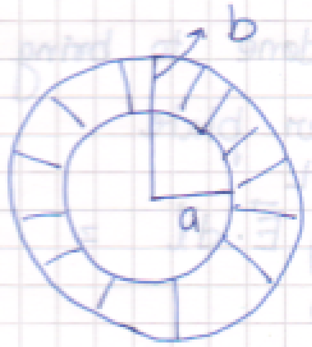
$$\therefore \boxed{\vec{E}_{out}(\vec{r}) = 0}$$

Note that $\quad \vec{P} = K\vec{r}, \quad \vec{E} = -\frac{K}{\epsilon_0}\vec{r}$

$$\boxed{\vec{D} = \epsilon_0 \vec{E} + \vec{P} = 0}$$

Also $\quad \vec{P} = -\epsilon_0 \vec{E}$

Q.2 Page-3



$$\vec{E}(\vec{r}) = 0 \qquad r < a$$

$$\vec{E}(r) = \frac{Q}{4\pi\epsilon}\frac{\hat{r}}{r^2} \qquad a < r < b$$

$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon_0}\cdot\frac{\hat{r}}{r^2} \qquad r > b$$

Potential at the centre = Work done to bring a point charge from infinity to the centre.

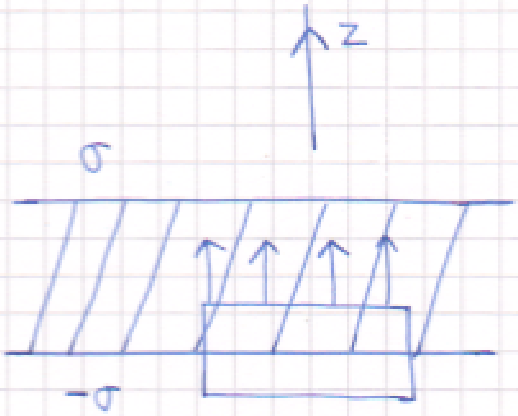
$$V = -\int_{\infty}^{0} \vec{E}\cdot d\vec{r} = -\int_{-\infty}^{b} \vec{E}\cdot d\vec{r} - \int_{b}^{a} \vec{E}\cdot d\vec{r} - \int_{a}^{0} \vec{E}\cdot d\vec{r}$$

$$= -\frac{Q}{4\pi\epsilon_0}\int_{-\infty}^{b}\frac{dr}{r^2} - \frac{Q}{4\pi\epsilon}\int_{b}^{a}\frac{dr}{r^2}$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{b}\right] - \frac{Q}{4\pi\epsilon}\left[-\frac{1}{a}+\frac{1}{b}\right]$$

$$= \frac{Q}{4\pi}\left[\frac{1}{\epsilon_0 b} + \frac{1}{\epsilon a} - \frac{1}{\epsilon b}\right]$$

Q.3



Electric field outside the plate = 0.

Electric field inside

$$E \cdot A = -\frac{\sigma}{\epsilon_0}\cdot A$$

$$\therefore \vec{E} = -\frac{\sigma}{\epsilon}\hat{z}$$

Work done to bring a unit charge from upper plate to lower plate

$$W = \int_U^L \vec{E} \cdot d\vec{l} = E \cdot d = \frac{\sigma d}{\epsilon} = V$$

$$C = \frac{Q}{V} = \frac{A\sigma\epsilon}{\sigma d} = \frac{A}{d}\epsilon$$

$$\therefore \quad C = \frac{A}{d}\epsilon_0 \cdot \frac{\epsilon}{\epsilon_0}$$

$$\boxed{C = C_0 \epsilon_r}$$

$$C_0 = \frac{A\epsilon_0}{d}$$ Capacitance of ~~when there is~~ two parallel plates when there is no dielectric inside.