

Assignment - 6

Solution

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Q. 1

$$\vec{P} = \kappa \vec{r}$$

a) $f_b = -\nabla \cdot \vec{P} = -\left[\frac{1}{r^2} \cdot \frac{\partial}{\partial r} (r^2 P_r) + \frac{1}{r \sin \theta} \cdot \frac{\partial}{\partial \theta} (\sin \theta P_\theta) \right]$

(for bound P, effect of net) regards terms $\frac{1}{r \sin \theta} \cdot \frac{\partial}{\partial \theta} P_\theta$ as zero

$$= -\frac{1}{r^2} \frac{\partial}{\partial r} [r^2 \kappa r] = \kappa r$$

$$= -3\kappa$$

Hence volume bound charge density

$$f_b = -3\kappa$$

Surface bound charge density

$$\sigma_b = \vec{P} \cdot \hat{n} \quad \text{for spherical surface } \hat{n} = \hat{r}$$

Hence $\sigma_b = (\vec{P} \cdot \hat{r}) \Big|_{r=R} = \kappa R$

$$\sigma_b = \kappa R$$

b) Electric field inside \vec{E}_{in}

\vec{E}_{in} is due to f_b .

We consider a spherical Gaussian surface ~~out~~ of radius r ($r < R$) and apply Gauss law


$$4\pi r^2 E(r) = \frac{1}{\epsilon_0} \frac{A}{\ell} r^3 (-\beta \kappa)$$

$$\Rightarrow \vec{E}_{in}(r) = -\frac{\kappa}{\epsilon_0} \vec{r}$$

Electric field outside: \vec{E}_{out}

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$$

Consider a Gaussian surface of radius r ($r > R$)

Apply Gauss Law

$$(890 \text{ me}) 4\pi r^2 E(r) = \frac{1}{\epsilon_0} Q_{\text{in}} \quad \Rightarrow \quad q \cdot \vec{E} = \frac{q}{\epsilon_0 r^2} \vec{r}$$

Now Q_{in} is total bound charges (due to both S_b and σ_b)

$$Q_{\text{in}} = \int S_b d\sigma + \int \sigma_b dS \quad \Rightarrow \quad =$$

$$= -\rho_k \cdot \frac{4}{3}\pi R^3 + 4\pi R^2 K R$$

~~total charge enclosed within Gaussian surface~~

$$= 0$$

$$\therefore \vec{E}_{\text{out}} = 0$$

Note that $\vec{P} = K\vec{r}$, $\vec{E} = -\frac{K}{\epsilon_0} \vec{r} = \frac{1}{\epsilon_0} \vec{P}$ until $r = R$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = 0$$

$$\text{Also } \vec{P} = -\epsilon_0 \vec{E}$$

so that electric field inside a conductor is
total charge within $(R > r)$ is zero

$$(K\vec{r}) \cdot \vec{r} \cdot \frac{A \cdot \vec{r}}{4\pi} = (1/\epsilon_0) r^2 \pi R^2$$

$$\frac{\epsilon_0}{2} r^2 = (1/\epsilon_0) r^2$$

A.2

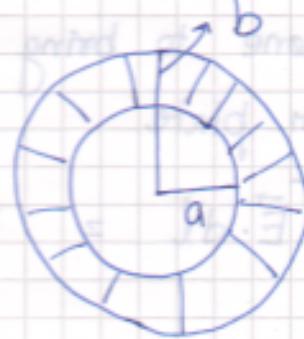
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$$\vec{E}(\vec{r}) = 0 \quad r < a$$

$$\vec{E}(r) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{\hat{r}}{r^2} \quad a < r < b$$

$$\vec{E}(r) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{\hat{r}}{r^2} \quad r > b$$



Potential at the centre = Work done to bring a point charge from infinity to the centre.

$$\begin{aligned}
 V &= - \int_{-\infty}^{0} \vec{E} \cdot d\vec{r} = - \int_{-\infty}^{b} \vec{E} \cdot d\vec{r} - \int_b^a \vec{E} \cdot d\vec{r} - \int_a^0 \vec{E} \cdot d\vec{r} \\
 &= - \frac{Q}{4\pi\epsilon_0} \cdot \int_{-\infty}^b \frac{dr}{r^2} - \frac{Q}{4\pi\epsilon_0} \cdot \int_b^a \frac{dr}{r^2} \\
 &= - \frac{Q}{4\pi\epsilon_0} \left[-\frac{1}{b} \right] - \frac{Q}{4\pi\epsilon_0} \left[-\frac{1}{a} + \frac{1}{b} \right] \\
 &= \frac{Q}{4\pi} \left[\frac{1}{\epsilon_0 b} + \frac{1}{\epsilon_0 a} - \frac{1}{\epsilon_0 b} \right]
 \end{aligned}$$

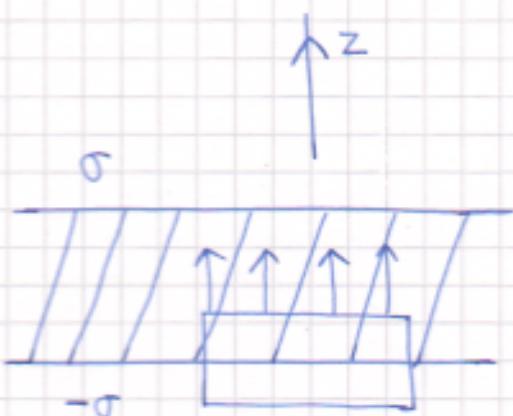
A.3

Electric field outside the plate = 0.

Electric field inside

$$E \cdot A = - \frac{\sigma}{\epsilon_0} \cdot A$$

$$\therefore \vec{E} = \sigma \frac{\epsilon_0}{2} \hat{z} - \frac{\sigma}{\epsilon_0} \hat{z}$$



Work done to bring a unit charge from upper plate to lower plate

$$W = \int_U^L \vec{E} \cdot d\vec{l} = E \cdot d = \frac{\sigma d}{\epsilon} = \frac{Q}{V} \quad \text{if } (\gamma) \text{ is}$$

$$C = \frac{Q}{V} = \frac{A\sigma \epsilon}{\sigma d} = \frac{A \epsilon}{d}$$

$$\therefore C = \frac{A}{d} \epsilon_0 \cdot \frac{\epsilon}{\epsilon_0}$$

$$C_{\text{ext}} = C_0 \epsilon_0$$

$$C_0 = \frac{A \epsilon_0}{d}$$

Capacitance of two parallel plates when there is no dielectric inside.

$$\left[\frac{1}{d} + \frac{1}{d} - \frac{1}{d} \right] \cdot \frac{Q}{\epsilon_0} =$$

$$\left[\frac{1}{d} + \frac{1}{d} - \frac{1}{d} \right] \cdot \frac{Q}{\epsilon_0} =$$

$$d \left[\epsilon_0 \left(\frac{1}{d} + \frac{1}{d} \right) \frac{Q}{\epsilon_0} \right] =$$

ΔA

Δ

Δ

At distance Δ $\epsilon_0 \rightarrow \epsilon$

$\epsilon = \epsilon_0 \epsilon_r$

Electric field due to Δ

$$A \cdot \frac{\Delta}{\epsilon} = A \cdot E$$

$$\frac{1}{2} \epsilon_0 \epsilon_r \frac{Q^2}{\Delta^2} = \frac{1}{2} \epsilon_0 \epsilon_r \frac{Q^2}{d^2} \therefore$$