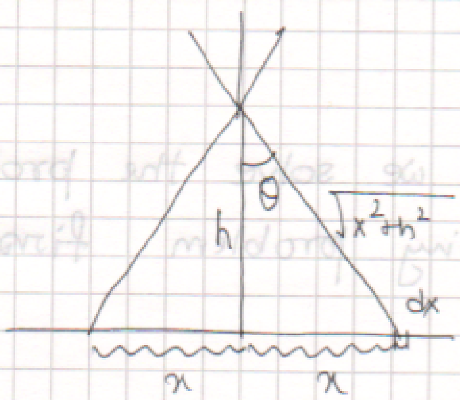
Prob

(2)



$dq = \lambda\, dx$

$$dE_z = \frac{1}{4\pi\epsilon_0} \cdot \frac{2\,dq}{x^2+h^2} \cos\theta$$

$$\cos\theta = \frac{h}{\sqrt{x^2+h^2}}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{2\lambda\, dx}{x^2+h^2} \cdot \frac{h}{\sqrt{x^2+h^2}}$$

$$= \frac{2\lambda h}{4\pi\epsilon_0} \cdot \frac{dx}{(x^2+h^2)^{3/2}}$$

$$E_z = \frac{2\lambda h}{4\pi\epsilon_0} \cdot \int_0^{\infty} \frac{dx}{(x^2+h^2)^{3/2}}$$

$$\cancel{x^2+h^2 = y^2}$$
$$\cancel{x\,dx = y\,dy}$$
$$\cancel{dx = \frac{y\,dy}{\sqrt{y^2-h^2}}}$$

$x = h\tan\theta$

$dx = h\sec^2\theta\, d\theta$

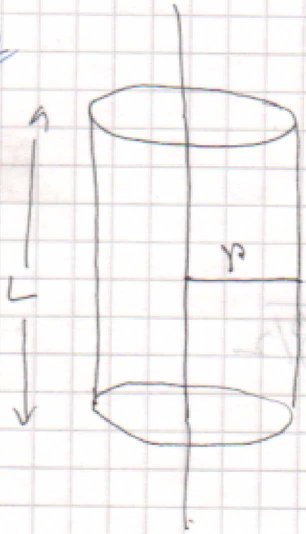
$$= \frac{2\lambda h}{4\pi\epsilon_0} \cdot \int_0^{\pi/2} \frac{h\sec^2\theta\, d\theta}{h^3 \sec^3\theta}$$

$$= \frac{2\lambda}{4\pi\epsilon_0} \cdot \frac{1}{h} \cdot \int_0^{\pi/2} \cos\theta\, d\theta$$

$$= \frac{\lambda}{2\pi\epsilon_0} \cdot \frac{1}{h}$$

Prob

(4)



$d\vec{s} = r\, d\theta\, dz\, \hat{r}$

$\vec{E} \cdot d\vec{s} = E(r)\, r\, d\theta\, dz$

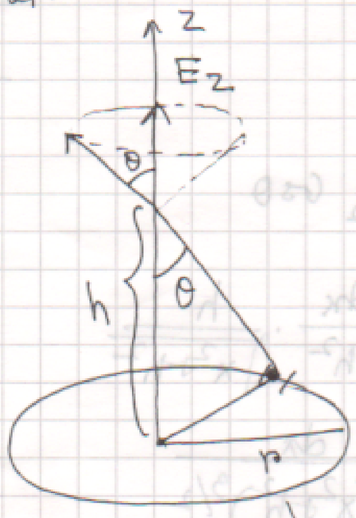
$$\oint \vec{E}\, d\vec{s} = \frac{L\lambda}{\epsilon_0}$$

$$\Rightarrow \quad E(r)\, r \int_0^{2\pi} d\theta \int_0^{L} dz = \frac{L\lambda}{\epsilon_0}$$

$$\boxed{E(r) = \frac{\lambda}{2\pi\epsilon_0} \cdot \frac{1}{r}}$$

3. Prob

Before we solve the problem let us solve the following problem first.



$r\,d\theta$, $dq = \lambda r\, d\theta$

total charge is

$q = 2\pi r \lambda$

$\lambda = \frac{q}{2\pi r}$

$$\cos\theta = \frac{h}{\sqrt{h^2+r^2}}$$

$$d|\vec{E}| = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{h^2+r^2}$$

$$dE_z = |\vec{E}|\cos\theta = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{h^2+r^2} \cdot \frac{h}{\sqrt{h^2+r^2}}$$

$$= \frac{\lambda}{4\pi\epsilon_0} \cdot \frac{rh}{(h^2+r^2)^{3/2}} \cdot d\theta$$

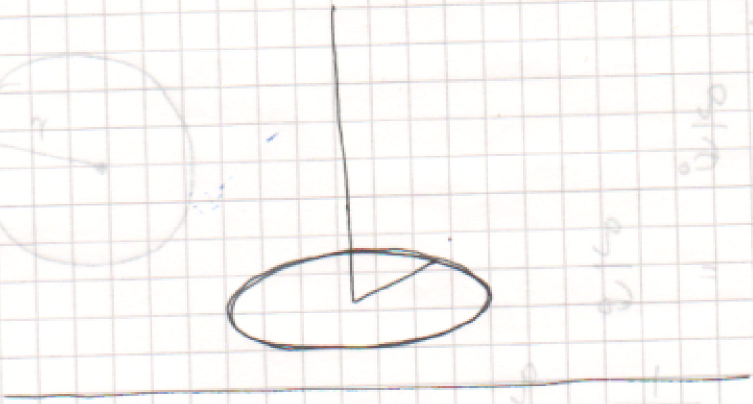
$$E_z = \int_0^{2\pi} dE_z = \frac{\lambda}{4\pi\epsilon_0} \int_0^{2\pi} \frac{rh}{(h^2+r^2)^{3/2}}\, d\theta$$

$$= \frac{\lambda}{2\epsilon_0} \cdot \frac{rh}{(h^2+r^2)^{3/2}}$$

$$= \frac{q}{4\pi\epsilon_0} \cdot \frac{h}{(h^2+r^2)^{3/2}}$$

2

Prob 3 Contin.



The circular ~~disc~~ strip has charge $\boxed{dQ = \sigma \cdot 2\pi r\, dr}$

Electric field is along z direction

$$dE_z = \frac{dQ}{4\pi\epsilon_0} \cdot \frac{h}{(h^2+r^2)^{3/2}}$$

$$= \frac{\sigma\, 2\pi\, r\, dr}{4\pi\, \epsilon_0} \cdot \frac{h}{(h^2+r^2)^{3/2}}$$

$$E_z = \int_0^{\infty} \frac{\sigma h}{2\epsilon_0} \cdot \frac{r\, dr}{(h^2+r^2)^{3/2}}$$

$$r^2 + h^2 = x^2$$

$$r\, dr = x\, dx$$

$$E_z = \frac{\sigma h}{2\epsilon_0} \int \frac{x\, dx}{x^3} = \frac{\sigma h}{2\epsilon_0}\left(-\frac{1}{x}\right) = -\frac{\sigma h}{2\epsilon_0} \cdot \frac{1}{\sqrt{r^2+h^2}}\Bigg|_0^{\infty}$$

$$= +\frac{\sigma h}{2\epsilon_0} \frac{1}{h}$$

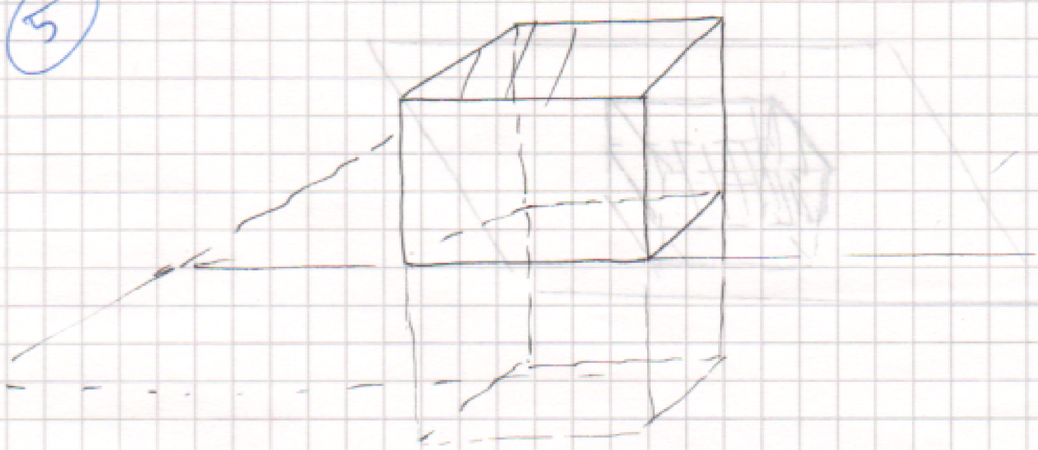
$$= \frac{\sigma}{2\epsilon_0}$$

$$\boxed{E_z = \frac{\sigma \cancel{h}}{2\epsilon_0} \cdot \frac{1}{\cancel{h}}}$$

Prob 5

3

A



$$\oint \vec{E}\cdot d\vec{a} = \frac{Q}{\epsilon_0}$$

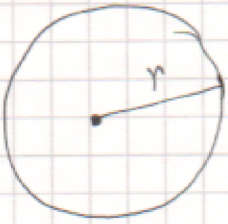
$$E\cancel{A} + E\cdot\cancel{A} = \frac{\cancel{A}\cdot\sigma}{\epsilon_0}$$

$$2E = \frac{\sigma}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}.$$

6 In 2d

$$\oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$



$$E(r)\cdot 2\pi r = \frac{q}{\epsilon_0}$$

$$\Rightarrow E(r) = \frac{1}{2\pi\epsilon_0}\cdot\frac{q}{r}$$

$$\therefore \boxed{\vec{E}(\vec{r}) = \frac{1}{2\pi\epsilon_0}\cdot\frac{q}{r}\cdot\hat{r}}$$