PHY102: Assignment 3

1. Show that, $\vec{E} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is conservative *i.e.* $\vec{\nabla} \times \vec{E} = 0$. Therefore, the vector $\vec{E}$ can be written as gradient of a scalar (as we discussed in the class) : $\vec{E} = \vec{\nabla}\phi$. Find ϕ.

Ans. The first part is easy. $\vec{E} = \vec{\nabla}\phi \Rightarrow \frac{\partial\phi}{\partial x} = 2xy + z^3$, $\frac{\partial\phi}{\partial y} = x^2$, $\frac{\partial\phi}{\partial z} = 3xz^2$. Hence, from these equations we get, $\phi = x^2y + xz^3 + f_1(y,z), \phi = yx^2 + f_2(x,z), \phi = xz^3 + f_3(x,y)$. Comparing these, we get, $\phi = x^2y + xz^3 + yx^2 + c$

2. Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along
(a) the straight line from $(0,0,0)$ to $(2,1,3)$.
(b) the space curve $x = 2t^2, y = t, z = 4t^2 - t$ from $t = 0$ to $t = 1$.
(c) the curve defined by $x^2 = 4y, 3x^3 = 8z$ from $x - 0$ to $x = 2$.

3. A force is given by $\vec{F} = -3x^2\hat{i} + 5xy\hat{j}$. Calculate work done by the force along the curve in the $x - y$ plane, $y = 2x^2$ from a point $(0,0,0)$ to $(1,2,0)$.

4. Consider a vector field $\vec{V} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$. Compute

$$\oint_S \vec{V} \cdot d\hat{s}$$

over the surface of a cube of side 1 as shown in figure.