PHY102: Assignment 2

1. $\vec{R} = (3\cos t)\hat{i} + (2\sin t)\hat{j} - (2t^2)\hat{k}$. Find a) $\frac{d\vec{R}}{dt}$, b) $\frac{d^2\vec{R}}{dt^2}$, c) $\left|\frac{d\vec{R}}{dt}\right|$ and d) $\vec{R}\cdot\frac{d^2\vec{R}}{dt^2}$

Ans. (a) $\frac{d\vec{R}}{dt} = -3\sin t\ \hat{i} + 2\cos t\ \hat{j} - 4t\ \hat{k}$
(b) $\frac{d^2\vec{R}}{dt^2} = (-3\cos t)\hat{i} + (-2\sin t)\hat{j} - 4\hat{k}$
(c) $\left|\frac{d\vec{R}}{dt}\right| = \sqrt{9\sin^2 t + 4\cos^2 t + 16t^2}$
(d) $5\sin t\cos t + 16t$

2. Suppose a particle is moving along a curve which is parametrised by, $x = a\cos t, y = b\sin t, z = ct$. Find the velocity $\vec{v}(t)$ and acceleration $\vec{f}(t)$ of the particle at a time t. What is $\vec{v}(t)\cdot\vec{f}(t)$?

Ans. $\vec{r}(t) = a\cos t\ \hat{i} + b\sin t\ \hat{j} + ct\ \hat{k}$. Velocity is $\vec{v}(r) = \frac{d\vec{r}(t)}{dt}$, acceleration is $\vec{f}(t) = \frac{d\vec{v}(t)}{dt}$. Calculate these quantities.

3. Show that $\vec{\nabla}(FG) = F\vec{\nabla}G + G\vec{\nabla}F$

Ans. $\vec{\nabla}(FG) = \hat{i}\frac{\partial(FG)}{\partial x} + \hat{j}\frac{\partial(FG)}{\partial y} + \hat{k}\frac{\partial(FG)}{\partial y}$. Now use the fact that $\frac{\partial(FG)}{\partial x} = F\frac{\partial G}{\partial x} + G\frac{\partial F}{\partial x}$ and similarly for y and z derivatives. You will get the result.

4. Given $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that, a) $\vec{\nabla}(\ln|\vec{r}|) = \frac{\hat{r}}{r}$, b) $\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\hat{r}}{r^2}$ and c) $\vec{\nabla}r^n = nr^{n-1}\vec{r}$ where $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

5. $\vec{A} = 3x^2y^3\hat{i} - (2z + 7x^2)\hat{j} + xyz\hat{k}$. Find $\vec{\nabla}\cdot\vec{A}$ at a point $P = (1, -3, 5)$.

6. Show that $\vec{\nabla}^2\left(\frac{1}{r}\right) = 0$.

7. A vector whose divergence is zero is called solenoidal. Determine the constant a so that the following vector is solenoidal

$$\vec{V} = (4x + 2y - 3z)\hat{i} + (x - y)\hat{j} + (5x - 7y + az)\hat{k}$$

8. Suppose $\vec{A}(x, y, z) = A_1(x, y, z)\hat{i} + A_2(x, y, z)\hat{j} + A_3(x, y, z)\hat{k}$. Show that,

$$d\vec{A} = (\vec{\nabla}A_1\cdot d\vec{r})\hat{i} + (\vec{\nabla}A_2\cdot d\vec{r})\hat{j} + (\vec{\nabla}A_3\cdot d\vec{r})\hat{k}.$$

Ans. $d\vec{A}(x, y, z) = dA_1(x, y, z)\hat{i} + dA_2(x, y, z)\hat{j} + dA_3(x, y, z)\hat{k}$. Now, $dA_1(x, y, z) = \vec{\nabla}A_1\cdot d\vec{r}$ (showed in class).

9. Show that $\vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right) = 0$, for $r \neq 0$ where, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

10. Show that $\vec{\nabla}f(x, y, z)$ is a vector perpendicular to the surface $f(x, y, z) = constant$.

11. Evaluate $\vec{\nabla} \times \frac{\vec{r}}{r^2}$, where, $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$.