PHY102: SOLUTION : Assignment 1

1. $\vec{A} = 3\hat{i} + 2\hat{j} - 2\hat{k}, \vec{B} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{C} = 4\hat{i} - 3\hat{j} + 2\hat{k}$.

- (i) Find the resultant $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.
- (ii) What is the magnitude of the resultant vector?
- (iii) Are these three vectors linearly independent?

Ans. (i) $\vec{R} = 8\hat{i} - 2\hat{k}$. (ii) $|\vec{R}| = 2\sqrt{17}$

(iii) Calculate the determinant given in problem 4.

2. $\vec{A} = 3\hat{i} - \hat{j} + 5\hat{k}, \ \vec{B} = 2\hat{i} + \hat{j} - \hat{k} \text{ and } \vec{C} = 5\hat{i} - 2\hat{j} - 1\hat{k}.$ Find the unit vector along $\vec{A} + 2\vec{B} + 3\vec{C}$. **T**

3. The initial point P and terminal point Q of a vector \vec{PQ} is given by (1, 2, 3) and (-2, 3, -4) respectively. Find out the vector \vec{PQ} . **T**

4. Show that the necessary and sufficient condition that the vectors $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$, $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$ and $\vec{C} = C_1\hat{i} + C_2\hat{j} + C_3\hat{k}$ be linearly independent is that the determinant

$$\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

is non-zero. \mathbf{T}

5. Show that $|\vec{A} + \vec{B} + \vec{C}| \le |\vec{A}| + |\vec{B}| + |\vec{C}|$.

Ans. Hint: You know that the sum of two arms of a triangle always greater than or equals to the third arm. Try to generalise it for polygone.

6. $\vec{A} = 2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{B} = \hat{i}-3\hat{j}+\hat{k}$. Find the angle between these two vectors.

Ans. $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$. Calculate $\vec{A} \cdot \vec{B}$ and $|\vec{A}|$ and $|\vec{B}|$ and equate.

7. $\vec{A} = 3\hat{i} - \hat{j} + 5\hat{k}, \ \vec{B} = 2\hat{i} + \hat{j} - \hat{k}$. Find the projection of \vec{A} on \vec{B} and vice-versa.

8. Find the value of a for which the vector $\vec{A} = a\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{B} = 2a\hat{i}+a\hat{j}-2\hat{k}$ are perpendicular.

Ans. Calculate $\vec{A} \cdot \vec{B}$ and equate it to zero. Solve *a* from that equation.

9. Evaluate : (i) $2\hat{i} \times (2\hat{i} - 3\hat{k})$, (ii) $(\hat{i} - 3\hat{k}) \times (\hat{i} + 2\hat{j} - 5\hat{k})$.

10. $\vec{A} = 3\hat{i} + 2\hat{j} - 2\hat{k}, \ \vec{B} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{C} = 4\hat{i} - 3\hat{j} + 2\hat{k}$. Find (i) $(\vec{A} \times \vec{B}) \times \vec{C}$, (ii) $\vec{A} \times (\vec{B} \times \vec{C})$, (iii) $\vec{A} \cdot (\vec{B} \times \vec{C})$

 \mathbf{T} problems will be discussed in the tutorials. However, it is advised that you should solve all the problems for your practice. If you have any problem to solve other problems you are free to discuss with me or any other tutors.