PHY102: Assignment 2

1. $\vec{R} = (3\cos t)\hat{i} + (2\sin t)\hat{j} - (2t^2)\hat{k}$. Find a) $\frac{d\vec{R}}{dt}$, b) $\frac{d^2\vec{R}}{dt^2}$, c) $\left|\frac{d\vec{R}}{dt}\right|$ and d) $\vec{R}\cdot\frac{d^2\vec{R}}{dt^2}$

2. Suppose a particle is moving along a curve which is parametrised by, $x = a\cos t, y = b\sin t, z = ct$. Find the velocity $\vec{v}(t)$ and acceleration $\vec{f}(t)$ of the particle at a time t. What is $\vec{v}(t)\cdot\vec{f}(t)$?

3. Show that $\vec{\nabla}(FG) = F\vec{\nabla}G + G\vec{\nabla}F$

4. Given $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that, a) $\vec{\nabla}(\ln|\vec{r}|) = \frac{\hat{r}}{r}$, b) $\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\hat{r}}{r^2}$ and c) $\vec{\nabla}r^n = nr^{n-1}\vec{r}$ where $r = |\vec{r}| = \sqrt{x^2+y^2+z^2}$.

5. $\vec{A} = 3x^2y^3\hat{i} - (2z+7x^2)\hat{j} + xyz\hat{k}$. Find $\vec{\nabla}\cdot\vec{A}$ at a point $P = (1,-3,5)$.

6. Show that $\vec{\nabla}^2\left(\frac{1}{r}\right) = 0$.

7. A vector whose divergence is zero is called solenoidal. Determine the constant a so that the following vector is solenoidal

$$\vec{V} = (4x+2y-3z)\hat{i} + (x-y)\hat{j} + (5x-7y+az)\hat{k}$$

8. Suppose $\vec{A}(x,y,z) = A_1(x,y,z)\hat{i} + A_2(x,y,z)\hat{j} + A_3(x,y,z)\hat{k}$. Show that,

$$d\vec{A} = (\vec{\nabla}A_1\cdot d\vec{r})\hat{i} + (\vec{\nabla}A_2\cdot d\vec{r})\hat{j} + (\vec{\nabla}A_3\cdot d\vec{r})\hat{k}.$$

9. Show that $\vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right) = 0$, for $r \neq 0$ where, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

10. Show that $\vec{\nabla}f(x,y,z)$ is a vector perpendicular to the surface $f(x,y,z) = constant$.

11. Evaluate $\vec{\nabla}\times\frac{\vec{r}}{r^2}$, where, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.