Analytic func^n

Single valued func^n

For every value of z the is a single number $f(z)$.
$f(z) \rightarrow$ Single valued func^n.

More than one number to be associated with each z. $f(z) \longrightarrow$ multivalued func^n.

A func^n is analytic at a given point of the complex plane if $\exists$ a neighbourhood of this point such that the func^n is single valued and differentiable at all points of this neighbourhood.

If a func^n is analytic at all points of some region of the complex plane; $\rightarrow$ Called analytic throughout the region.

Set of points where the func^n is analytic is called domain of analyticity of the func^n.

If the domain of analyticity is the entire complex plane $\longrightarrow$ the func^n is called entire func^n.

Neighbourhood: $d(p, p')$ distance between p and p'.
Neighbourhood of a point p means set of all points p' satisfying $d(p,p') < R$ for any positive no. R.

<u>Regular point</u> : A point where the func^n is analytic.

<u>Singular point</u> : A point where the func^n is not analytic.

<u>Important</u> : It may happen that a func^n is analytic in the complex neighbourhood of ~~the~~ a point ~~about~~ but not analytic at the point itself.

<u>Example</u>: $f(z) = \frac{1}{z}$. The func^n is analytic in the neighbourhood of $z=0$ but not differentiable at $z=0$. Such singlar point is called isolated singular point.

<u>The Cauchy Theorem</u>

We have seen that : $\int_a^b f(z)\,dz = F(b) - F(a)$

Where $F(z)$ is primitive func^n ↓ Independent of choice of contour.

This is true only when $f(z)$ posses a primitive func^n.
In general the contour integration depends on the choice of contour.

If a function is analytic in some domain D, then they always posses a primitive func^n in that domain. This means the contour integration of an analytic func^n along two different contours, both lying in the domain of analyticity and having the same end points yield the same result, provided one can deform one contour continuously to bring it into the other without crossing any singular point.

Theorem: Let C denote a piecewise, regular closed curve in the complex plane and let $f(z)$ be analytic on C and within the whole region enclose by C. Then

$$\oint_C f(z)\, dz = 0$$

I leave the proof as a home work.

Cauchy's Integral representation

From Cauchy's theorem we shall prove an important theorem which will be very useful for us.

Theorem: Let $f(z)$ be analytic throughout a simply connected region R.

A region in complex plane without any hole. Any closed curve can be shrink to a point.

If C is a closed picewise regular curve within R and z a point not on C, then

$$\frac{1}{2\pi i}\oint_C \frac{f(z')}{z-z'}\, dz' = \begin{cases} f(z) & \text{if } z \text{ is interior to } C \\ 0 & \text{if } z \text{ is exterior to } C \end{cases}$$

$\oint \rightarrow$ integration has taken counter clock wise.