PHY 601: Assignment 1

1. Find out the cube root of $-8i$.
2. Find out the square root of $1 - \sqrt{3}i$.
3. Show that the n^{th} roots of any complex number z_0 are given by,

$$c, c\ \omega_n, c\ \omega_n^2, \cdots, c\ \omega_n^{n-1}$$

where, c is any particular n^{th} root of the number z_0 and ω_n^k for $k = (0, 1, \cdots, n-1)$ are the distinct n^{th} roots of unity.

4. Show that

$$\sum_{k=0}^{n-1} \omega_n^k = 1$$

where, ω_n^k for $k = (0, 1, \cdots, n-1)$ are the distinct n^{th} roots of unity.

5. Let a be a fixed real number. Show that the two square roots of $a + i$ are

$$\pm\sqrt{A}\exp\left(i\frac{\alpha}{2}\right)$$

where, $A = \sqrt{a^2 + 1}$ and $\alpha = Arg(a + i)$.

6. Show that

$$\overline{z_1 + z_2 + \cdots + z_n} = \overline{z_1} + \overline{z_2} + \cdots + \overline{z_n}.$$

7. Show that,

$$\left|\frac{1}{z^4 - 4z^2 + 3}\right| \leq \frac{1}{3}.$$