Phys106, II-Semester 2018/19, Tutorial 9, Fri 29.3.

Work in teams of three. Do "Stages" in the order below. When all teams finished a stage, make sure all students at your table understand the solution and agree on one by using the board.

Stage 1

(i) Draw the following complex numbers as a dot in the complex plane: $z = 2 + i3$, $w = -2 + i$, $\xi = 1 - 4i$.

(ii) Evaluate the operations: $z + w$, $z - \xi$, zw, $z\xi$.

(iii) Express the following complex numbers as $z = r\exp(i\varphi)$, with $r \in \mathbb{R} > 0$, $\varphi \in \mathbb{R}, 0 < \varphi < 2\pi$. $z_1 = (1 + i)/\sqrt{2}$, $z_2 = i$, $z_3 = -1$, $z_4 = (1 + i)$.

(iv) Differentiate $\frac{d}{dx}\exp[i\beta x]$.

Stage 2

(i) Verify that the wave function $\Psi(x, t) = A\exp[i(kx - \frac{\hbar k^2}{2m}t)]$ solves the TDSE Eq. (85) without external potential $U(x, t) = 0$.

(ii) How does this have to be modified of the particle is moving in a *constant* external potential $U(x, t) = U_0 = const.$?

(iii) For both cases identify momentum, total energy and kinetic energy of the particle.

Stage 3 Read the documentation for the online-app for solution of the TDSE on http://www.falstad.com/qm1d/ . Some of its functionality (energy levels), we will learn in the context of the SE only next week. For now we shall try to on the fly reproduce the *Example: Numerical solution of TDSE* in the lecture.

(i) Switch the setup to "harmonic oscillator". Then switch "mouse: create gaussian" below. In the probability density field, click somewhere away from the centre. What do you see?

(ii) Now click on "mouse: edit function" and change the potential landscape at will. You may try to realise the one given in the lecture. Again start in a Gaussian. Discuss on your table to what extend you can understand the motion with classical physics, and where you can't.

(iii) Checkout some of the other setups at will.

Bonus: The "phasors" at the bottom and the horizontal lines in the figure are **energy eigen-states**. *We will learn about these next week and then revisit this app. They are like the energy levels in the Bohr atom or the standing waves that fit into the box.*

Stage 4

(i) Assume a particle that can move on a ring of circumference L. The wave-function thus requires periodic boundary conditions like in the Bohr model. Normalize the wave function (at $t = 0$):

$$\Psi(x, t) = A[e^{i\frac{4\pi}{L}x}e^{-iE_1 t} + e^{i\frac{2\pi}{L}x}e^{-iE_2 t}]. \tag{1}$$

(ii) Find the expectation value of the momentum. Discuss your results.