Phys106, II-Semester 2018/19, Tutorial 5, Fri 8.2.

Work in teams of three. Do "Stages" in the order below. When all teams finished a stage, make sure all students at your table understand the solution and agree on one by using the board (in Studio-Air), or paper sheets (in L1).

This tutorial has a large online component. If you can bring a laptop, that would be helpful.

Stage 1

(i) Re-read the lecture notes regarding **beating** of two waves, that is section 2.3.1).

(ii) Now explore beating using this web applet . You can also hear the result for the example of acoustic waves. What does the beating effect sound like? What happens to the beating frequency if the two combined waves have a larger (smaller) frequency difference?

(iii) Revise the concept of group-velocity Eq. (53) and share with your table. You can now explore two moving combined waves with this web applet . How does it look if $v_g < v$, $v_g = v$, $v_g > v$, where v is the phase velocity? More control over the two constituent waves, like in the lecture, can be found in this app .

Stage 2

(i) Re-read the lecture notes regarding the **Fourier series** method to decompose any periodic function into cosines or sines, that is Eq. (43), (44), (46), (47). Make sure you all understand the gist of what this equation means (not yet why it works, see stage 4). Ask a TA otherwise.

(ii) Now explore this similarly to the pictures in the lecture using the online app on http://www.falstad.com/fourier/ . Select sequentially the triangle, sawtooth square function. Move the "Number of Terms" slider fully to the left, then add slowly term by term. The white dots appearing are the g_n (cosines), h_n (sines) coefficients from the lecture. What happens for larger n?. How can you tell if there are only cos or only sin? What happens if you switch $f(x)$ itself to be a cos or sin?

(iii) Also check out https://www.geogebra.org/m/EYhBXfmK for some additional choice of functions.

Stage 3 Discuss on the table what is meant by dispersion. Then explore dispersion for a square-ish wave made of four sine waves using this web applet . Read the page and follow the instructions on it. What happens to the square wave at late times when you pick change all frequencies a small random amount $\sim 20\%$ up or down? *The app above is part of a larger collection of online wave related apps that you might find useful for understanding week 3 or follow up courses.*

Stage 4 (bonus material)

(i) If you want to understand also **why** Eq. (43), (44), (46), (47) are valid, it is helpful to review the concept of **vectors** and of a **basis** for a vector space. Let $\mathbf{v} = [v_x, v_y, v_z]^T$ be a 3-component (3D) vector. We can write it in the basis $\mathbf{i} = [1,0,0]^T$, $\mathbf{j} = [0,1,0]^T$, $\mathbf{k} = [0,0,1]^T$ as $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$. But we can choose another basis as well, for example $\mathbf{i}' = (\mathbf{i}+\mathbf{k})/\sqrt{2}$, $\mathbf{j}' = \mathbf{j}$, $\mathbf{k}' = (-\mathbf{i}+\mathbf{k})/\sqrt{2}$. Use the board to draw both bases into a 3D coordinate system.

(ii) We want to now express any vector $\mathbf{v}$ in the new basis as $\mathbf{v} = v'_x\mathbf{i}' + v'_y\mathbf{j}' + v'_z\mathbf{k}'$, where $v'_x = \mathbf{i}' \cdot \mathbf{v}$ etc. Here the symbol $\cdot$ denotes the *scalar product* of two vectors. It is calculated via $\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z$. Test all this using the example vector $\mathbf{v} = [1,1,2]$.

(iii) Now consider the following: (a) The above description works for any dimension d of the vector, $\mathbf{v} = [v_1, v_2, \cdots v_d]^T$, i.e. vectors can have 100 components, then the vector space is 100 dimensional. (b) It turns out functions $f(x)$ can be viewed as an ∞-dimensional vector. To heuristically understand this consider the figure below: We "discretize space" x by dividing it into lots and lots of points. We can then write the function into a vector $\mathbf{f} = [f(x_1), f(x_2), \cdots, f(x_j), \cdots f(x_J)]^T$. In the end we let $J \to \infty$. (c) It turns out the functions $\varphi_n(x) = \cos(2\pi n x/L)$ for $n = 0, 1, 2, \cdots$ form a **basis** for the vector space of all even functions with period L [proof, see math course]. After we realized this, in analogy to writing *any vector* as $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$, we can write *any even periodic function* as $f(x) = \sum_{n=0}^{\infty} g_n \varphi_n(x)$. The coefficients are $g_n \sim \int f(x)\varphi_n(x)dx$ which, if we discretize space x again, we can understand as formally analogous to a scalar product, since $g_n \sim \int f(x)\varphi_n(x)dx \approx \sum_j f(x_j)\varphi_n(x_j)\Delta x$, see Fig. 1.

(iv) In summary we can thus understand the Fourier series as "expanding the function f(x) in terms of a basis for the space of all functions, provided by the set of cosines with all possible wave lengths".

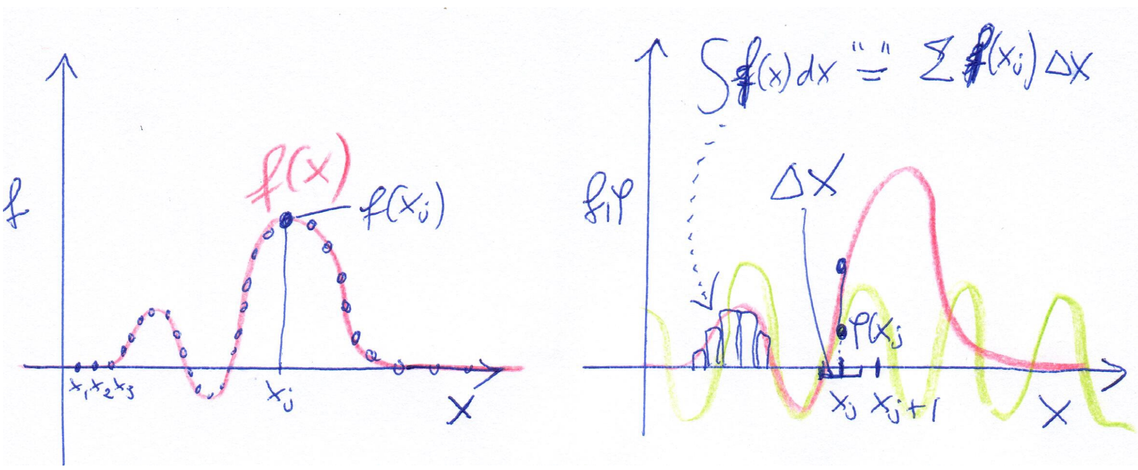

Figure 1: (left) We can view a function as a high-dimensional vector, where each vector element j is the function value $f(x_j)$ at a specific discrete point x_j. (right) When viewing the integration $\int dx f(x)\varphi(x)$ in the discretized form, it becomes structurally analogous to a scalar product.