Phys106, II-Semester 2018/19, Tutorial 10, Fri 5.4.

Work in teams of three. Do "Stages" in the order below. When all teams finished a stage, make sure all students at your table understand the solution and agree on one by using the board.

This tutorial has a large online component. If you can bring a laptop, that would be helpful. To make space for online apps, some learning calculations has been shifted to Assignments 9 and 10. Please make sure to go through those as well.

Stage 1 Particle in the box: Consider all the eigen-states $\phi_n(x)$ of the particle in an infinite box (Eq. 105 of lecture).

(i) Draw the box potential.

(ii) Write down the TISE <u>and</u> TDSE within the box only ($0 < x < L$), i.e. for $U(x) = 0$, that is Eq. 102 and Eq. 85 for $U = 0$.

(iii) Make the Ansatz $\Psi_n(x,t) = c_n(t)\phi_n(x)$ for the time-evolution of an eigen-state and insert into TDSE. Make use of the fact that time-derivatives only act on the time dependent functions, space derivatives only on the space dependent ones, to use the TISE and find $c_n(t)$. *Hint: All of this is following section 3.2.4) of the lecture.*

Stage 2 Particle in the box: In the previous stage you should have found $c_n(t) = \exp(-iE_n t/\hbar)$, where E_n is the particle-in-the-box energy of Eq. (105)

(i) For the ground- and first excited state in the box, explicitly write the time-dependence $\Psi_1(x,t)$ and $\Psi_2(x,t)$ based on stage1.

(ii) Now use the superposition principle, Eq. (88) with $d_1 = d_2 = 1/\sqrt{2}$ to explicitly combine the two into a new wave function $\Psi_3(x,t)$. *The choice of $c_{1,2}$ ensures the new wave function remains normalized correctly.*

(iii) Finally, calculate the probability density $|\Psi_3(x,t)|^2$ as a function of time. Discuss your answer.

Stage 3 Particle in the box: Read the documentation for the online-app for solution of the TDSE on http://www.falstad.com/qm1d/ . We can now use this app to verify your solution from stage 2. The default setting should be "setup: infinite well". There should be two white marked circles with rotating arrows at the bottom. These are the $c_n(t)$ from stage 1 for $n = 1, 2$ in the complex plane. The height of the curve in the centre panel is $|\Psi_3(x,t)|^2$. The color shading indicates the complex phase $\varphi(x,t)$ of the wave function $\Psi_3(x,t) \equiv n(x,t)e^{i\varphi(x,t)t}$. It turns out that lots of "color-stripes" indicate higher velocity, uni-color indicates small or no velocity.

(i) Inspect the default setting and see if this is consistent with your results from stage 1 and 2.

(ii) Interpret the particle dynamics that you see physically and discuss on your table.

(iii) The app now allows you to create superpositions of more state by clicking on further circles at the bottom. See what happens.

(iv) You can click on the centre panel to change the wavefunction into a Gaussian wave packet sitting at your mouse cursor. Do this together with significantly increasing the mass of the particle. See what happens. Try to understand why, based on other information available in the panels and description above.

Stage 4 Quantum tunneling: The same app as above also allows us to have a look at quantum tunneling.

(i) Change setup to "Well pair". You should see two square wells with a small barrier in between. This barrier is like the one discussed in section 3.3.2., the difference is that the space to the right and left is again limited by the other side of the square wells. Select "Mouse = Create Gaussian" and click on the middle (wavefunction) panel into the right well. Discuss on your table what is happening after some time. Interpret wrt. Lecture.

(ii) Now use the sliderbar "mass" to increase the mass of the particle and do the above steps again. Increase 2-3 times. Then make it lighter than at the beginning. Discuss what you see. Is this consistent with Eq. 113?