Phys106, II-Semester 2018/19, Assignment 9

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1. Find the roots of the following quadratic equations and express roots in compact notation of complex number as $z = r\exp(i\varphi)$, with $r \in \mathbb{R} > 0$, $\varphi \in \mathbb{R}, 0 < \varphi < 2\pi$:

   (i) $z^2 + 1/2 = 0$

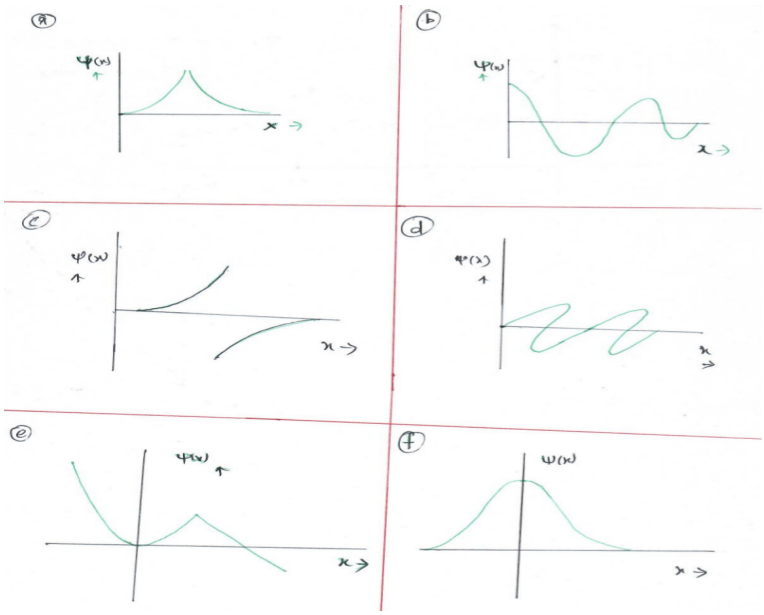
   (ii) $z^2 + 2z + 2 = 0$

   (iii) $2z^2 + 2z + 1/2 = -1/2$

2. Evaluate the operations for given $z_1 = 5 + i4$, $z_2 = \sqrt{2} + i3$, $z_3 = 6 - i\sqrt{3}$:

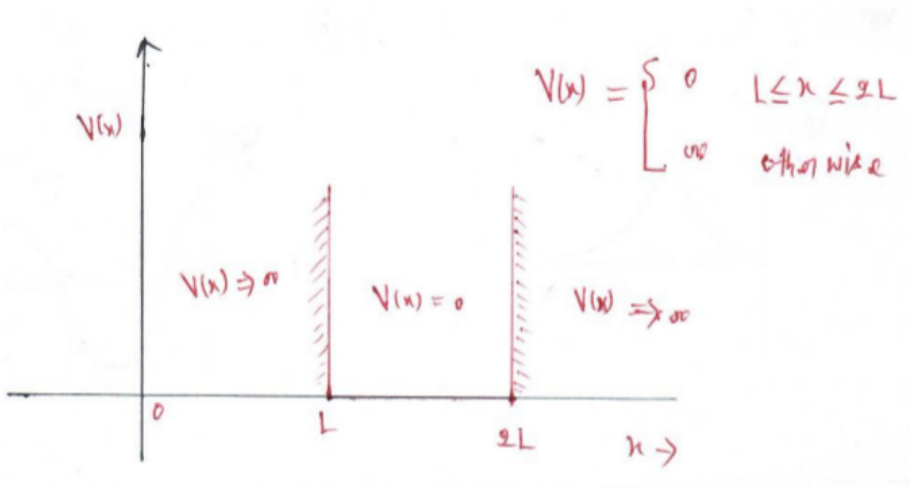
   - $z_1\, z_2$
   - $z_1\, z_3$
   - $\frac{z_1}{z_2}$
   - $\frac{z_1}{z_3}$
   - $z_2 + z_3$
   - $z_1 - z_3$

3. Which of the following are admissible wave functions as solution of the TDSE at some time?



4. Show that the position and momentum operators satisfy $\hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$.
   Hint: do this by showing $\hat{x}\hat{p}f(x) - \hat{p}\hat{x}f(x) = i\hbar f(x)$ for an arbitrary unspecified test function $f(x)$. We say that the position and momentum operator "do not commute", i.e. their ordering matters. What other mathematical objects to you know that do not commute?

5. A particle is in a square box potential that extends from $x = L$ to $x = 2L$ as in the sketch. The wavefunction can be given by $\Psi(x) = A \sin\left(\frac{n\pi}{L}x\right)$ for $n = 1, 2, 3, \ldots$. Find the normalisation constant and then the expectation value of position and momentum.



6. Find the expectation value of momentum for the wavefunction given in the lecture notes as, Eq. 97.