

# Phys106, II-Semester 2018/19, Assignment 2

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*Hint: For all these questions (and later ones) you may use the math software `mathematica`, available from CC. The little overhead in familiarising yourself with it now, will pay off manifold later, regardless of your major.*

1. A function  $f$  is called periodic with period  $T$  if  $f(t + T) = f(t)$ . The time average of a such a function is given by

$$\overline{f(t)} = \frac{1}{T} \int_0^T f(t) dt. \quad (1)$$

Calculate  $\overline{\cos(\omega t)}$ ,  $\overline{\sin(\omega t)}$ ,  $\overline{\cos^2(\omega t)}$ ,  $\overline{\sin^2(\omega t)}$ . What average would you get when averaging over a larger number of periods  $nT$ ?

2. Do the missing steps in the lecture for the derivation of the double slit interference pattern:
  - (i) Start with Fig. 2 in week 3, using geometry, express  $r_1$  and  $r_2$  through  $z$ ,  $L$ , and the angle  $\theta$  of the vectors connecting the slits and location  $z$  on the screen. These angles are slightly different, make the approximation that they are equal, as shown in the attached sketch Fig. 1.

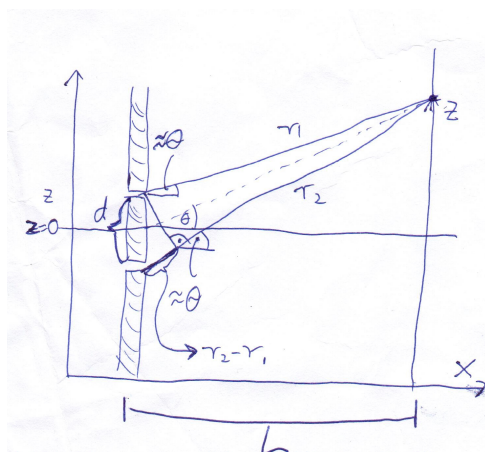


Abbildung 1: Sketch of variables for double slit geometry.

- (ii) Also approximate the  $r_{1,2}$  dependence of the prefactors of  $y_1$  and  $y_2$  (Fig. 2) as equal  $r_1 \approx r_2 \approx L$ . Do not do this approximation for  $r_{1,2}$  within the argument of the wave ( $\cos$ ) in Fig. 2.
- (iii) Within  $y(z, t)$ , split the space and time dependence in the trigonometric functions into the form  $f(t)g(x)$  using the trigonometric identity XXXX.
- (iv) Now calculate the intensity  $I(z, t) = |y(z, t)|^2$ , and perform the long time average over this using your results from question one.

(v) FINAL HELP TO GET THE EXPRESSION FROM LECTURE. SHOULD DEFINE  $I_0$  BUT AVOID ALL MESSY CONSTANTS.

3. The following wave equation is called “sine-Gordon equation”:

$$\frac{\partial^2}{\partial t^2} f(x, t) - \frac{\partial^2}{\partial x^2} f(x, t) + \sin [f(x, t)] = 0. \quad (2)$$

Determine if the superposition principle holds for this equation and why or why not.