Week 3

PHY 106 Quantum Physics
Instructor: Sebastian Wüster, IISER Bhopal, 2018

2) Waves and Particles

revision of movie:

Quantum physics is essentially all about
"things that ought to be particles are **also** waves"
and
"things that ought to be waves are **also** particles".

Thus, let's make sure we are all on the same page regarding waves.....

2.1) Introduction to wave mechanics

What **is** a wave?

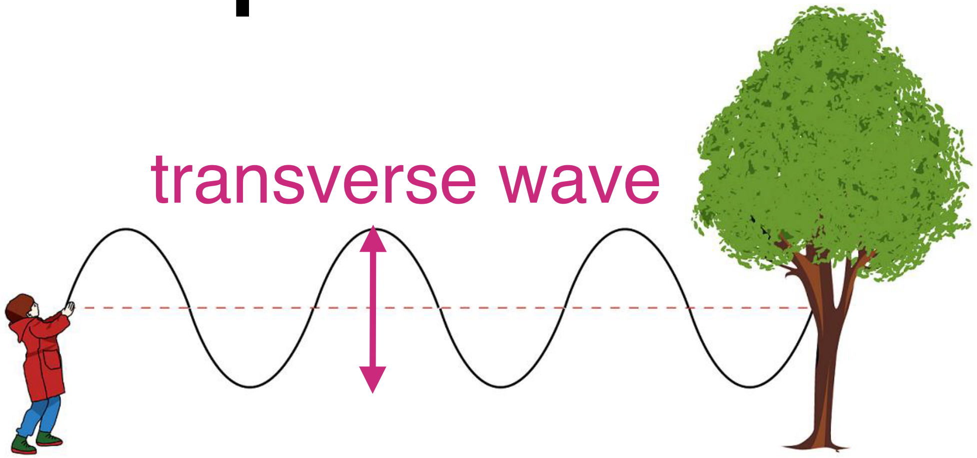
Definition of **wave:**

A perturbation of some **property** is transported through a **medium**, without transport of the medium itself

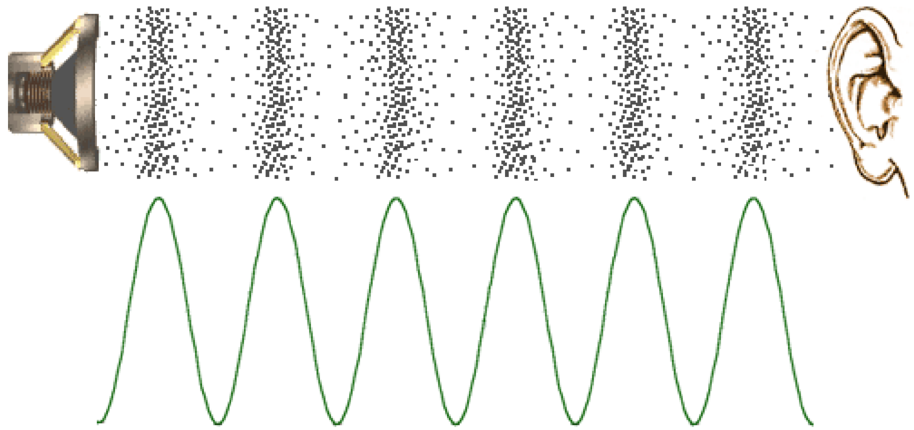
Book: A.P. French, "Vibrations and waves"

Waves

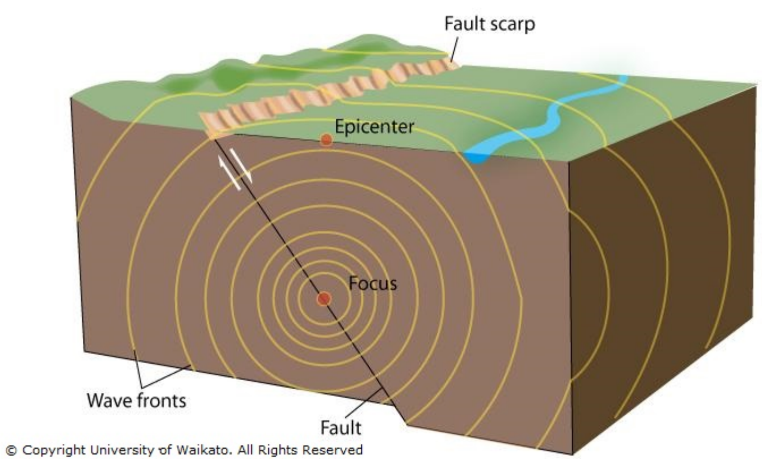
Examples:



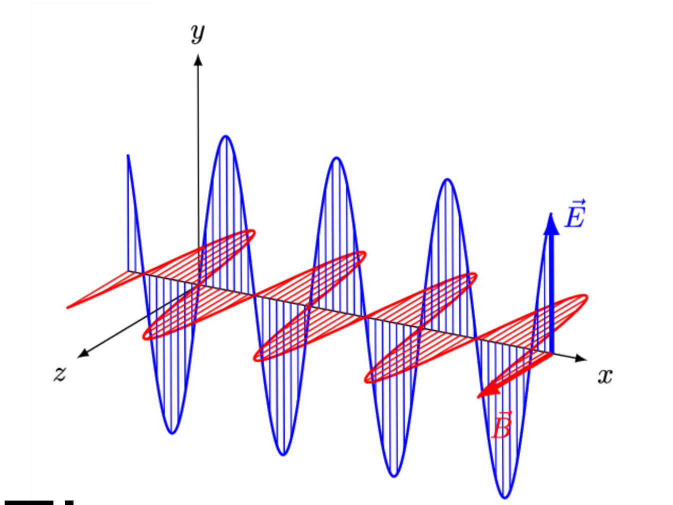
Rope waves



Sound wave



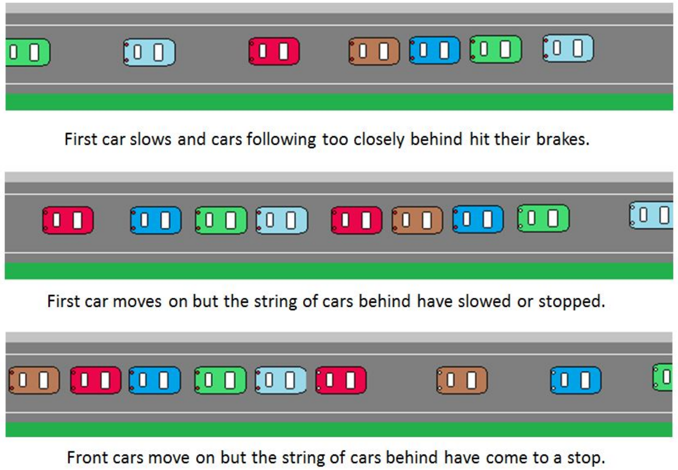
Seismic wave



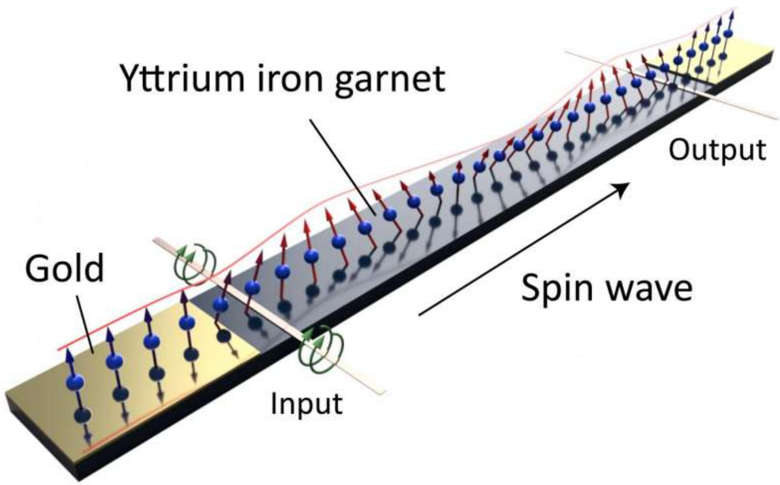
Elm wave

Waves

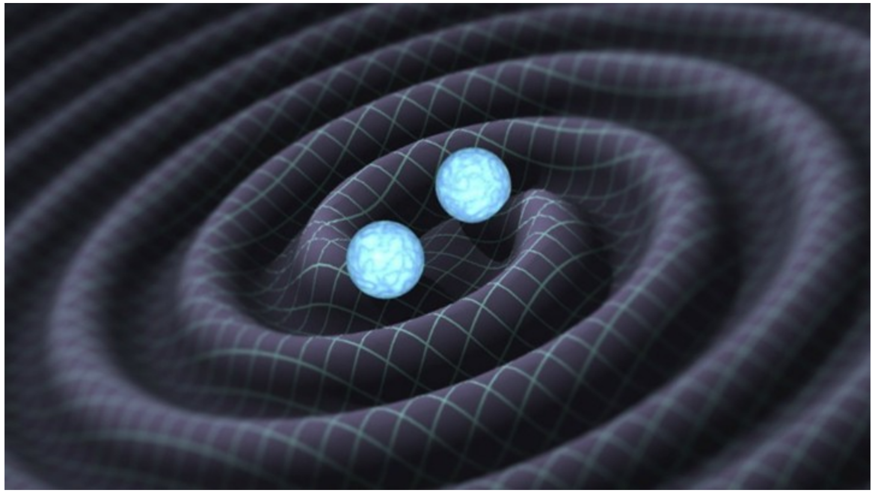
Water wave



Traffic waves

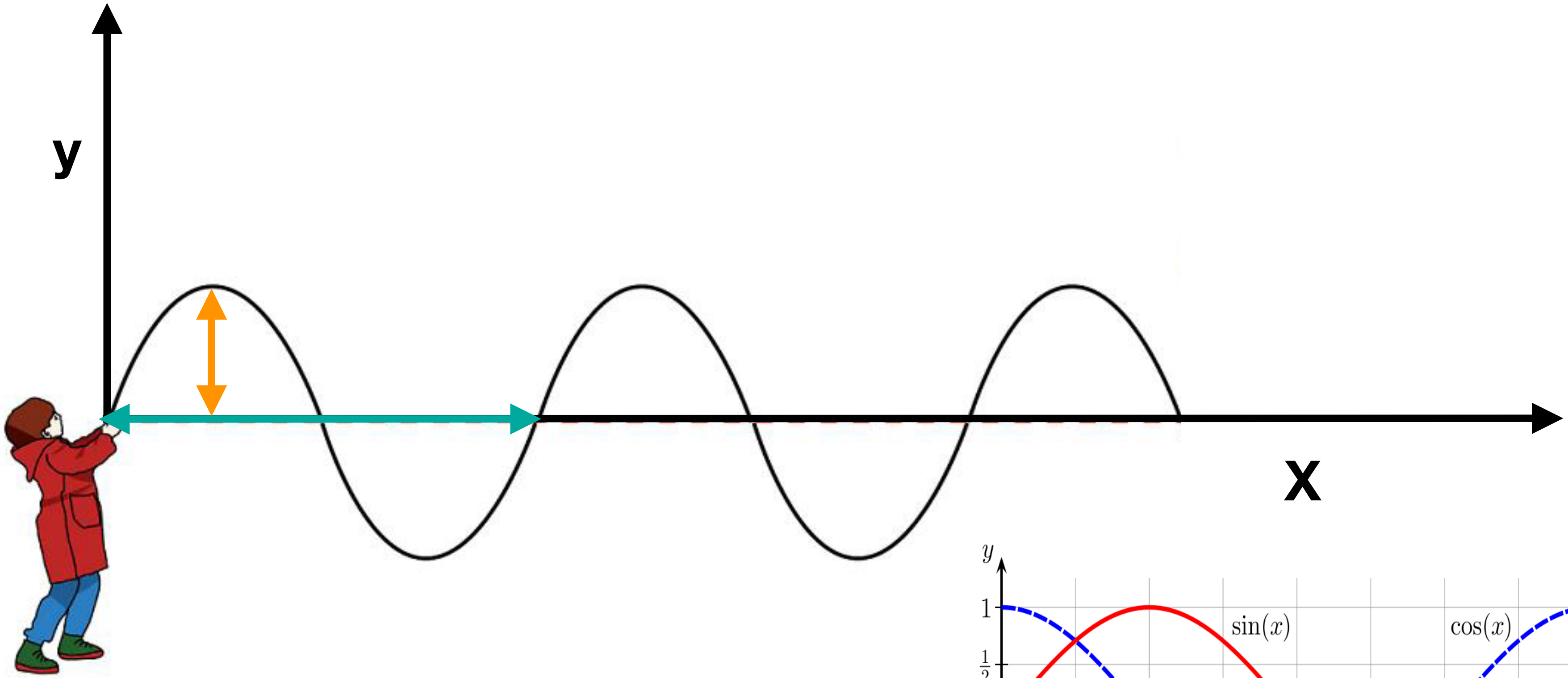


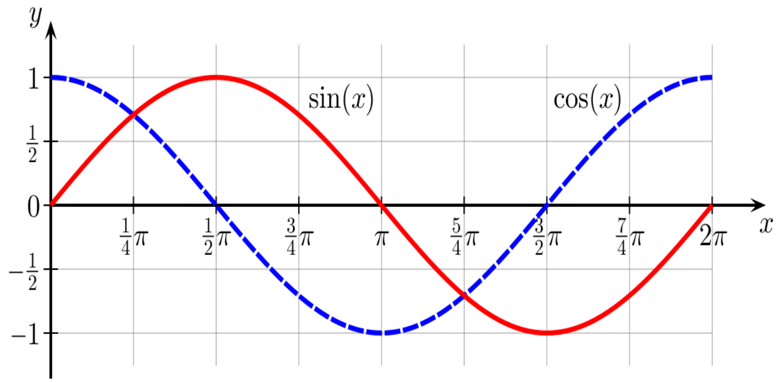
Spin waves



Gravitational wave

2.1.1) Waves, frequencies, wavelengths



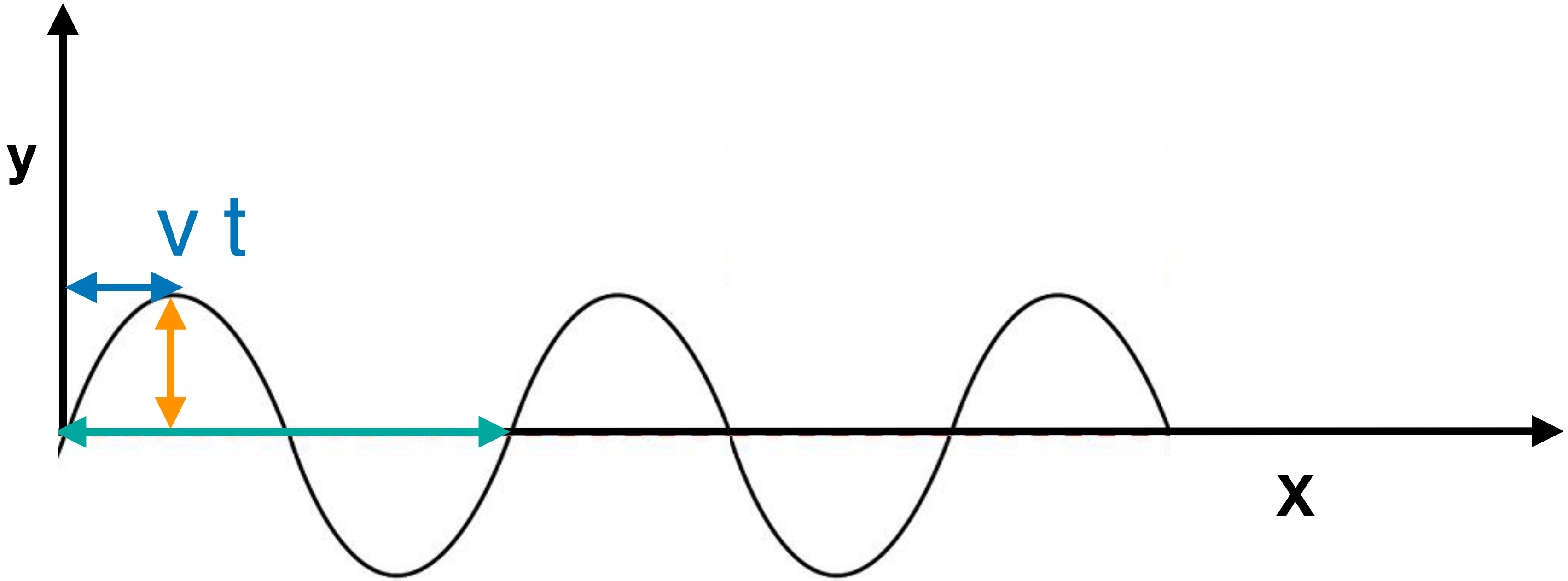


$$y(x) = \boxed{A} \sin\left(\frac{2\pi}{\boxed{\lambda}} x\right)$$

amplitude

wavelength

Waves



wave velocity

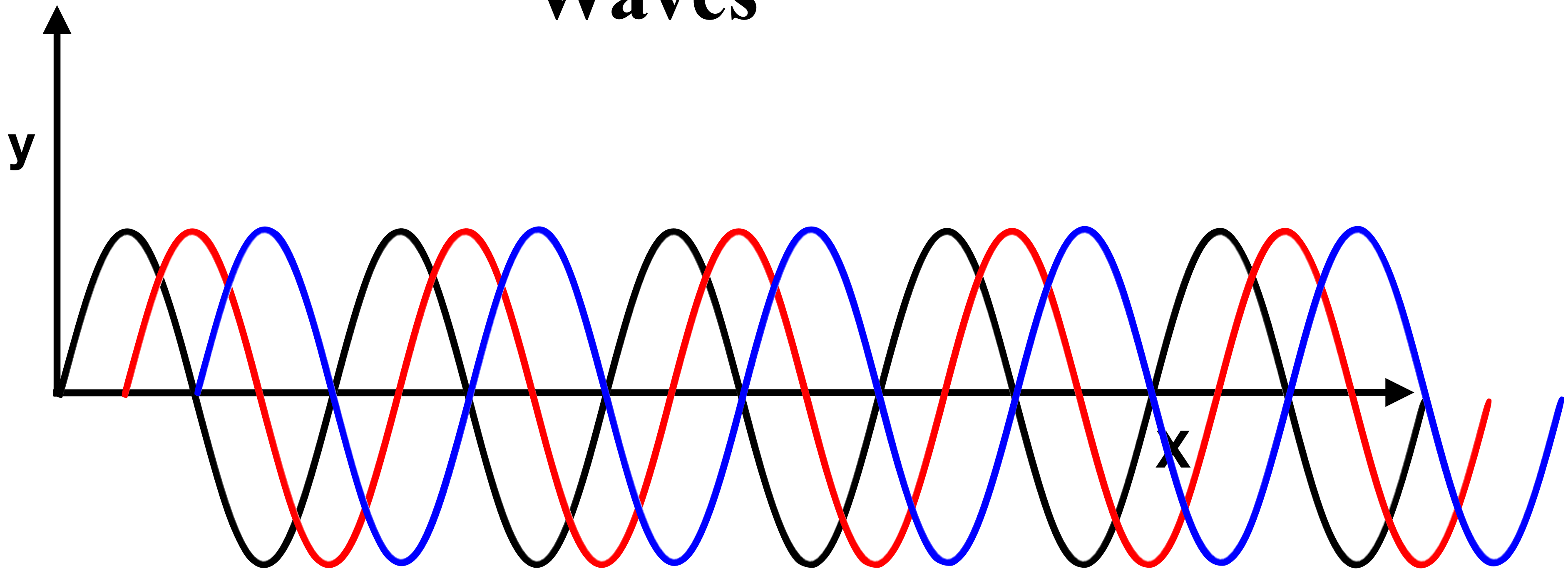
$$y(x, t) = \boxed{A} \sin\left(\frac{2\pi}{\boxed{\lambda}}(x - \boxed{V}t)\right) \quad (5)$$

amplitude

wavelength

Form of **progressive/travelling wave**

Waves



$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - Vt)\right)$$

$t = 0$

$t = \frac{\lambda}{4}\frac{1}{V}$

$t = \frac{\lambda}{2}\frac{1}{V}$

Argument of **sin** is called **phase**

V here is the **phase velocity**

Waves

Rewrite wave form:

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - Vt)\right)$$

$$k = \frac{2\pi}{\lambda} \quad \textbf{(7)}$$

$$y(x,t) = A\sin\left(kx - \omega t\right) \quad \textbf{(6)}$$

$$\boxed{\frac{\omega}{k} = V} \quad \textbf{(8)}$$

k is the **wave number** (unit 1/m)

ω is the **angular frequency**

ν is the **frequency** (unit Hz = 1/s)

$$\omega = 2\pi\nu \quad \textbf{(9)}$$

Waves

$$y(x,t) = A\sin\left(kx - \omega t\right) \quad \text{(6)}$$

$$k = \frac{2\pi}{\lambda} \quad \text{(7)}$$

Relation between **frequency**, **wave length** or **wave number** and **phase velocity** of any wave (unit m/s)

$$\frac{\omega}{k} = V \quad \text{(8)}$$

$$\nu\ \lambda = V \quad \text{(10)}$$

k is the **wave number**

ω is the **angular frequency**

$$\omega = 2\pi\nu \quad \text{(9)}$$

ν is the **frequency**

Wave velocities

$$\frac{\omega}{k} = V \quad \textbf{(8)} \qquad\qquad \nu\ \lambda = V \quad \textbf{(10)}$$

Examples:

sound in solid

$$V = \sqrt{\frac{Y}{\rho}} \approx 5000\ \textbf{m/s}$$

$$\nu = 440\ \textbf{Hz} \qquad \lambda = 11.4\ \textbf{m}$$

gravitational waves

$$V = c = 299792458\ \textbf{m/s}$$

$$\nu = 440\ \textbf{Hz} \qquad \lambda = 681\ \textbf{km}$$

Wave velocities

$$\frac{\omega}{k} = V \quad \textbf{(8)}$$

$$\nu \ \lambda = V \quad \textbf{(10)}$$

Examples II:

water wave (tsunami)

$V = 500$ **km/h**

$\nu = 3.3$ **/h**

$\lambda = 151$ **km**

light wave (elm)

$V = c = 299792458$ **m/s**

$\lambda = 700$ **nm**

$\nu = 4.2 \times 10^{14}$ **Hz**

2.1.2) The wave equation

Is there a general equation that governs wave behavior?

$$y(x,t) = A\sin\left(kx - \omega t\right)$$

We see:

$$\frac{\partial^2}{\partial x^2} y(x,t) = -k^2 A\sin\left(kx - \omega t\right) = -k^2\, y(x,t) \tag{11}$$

$$\frac{\partial^2}{\partial t^2} y(x,t) = -(-\omega)^2 A\sin\left(kx - \omega t\right) = -\omega^2\, y(x,t) \tag{12}$$

Wave equation

Any function:

$$y(x,t) = f(x - Vt)$$

Chain rule:

$$\frac{\partial^2}{\partial x^2} y(x,t) = (1)^2\, f''(x - Vt)$$

$$\frac{\partial^2}{\partial t^2} y(x,t) = (-V)^2\, f''(x - Vt)$$

Fulfills wave-equation: **(13)**

$$\frac{\partial^2}{\partial x^2} y(x,t) = \frac{1}{V^2} \frac{\partial^2}{\partial t^2} y(x,t)$$

The wave equation

$$\frac{\partial^2}{\partial x^2} y(x,t) = -k^2\, y(x,t) \quad \text{(11)}$$

$$\frac{\partial^2}{\partial t^2} y(x,t) = -\omega^2\, y(x,t) \quad \text{(12)}$$

$$y(x,t) = -\frac{1}{\omega^2}\frac{\partial^2}{\partial t^2} y(x,t)$$

With: $\frac{\omega}{k} = V$ (8) $\qquad \frac{k}{\omega} = \frac{1}{V}$

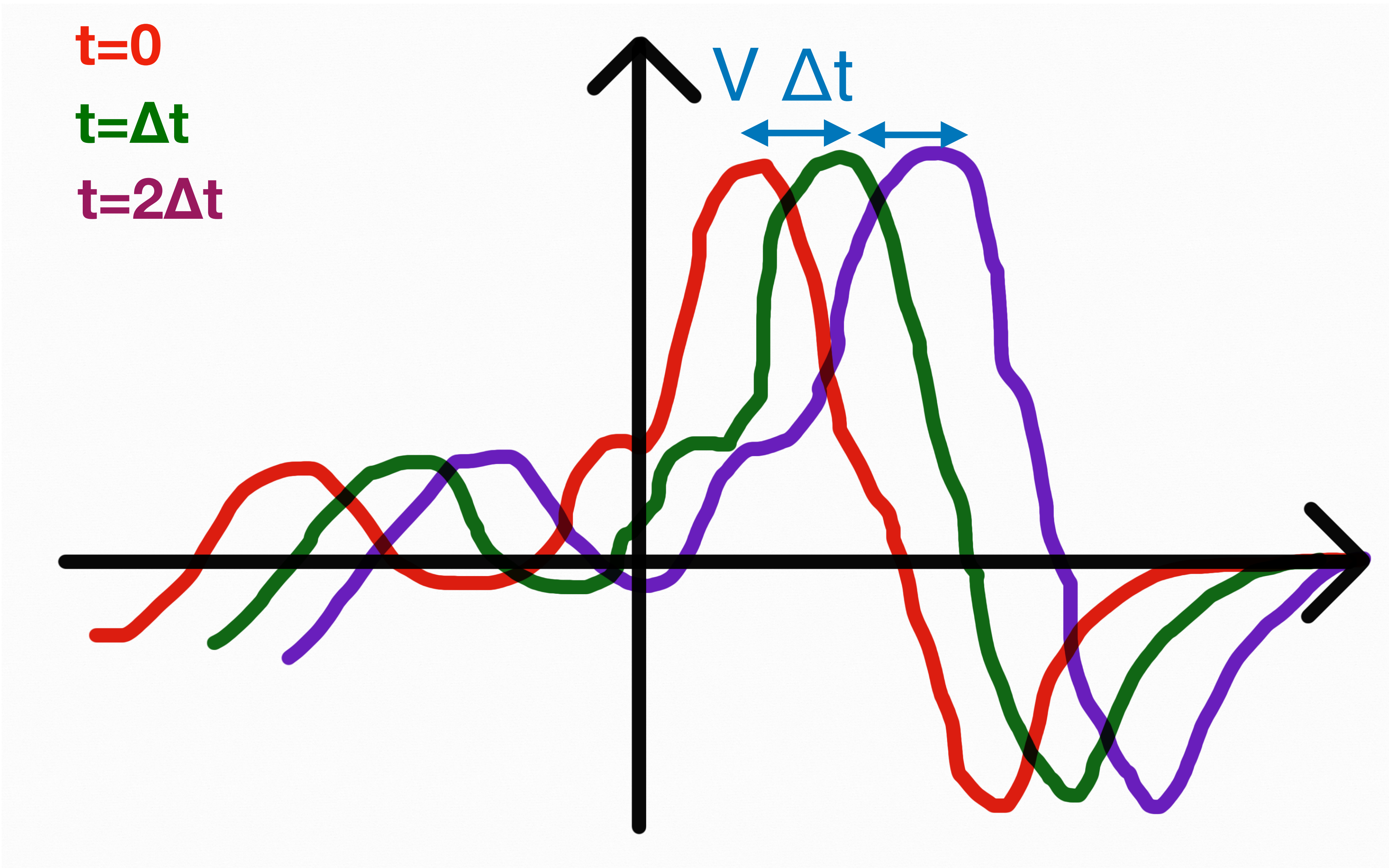
General **wave equation**

$$\frac{\partial^2}{\partial x^2} y(x,t) = \frac{1}{V^2}\frac{\partial^2}{\partial t^2} y(x,t) \quad \text{(13)}$$

- Change in time **causes** change in space **and vice versa**.

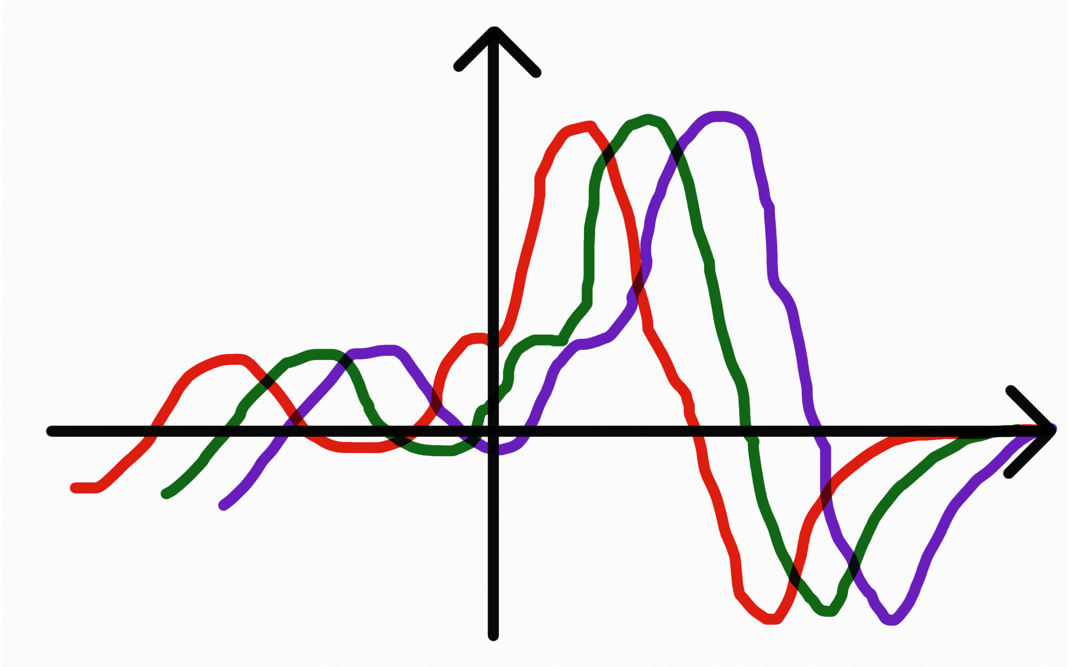
Wave equation

$y(x, t) = f(x - Vt)$ moves to the right with velocity V!!



Wave equation

$y(x,t) = f(x - Vt)$ moves to the right with velocity V!!



- $y(x,t) = f(x + vt)$ Moves to the left with velocity V, also fulfills wave equation

- Can be generalized to 2D, 3D

- There are many wave-equation**s**, one for each medium.

Superposition principle

The wave equation is linear. That means any **combination of waves** is also a **solution**

let:

$$\frac{\partial^2}{\partial x^2} y(x,t) = \frac{1}{V^2}\frac{\partial^2}{\partial t^2} y(x,t)$$

$$\frac{\partial^2}{\partial x^2} w(x,t) = \frac{1}{V^2}\frac{\partial^2}{\partial t^2} w(x,t)$$

Then:

$$\frac{\partial^2}{\partial x^2}\left[y(x,t) + w(x,t)\right] = \frac{1}{V^2}\frac{\partial^2}{\partial t^2}\left[y(x,t) + w(x,t)\right]$$

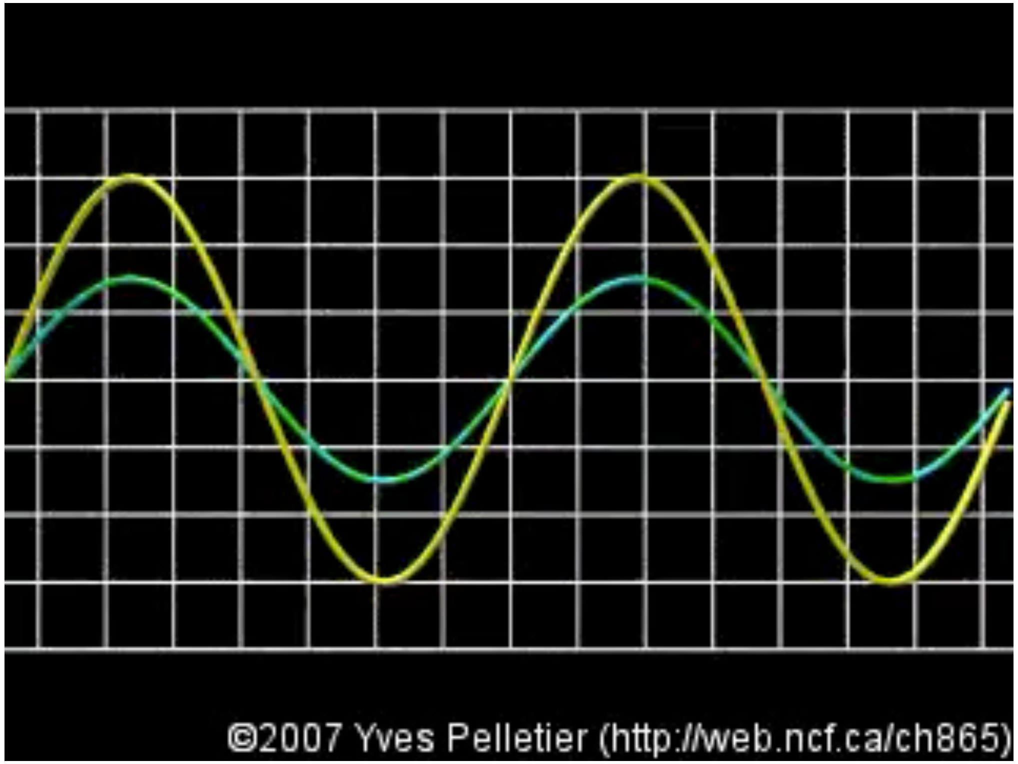
2.1.3) Standing waves

What happens if we combine two identical waves travelling in opposite directions?

https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html

2.1.3) Standing waves

What happens if we combine two identical waves travelling in opposite directions?



Animation from: https://www.youtube.com/watch?v=ic73oZoqr70

Using (6), we can write this as:

$$y(x,t) = A\sin\left(kx - \omega t\right) + A\sin\left(-kx - \omega t\right)$$

Standing waves

$$y(x,t) = A\sin\left(kx - \omega t\right) + A\sin\left(-kx - \omega t\right)$$

Trigonometric identity

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \qquad \textbf{(14)}$$

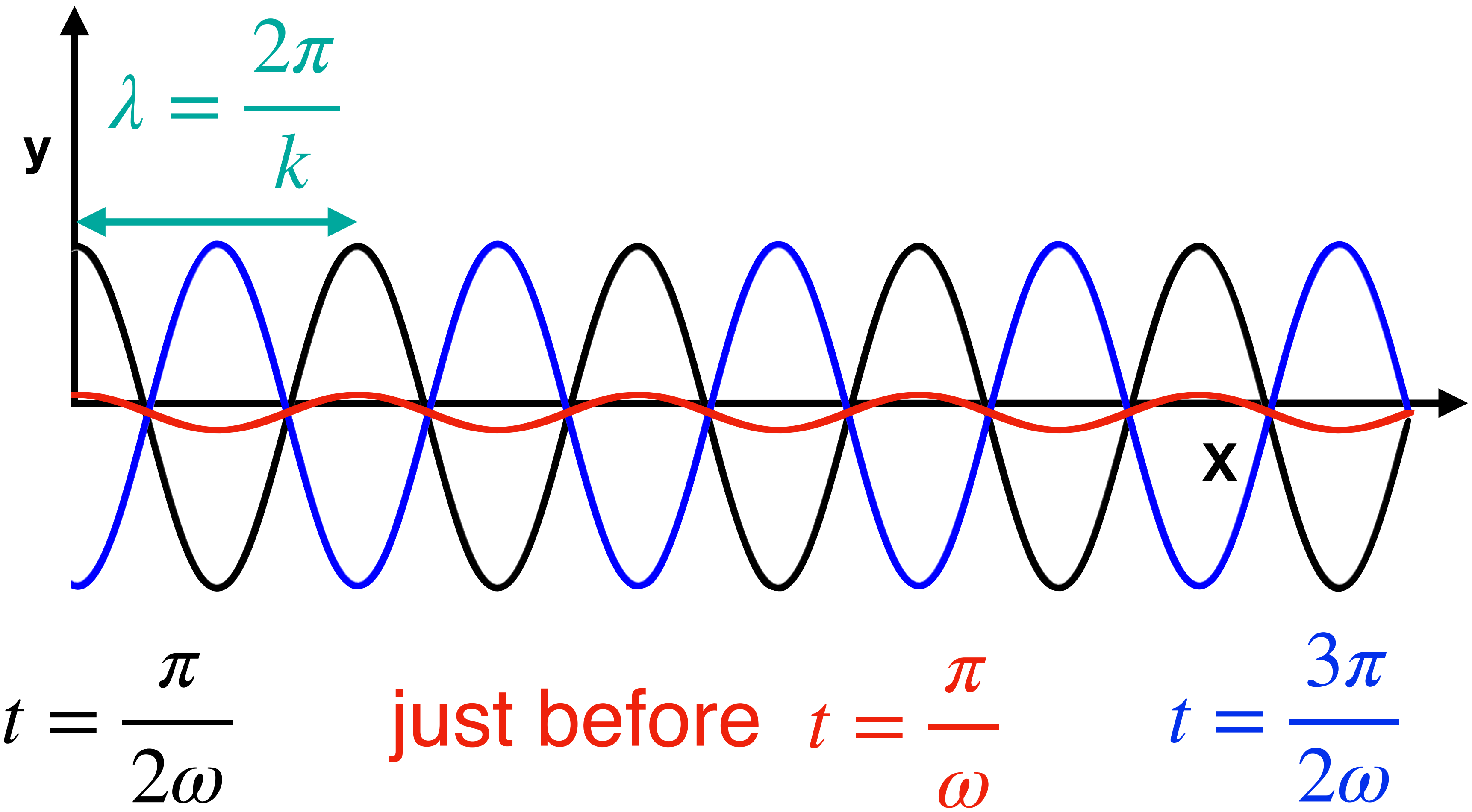
$$\begin{aligned} y(x,t) = A\Big[&\cancel{\sin(kx)\cos(\omega t)} - \cos(kx)\sin(\omega t) \\ &+ \cancel{\underbrace{\sin(-kx)}_{-\sin(kx)}\cos(\omega t)} - \underbrace{\cos(-kx)}_{\cos(kx)}\sin(\omega t)\Big] \end{aligned}$$

$$y(x,t) = -2A\cos(kx)\ \sin(\omega t)$$

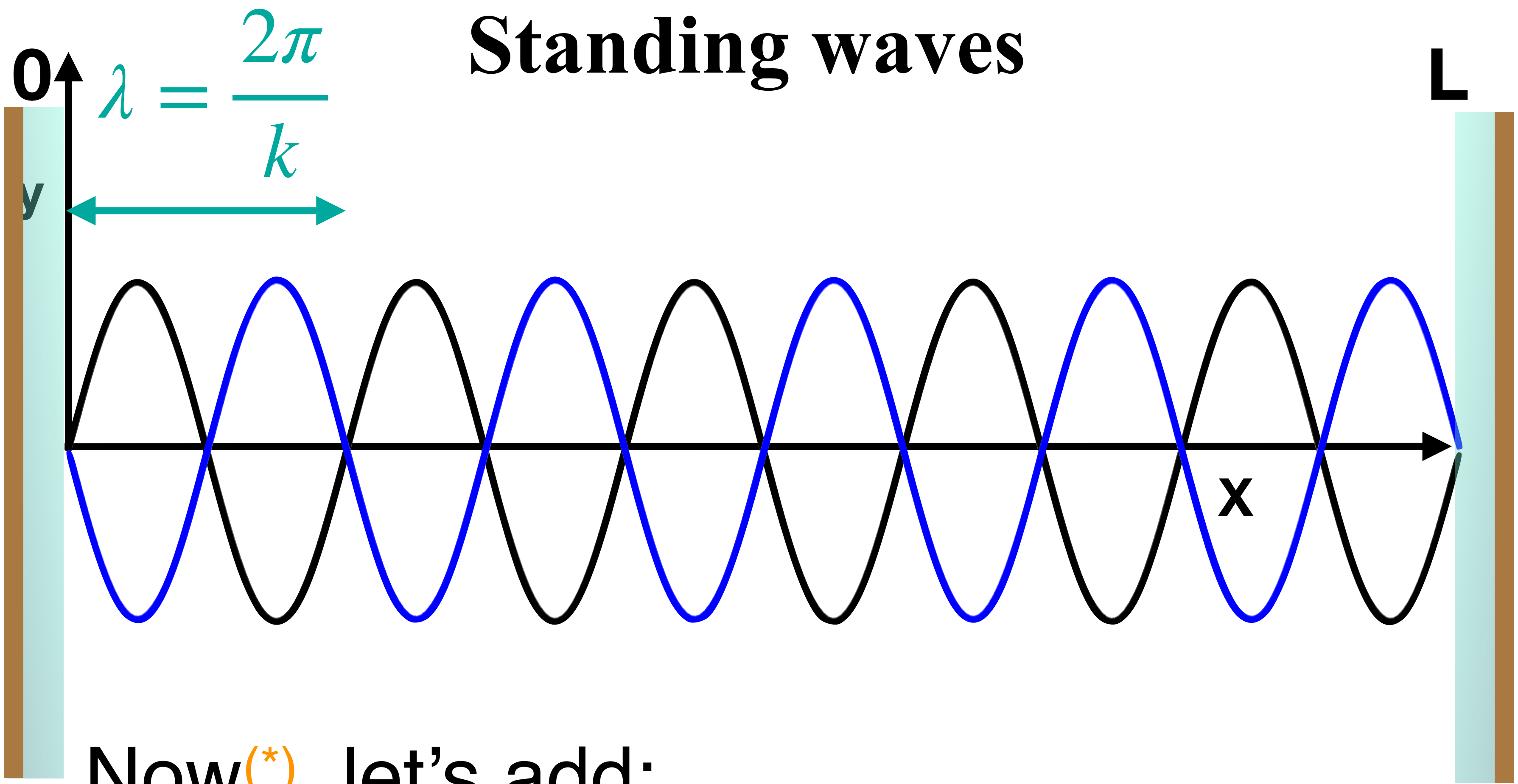
Standing waves

Formula for some **standing wave**

$$y(x,t) = \tilde{A}\cos(\boxed{k}x)\ \sin(\boxed{\omega}t) \quad (15)$$



Standing waves



Now(*) let's add:

Boundary condition: $y(0,t) = y(L,t) = 0$ **(16b)**

Resonance condition for **standing wave**

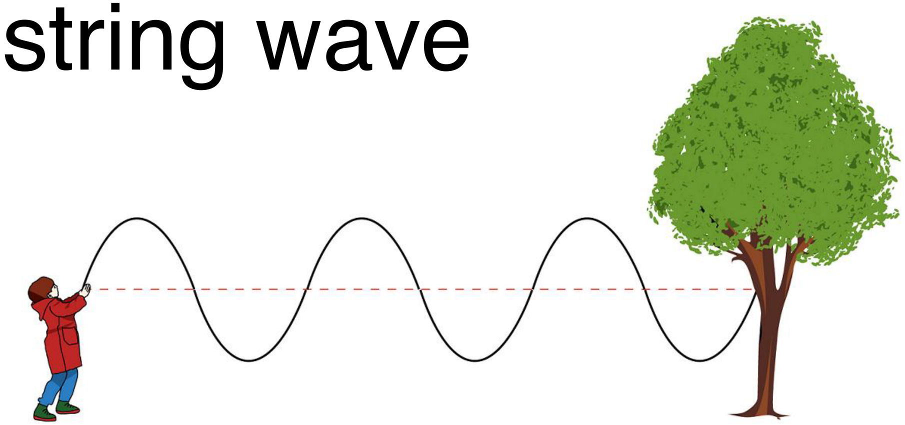
$$L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad n = 1,2,3\ldots \qquad (16)$$

* Q: Eq. (15) is an example that does not fulfill Eq. (16b). Find another example that does.

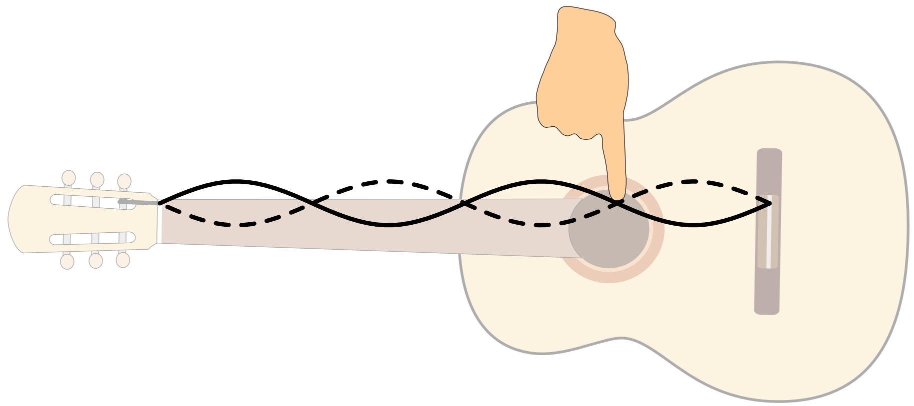
Standing waves

Examples:

Backreflected string wave



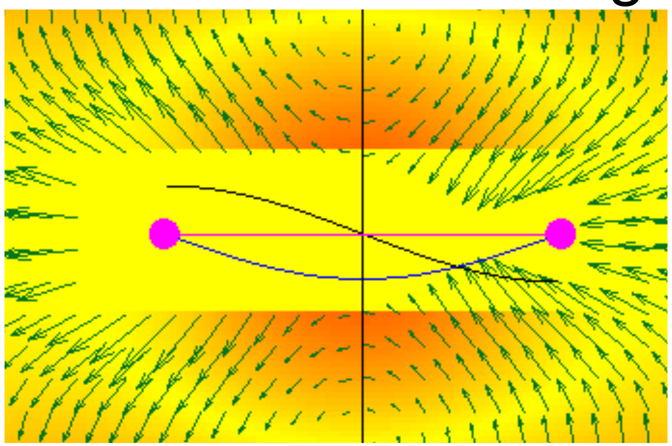
Musical instruments



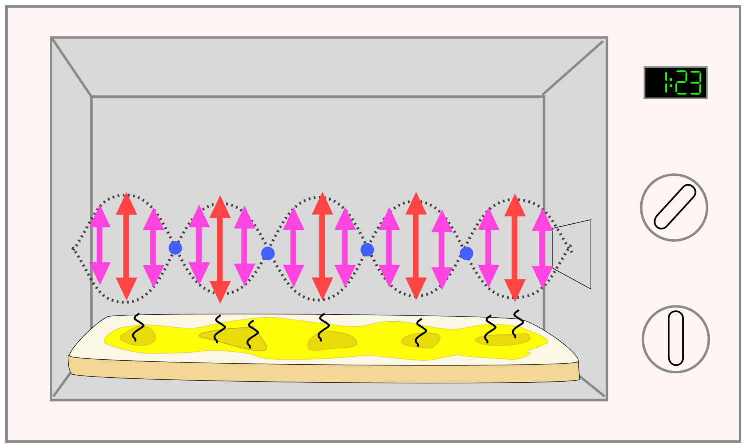
http://whatmusicreallyis.com/research/physics/

Antenna

current
charge



Micro-wave oven

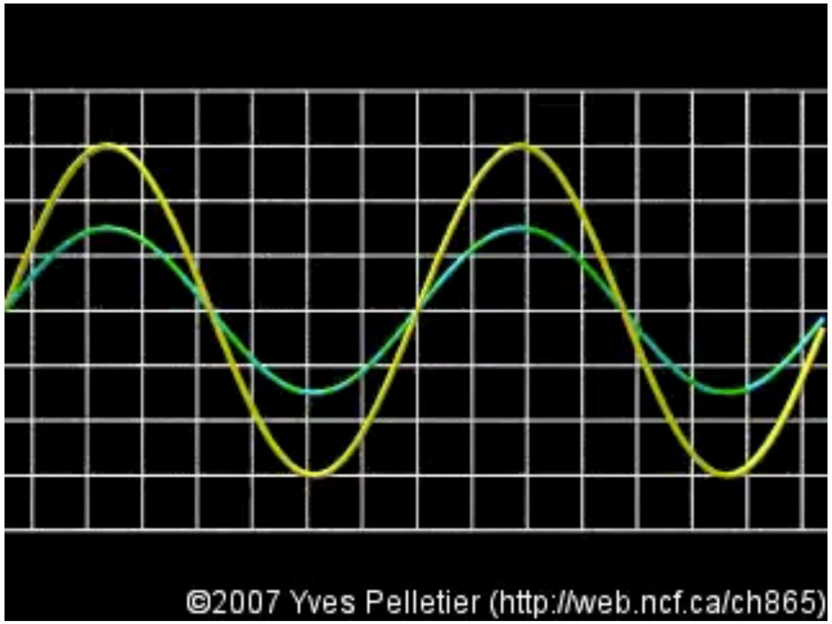


2.1.4) Phenomena characteristic for waves

Interference

Superposition principle: Waves taking different paths get added.

Standing wave: example where superimposed waves always cancel at anti-node:



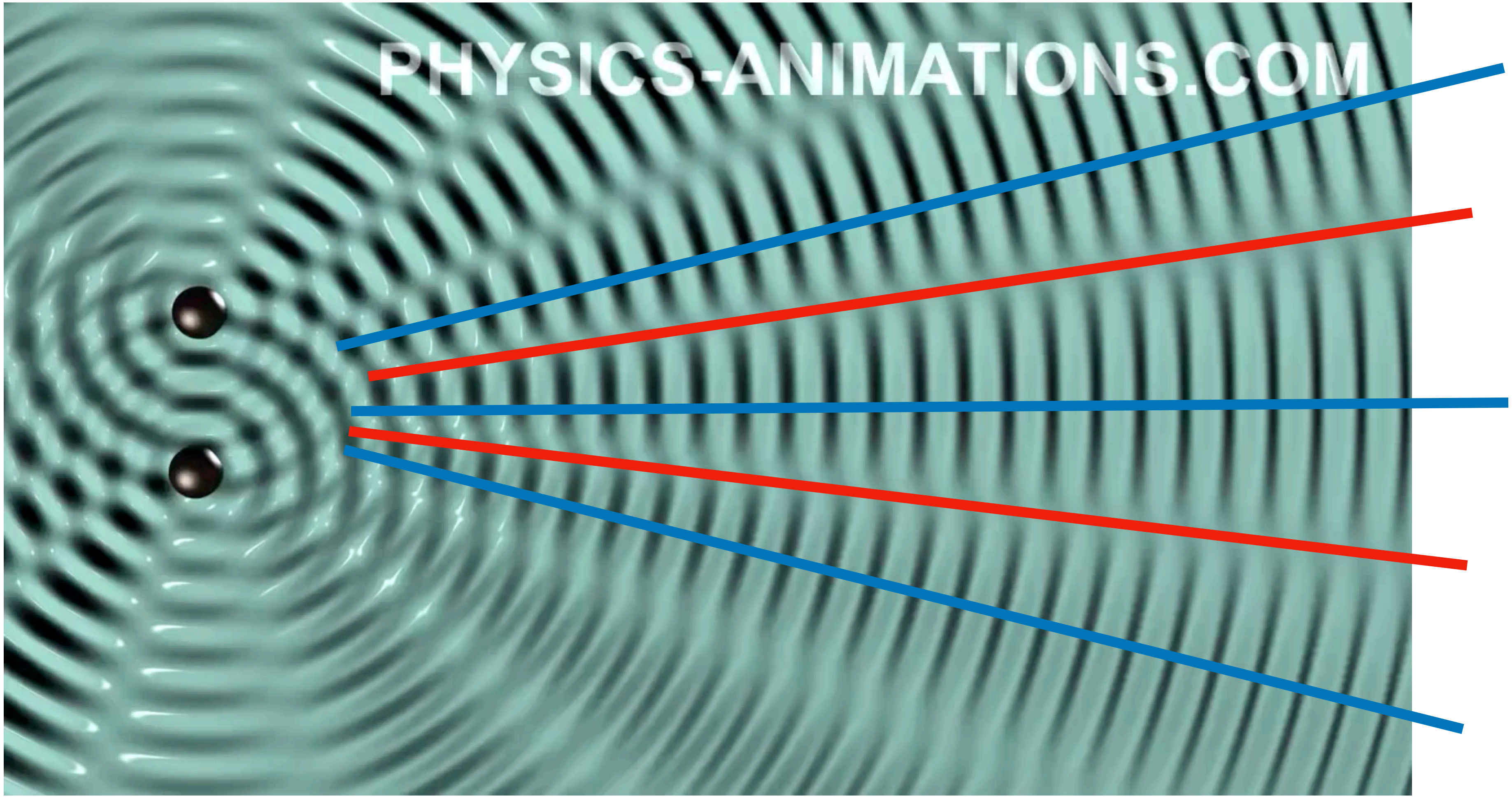
Interference

Usually (2D, 3D) more options:

Circular waves on a water surface

Interference

Usually (2D, 3D) more options:



https://www.youtube.com/watch?v=ovZkFMuxZNc

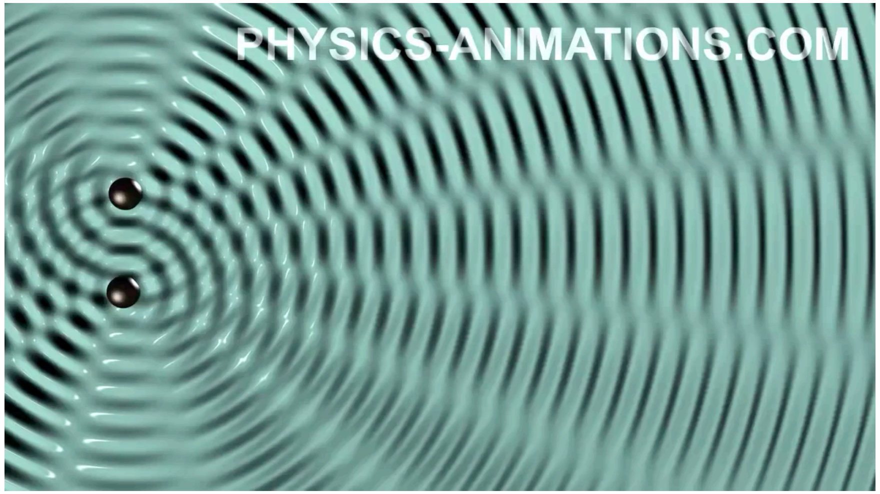
Two circular waves: strengthen cancel

Interference

Waves can show **interference**

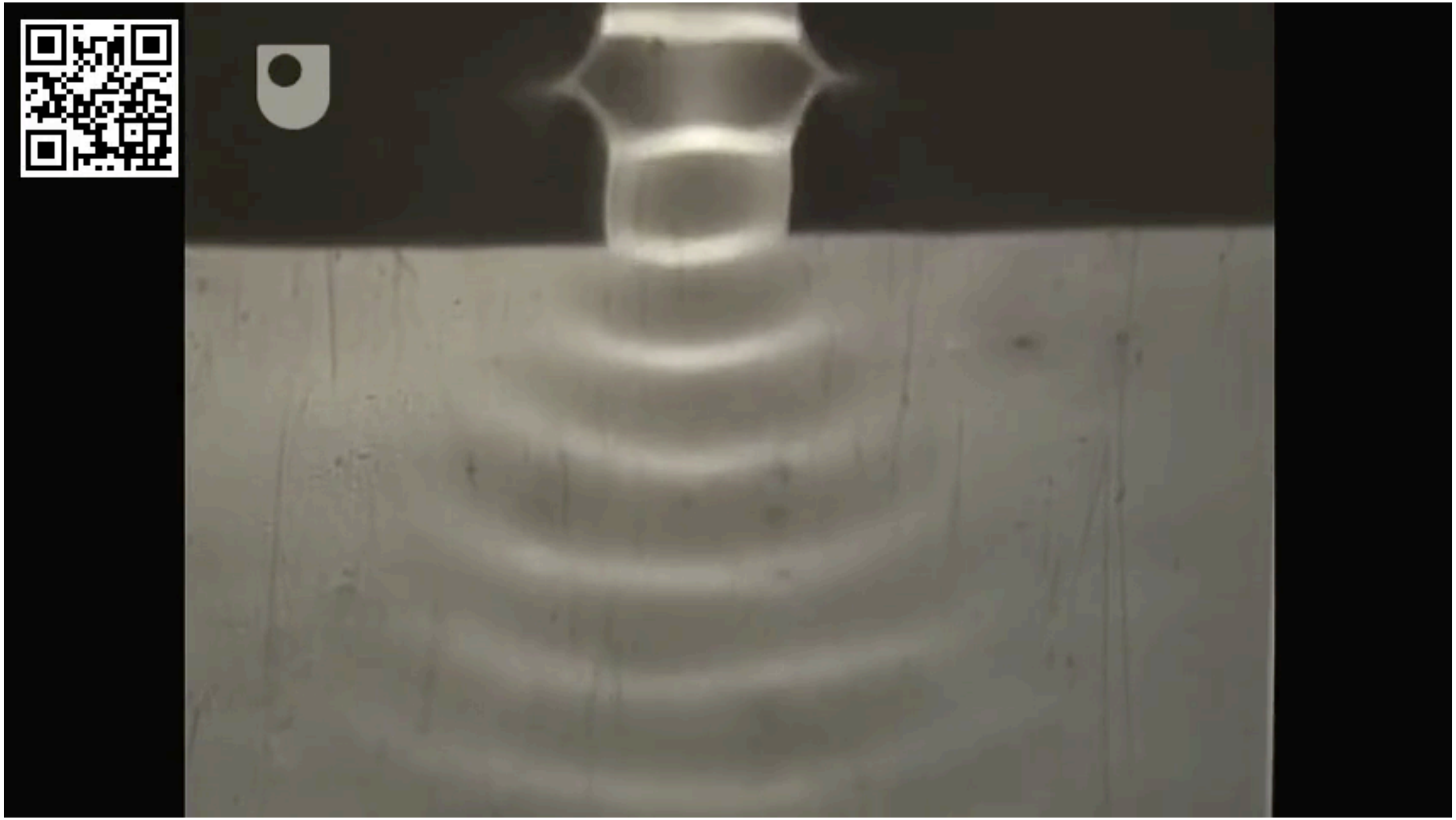
-strengthening in certain directions/ at certain times: **constructive** interference

-weakening in certain directions/ at certain times: **destructive** interference



Diffraction

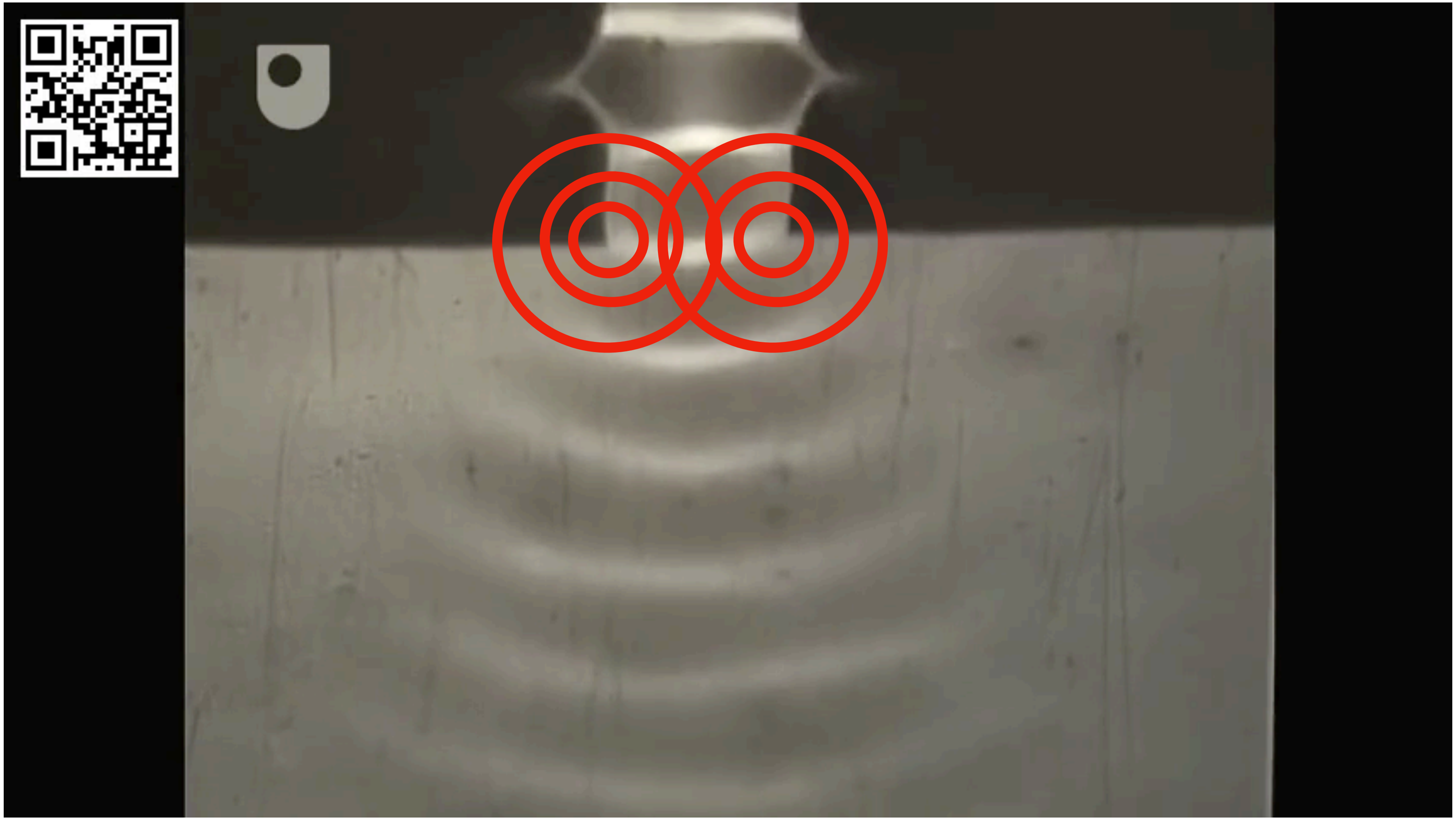
Waves can turn around corners:



https://www.youtube.com/watch?v=BH0NfVUTWG4

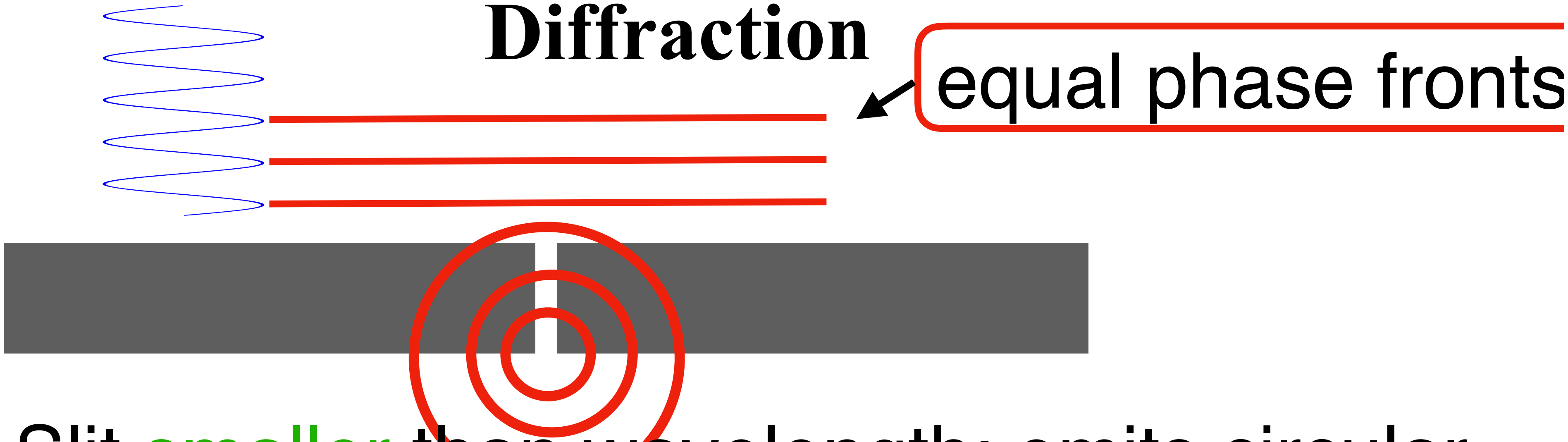
Diffraction

Decompose wave into lots of spherical waves:

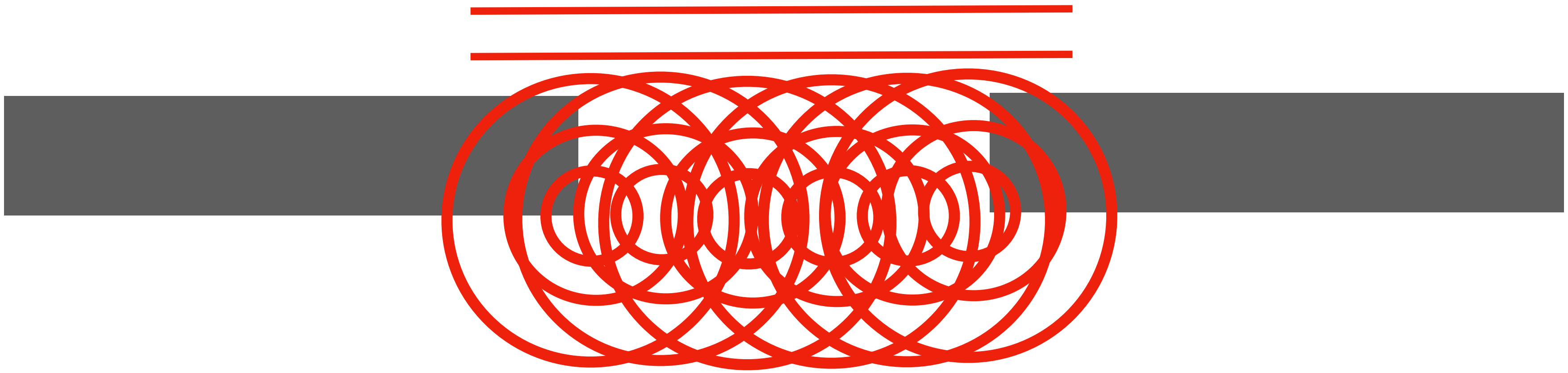


Could see this from 2D wave equation

Diffraction

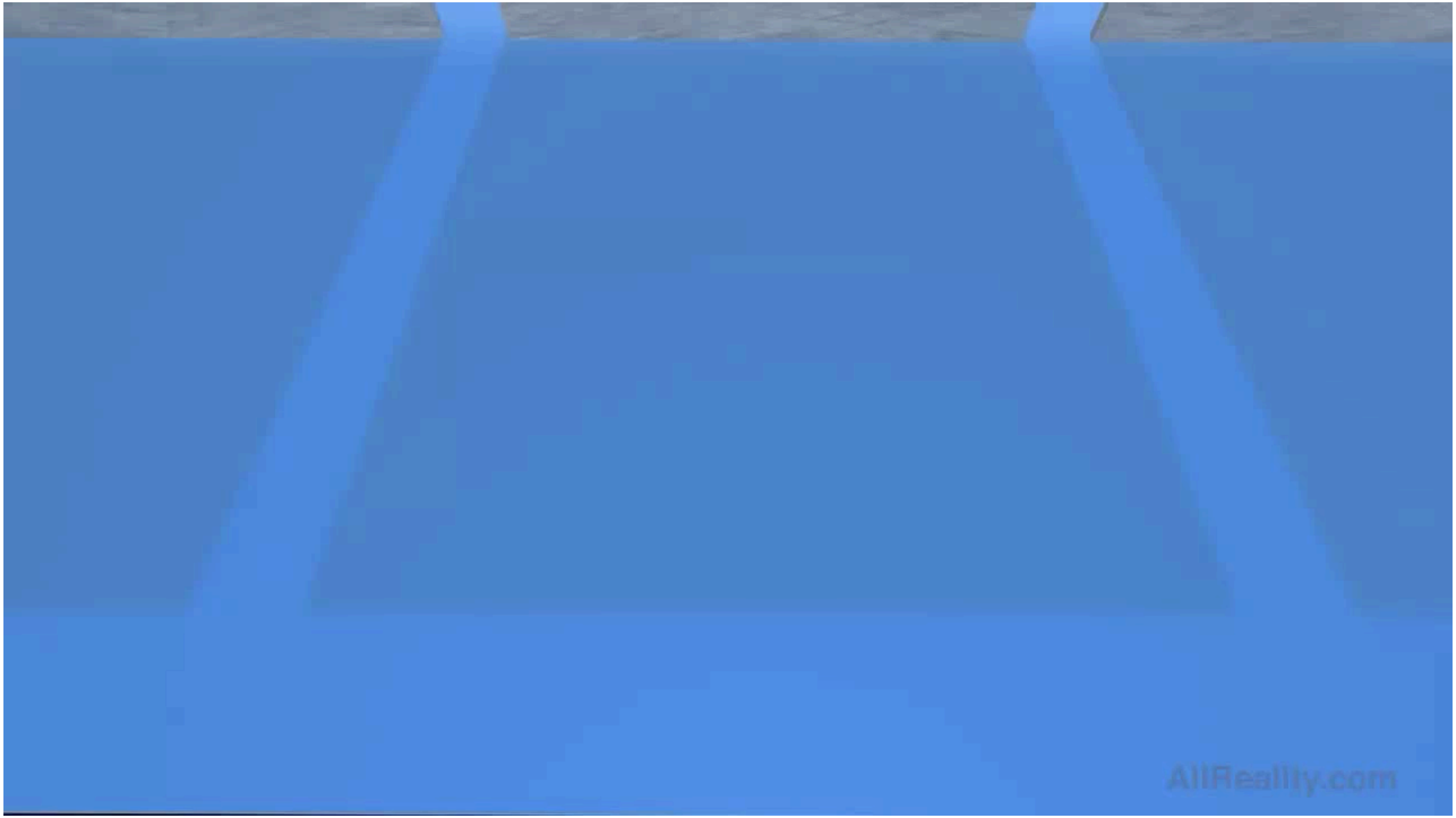


Slit smaller than wavelength: emits circular waves going in **ALL** directions



Slit larger than wavelength: waves destructively interfere if direction not almost forward *(tutorial, waves and optics course)*

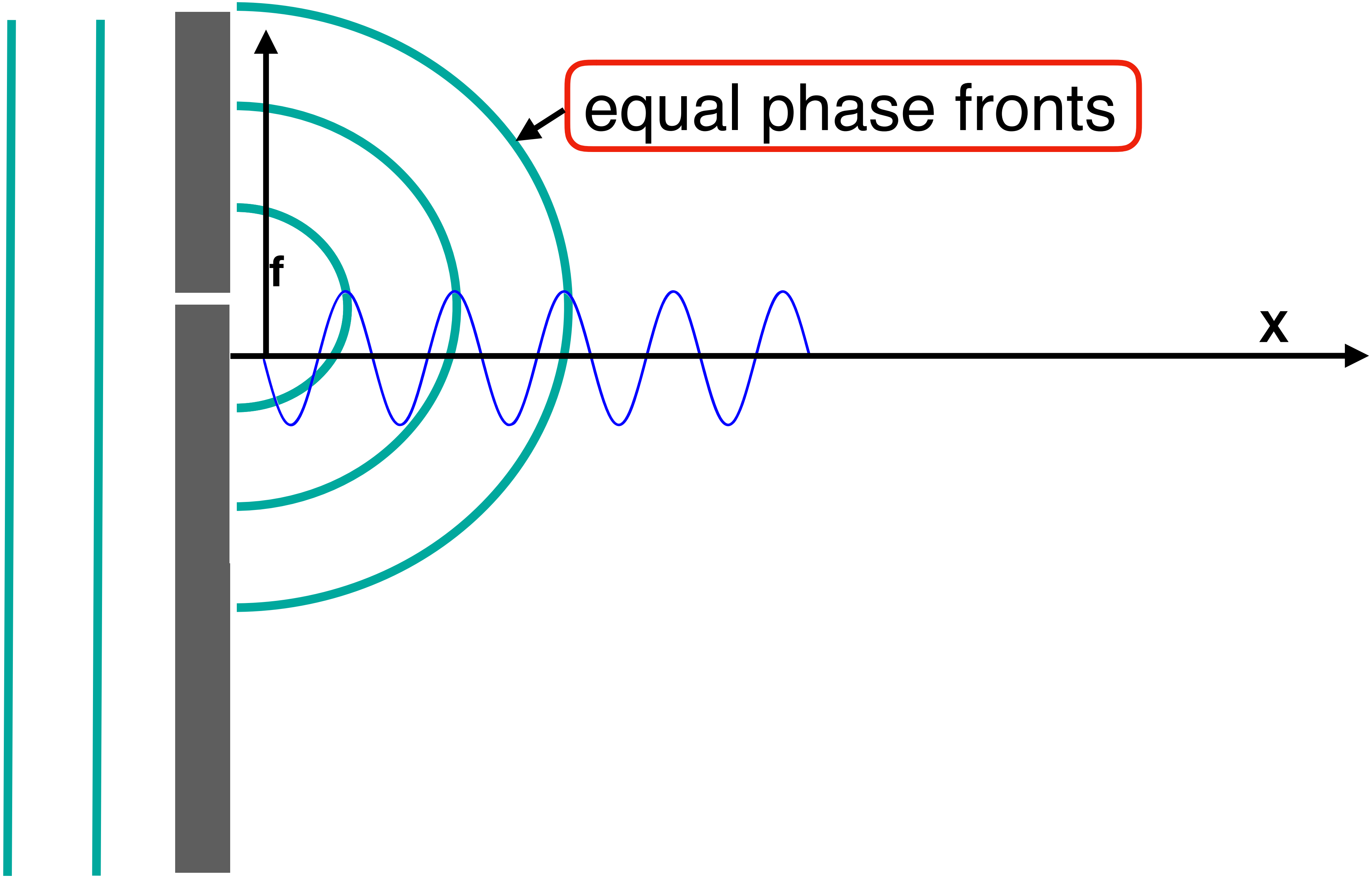
Diffraction and Interference



Double slit interference

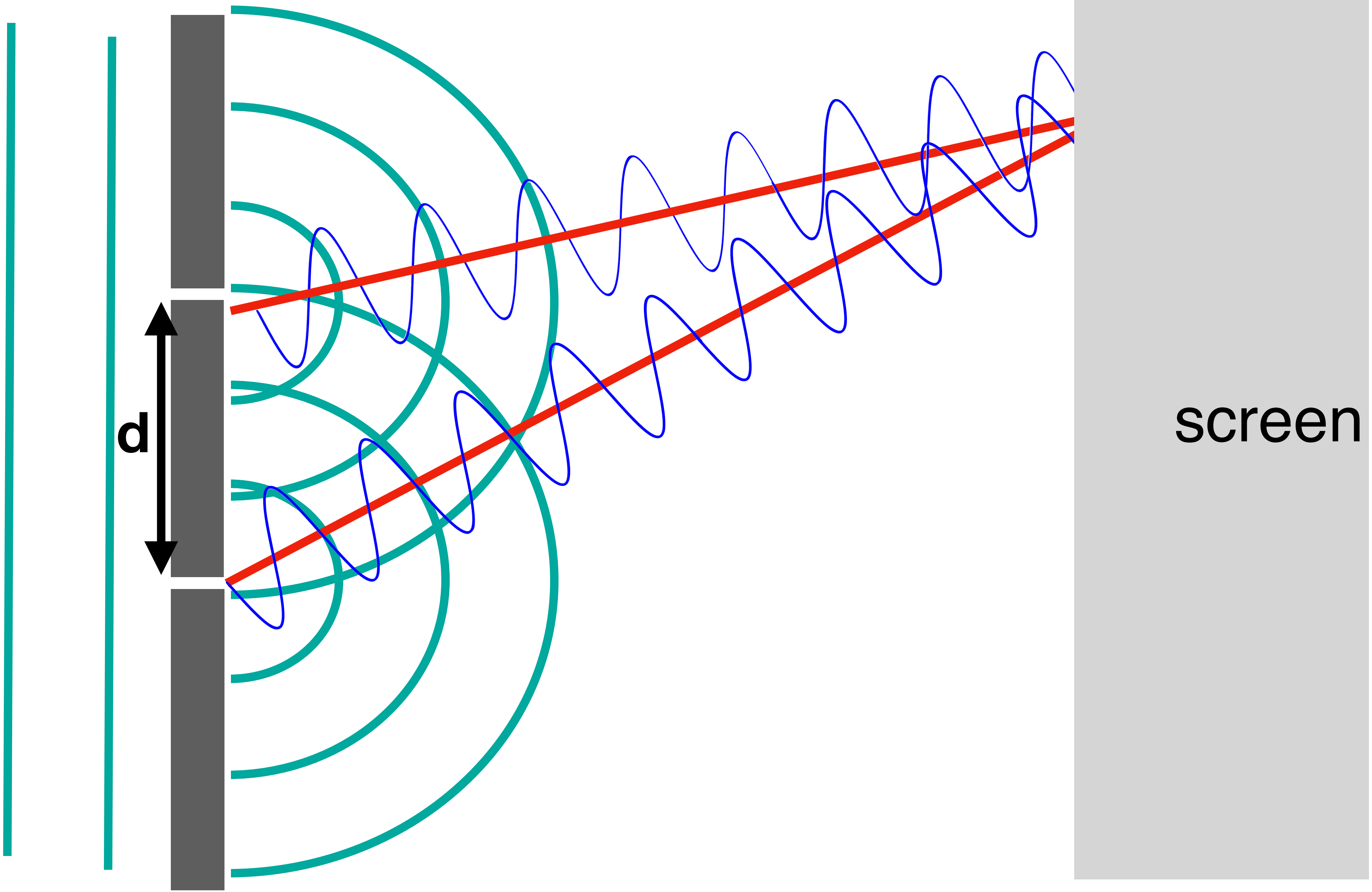
Diffraction and Interference

Double slit interference



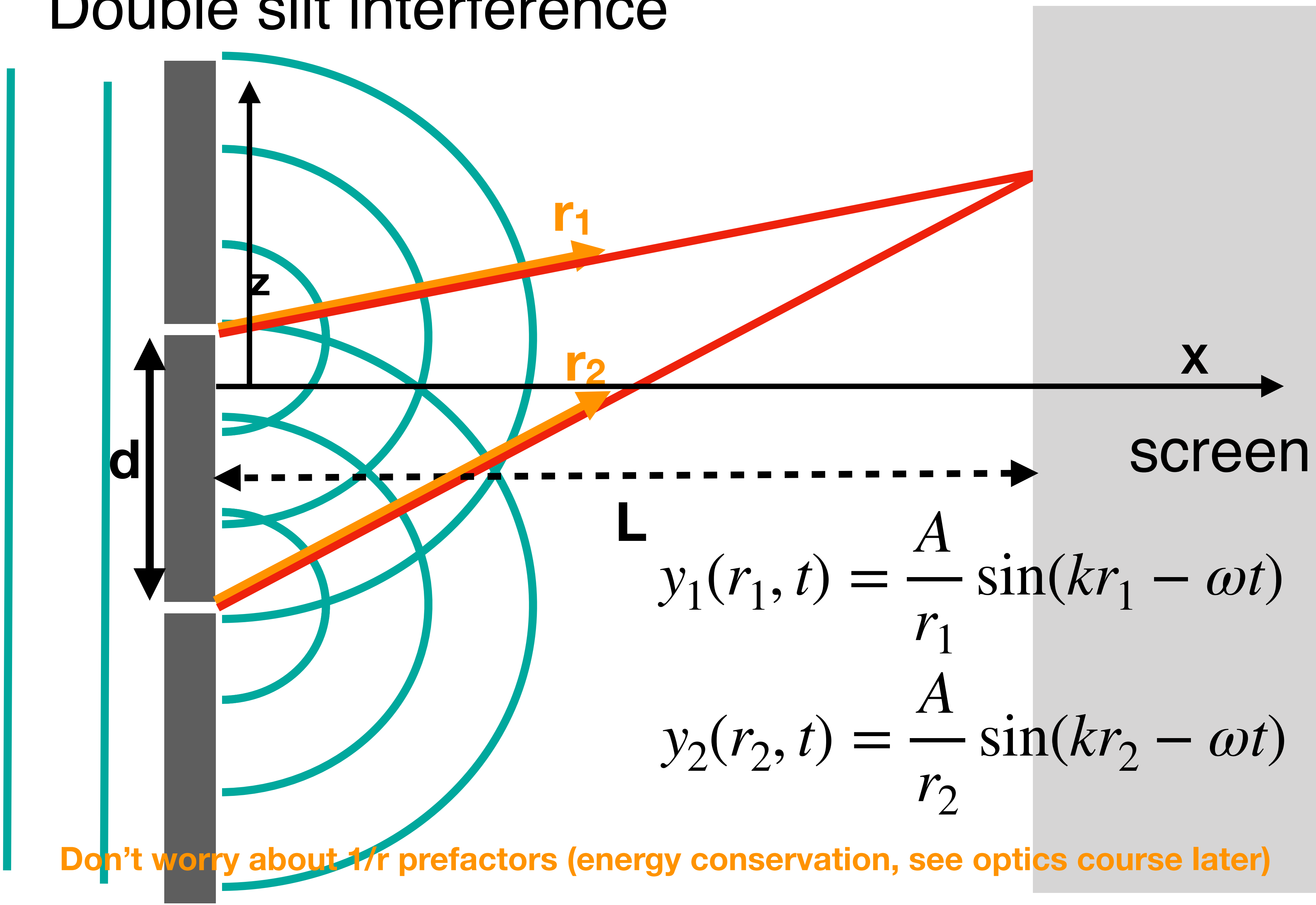
Diffraction and Interference

Double slit interference



Diffraction and Interference

Double slit interference



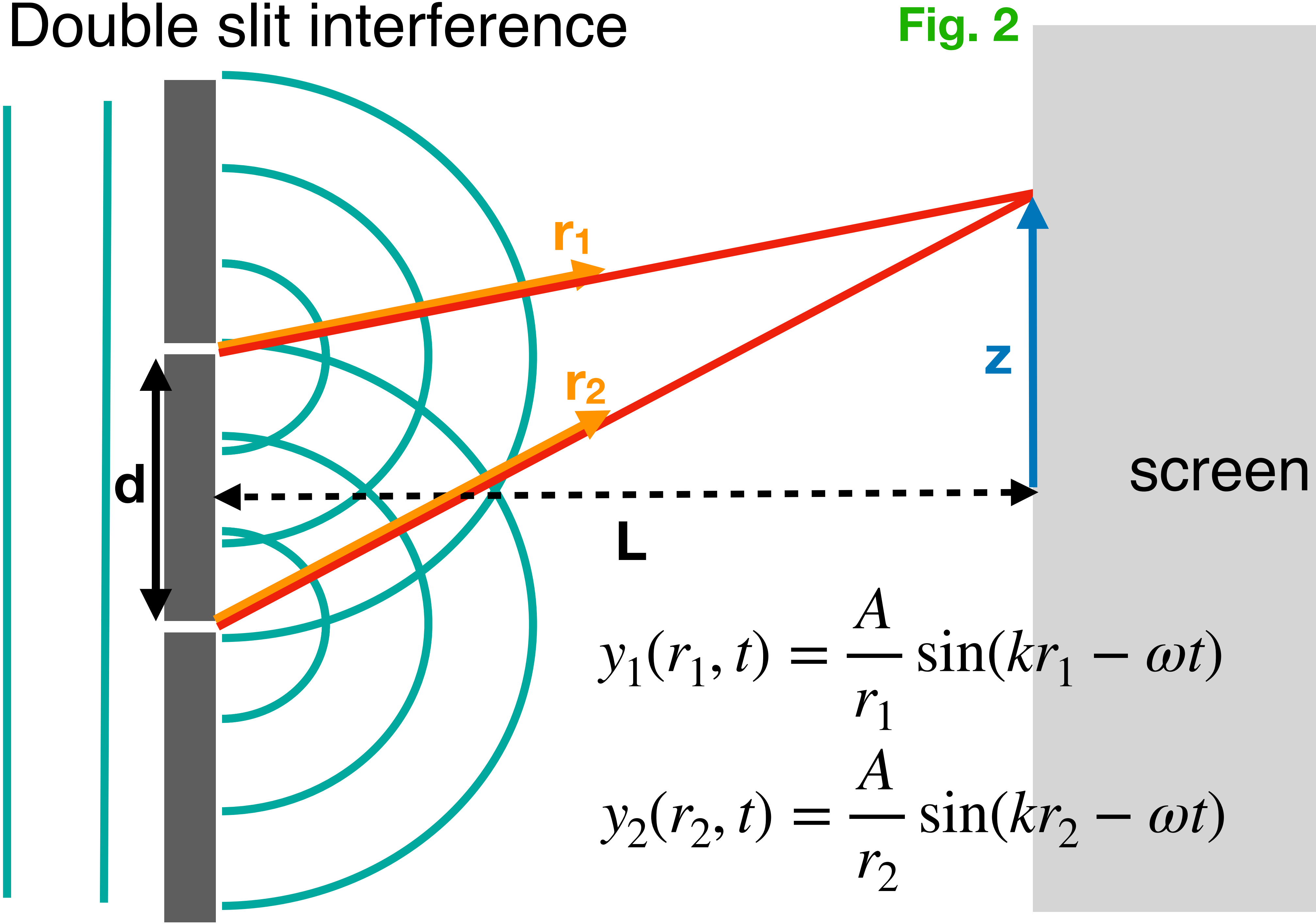
$$y_1(r_1, t) = \frac{A}{r_1}\sin(kr_1 - \omega t)$$

$$y_2(r_2, t) = \frac{A}{r_2}\sin(kr_2 - \omega t)$$

Don't worry about 1/r prefactors (energy conservation, see optics course later)

Diffraction and Interference

Double slit interference



$$y_1(r_1, t) = \frac{A}{r_1}\sin(kr_1 - \omega t)$$

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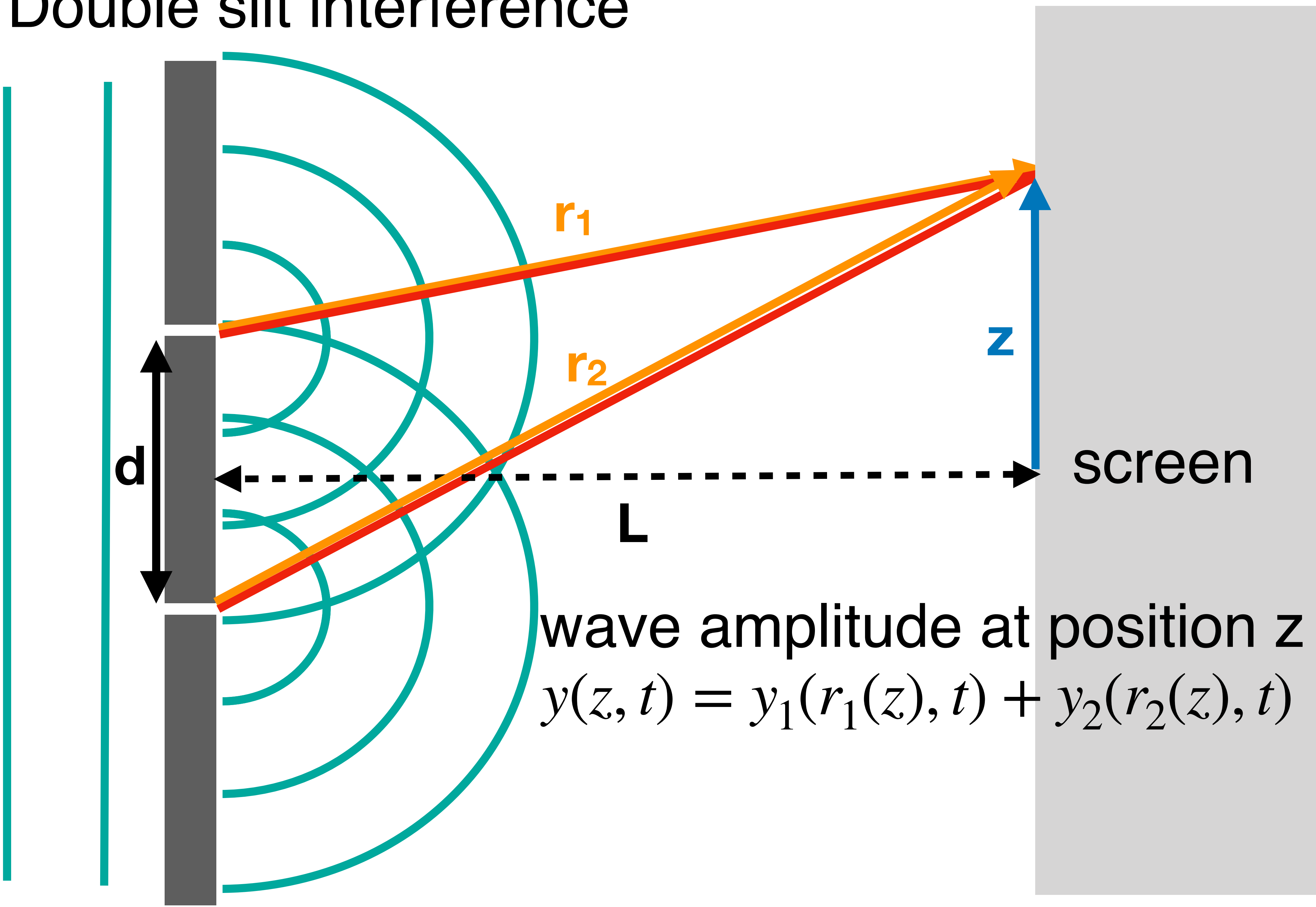
Diffraction and Interference

Double slit interference

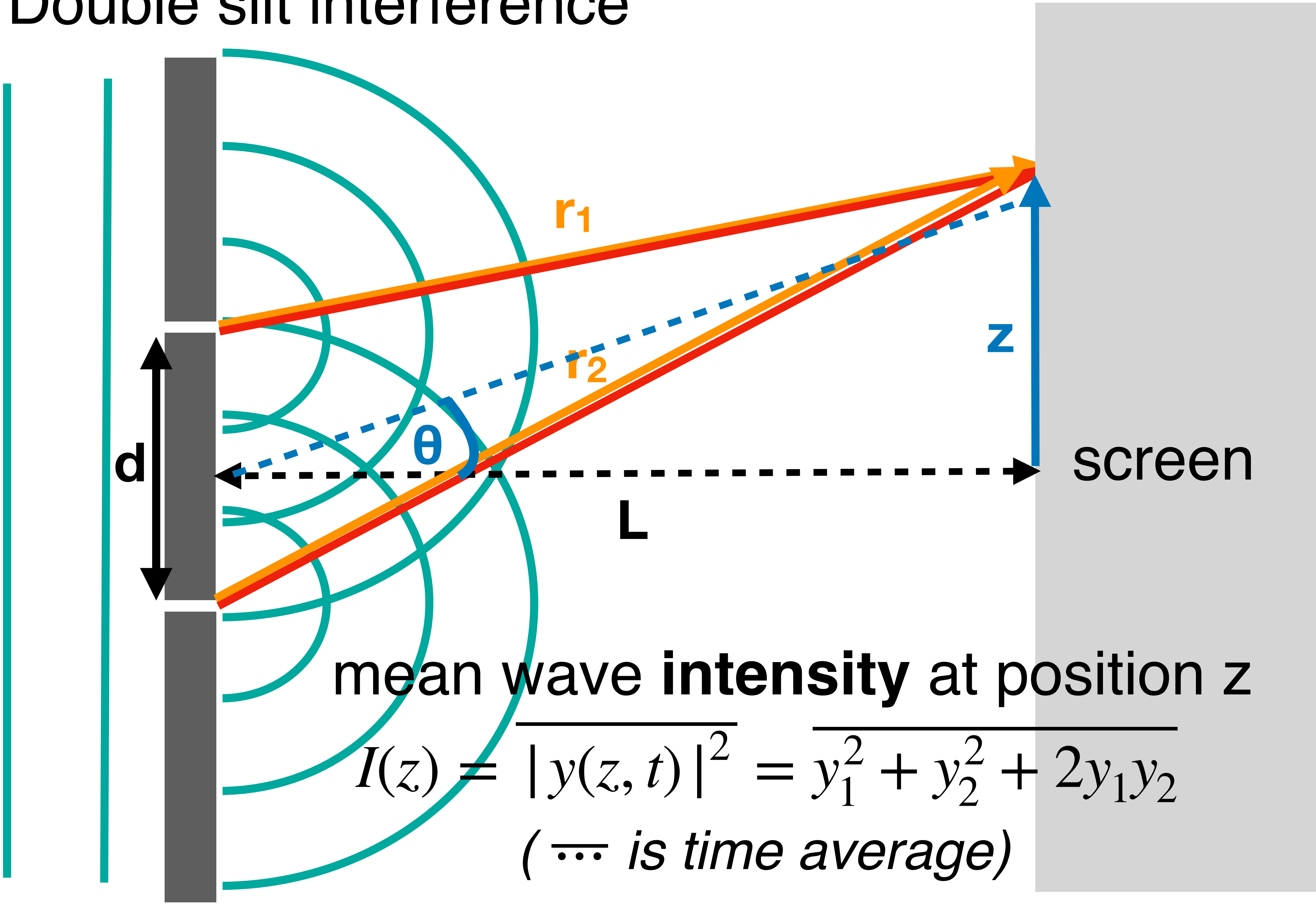


wave amplitude at position z

$$y(z, t) = y_1(r_1(z), t) + y_2(r_2(z), t)$$

Diffraction and Interference

Double slit interference

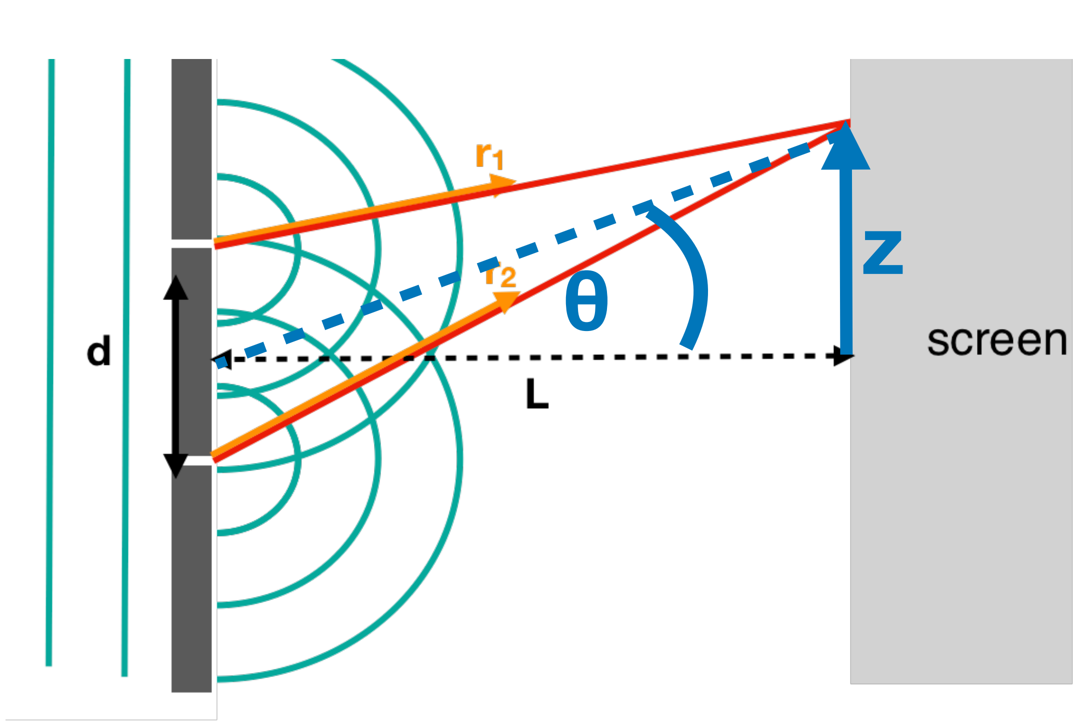


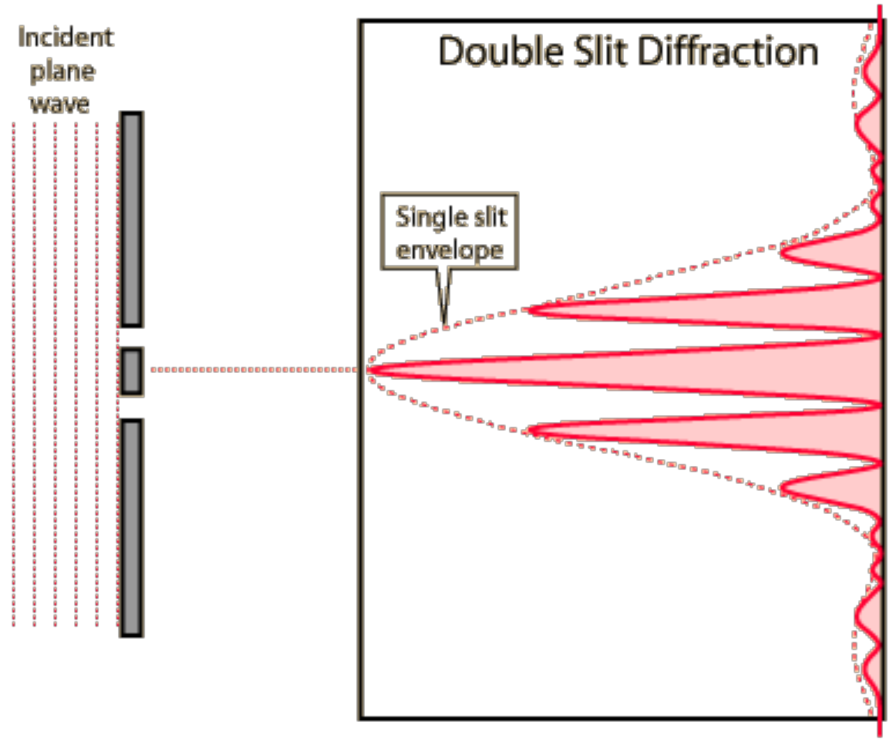
mean wave **intensity** at position z

$$I(z) = \overline{|y(z,t)|^2} = \overline{y_1^2 + y_2^2 + 2y_1y_2}$$

($\overline{\cdots}$ is time average)

Diffraction and Interference

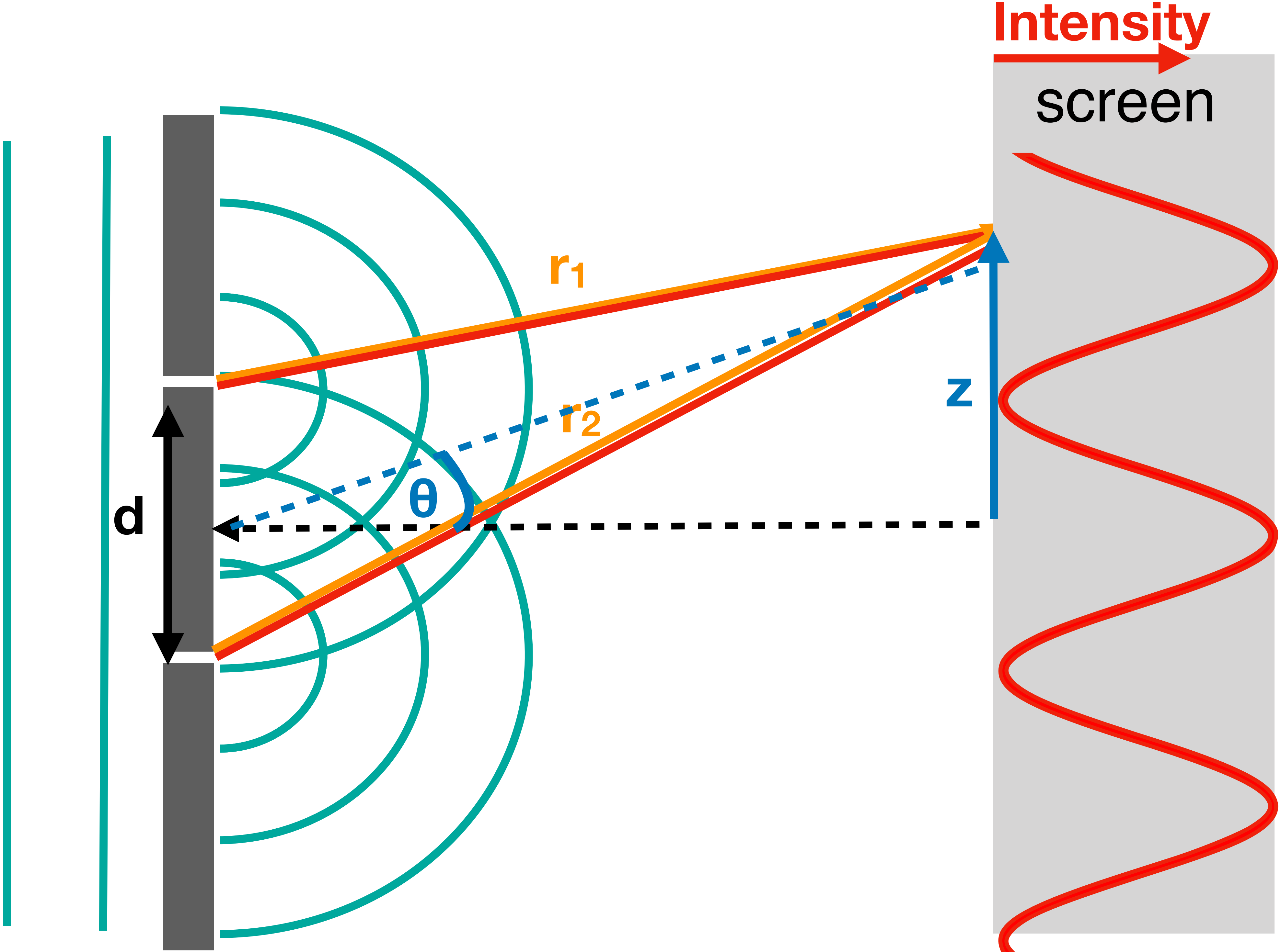




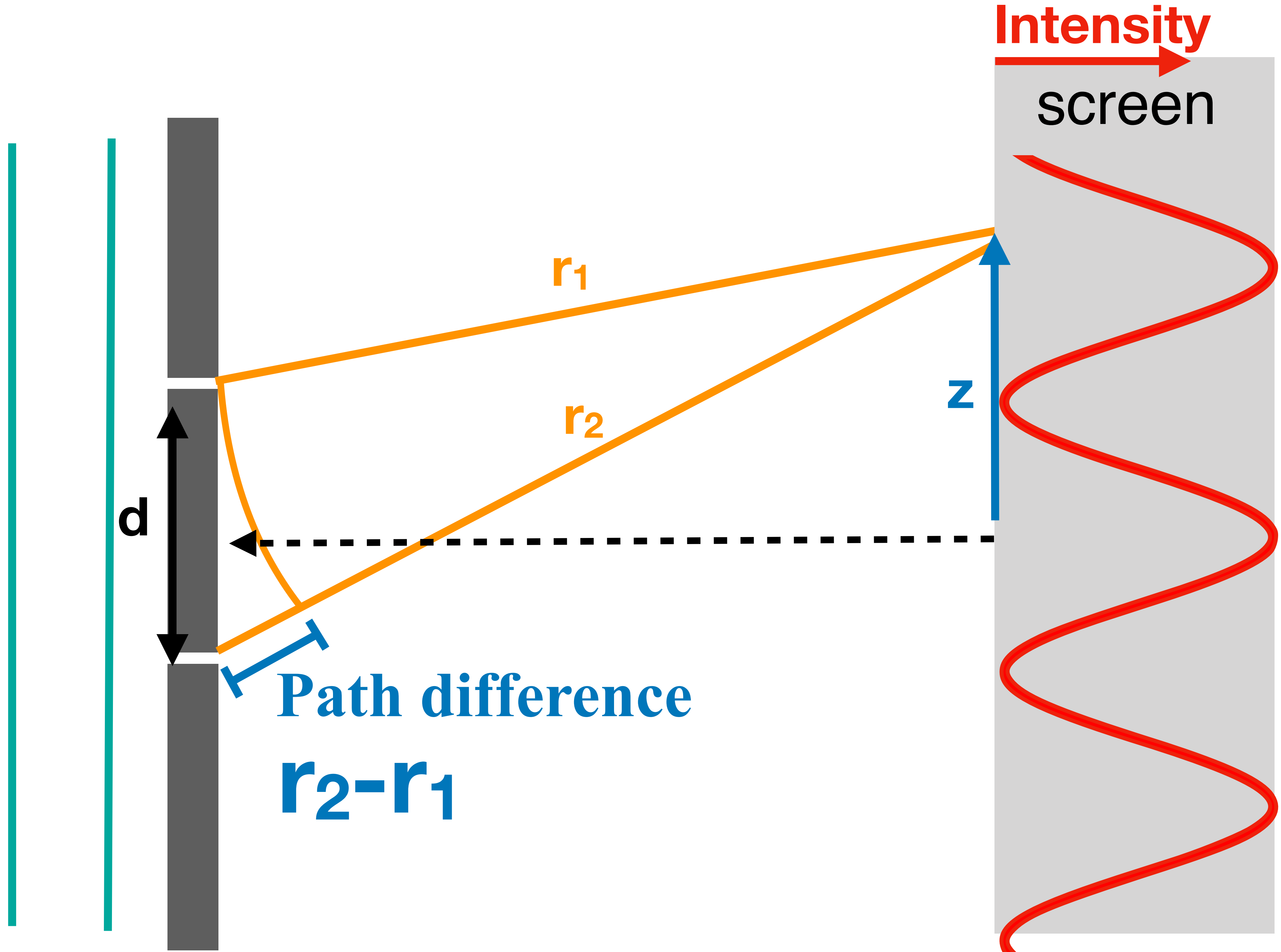
double slit **interference pattern**

$$I(\theta) \approx I_0 \cos(\pi d \frac{\sin\theta}{\lambda}) \qquad (17)$$

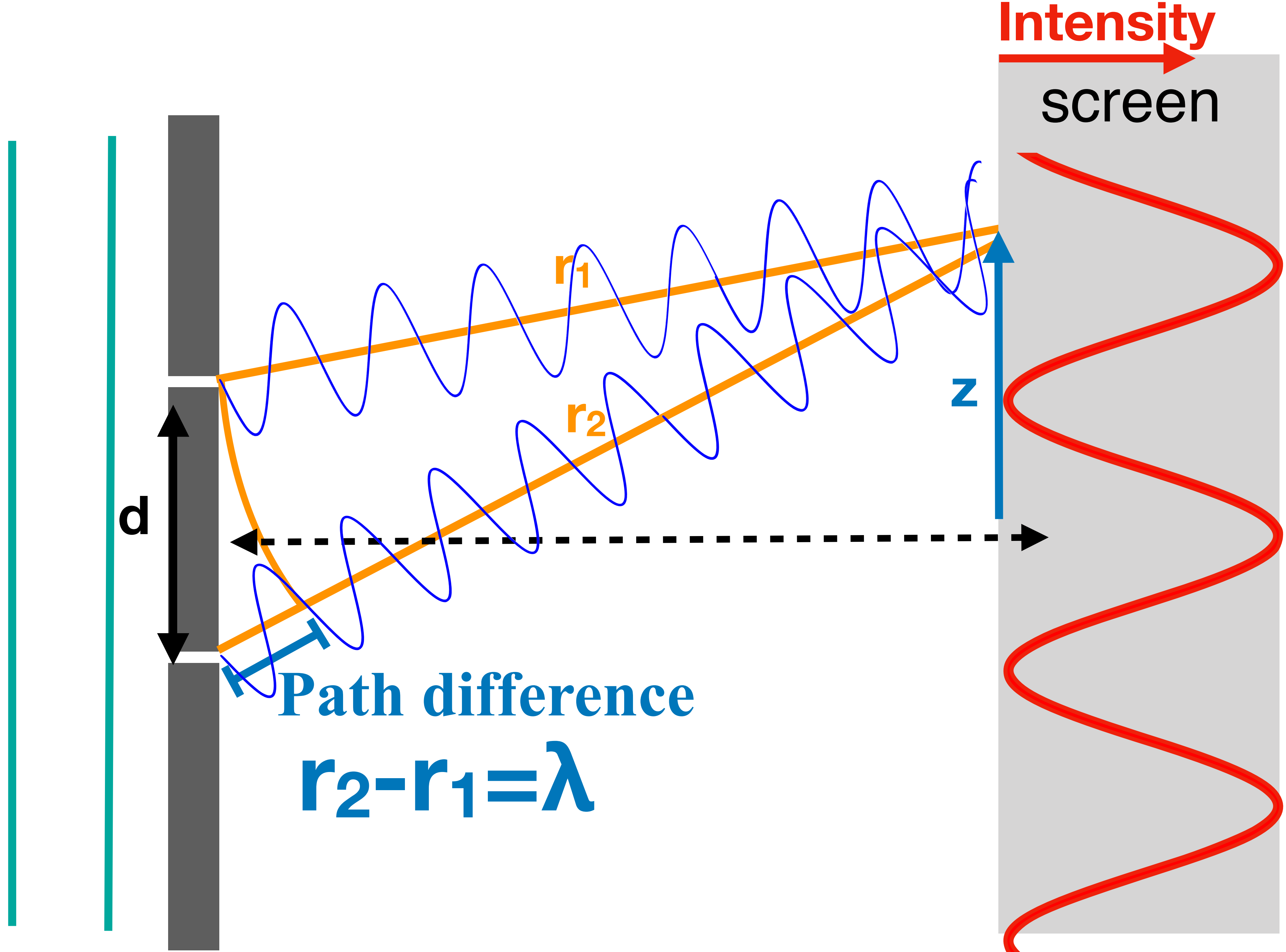
Diffraction and Interference



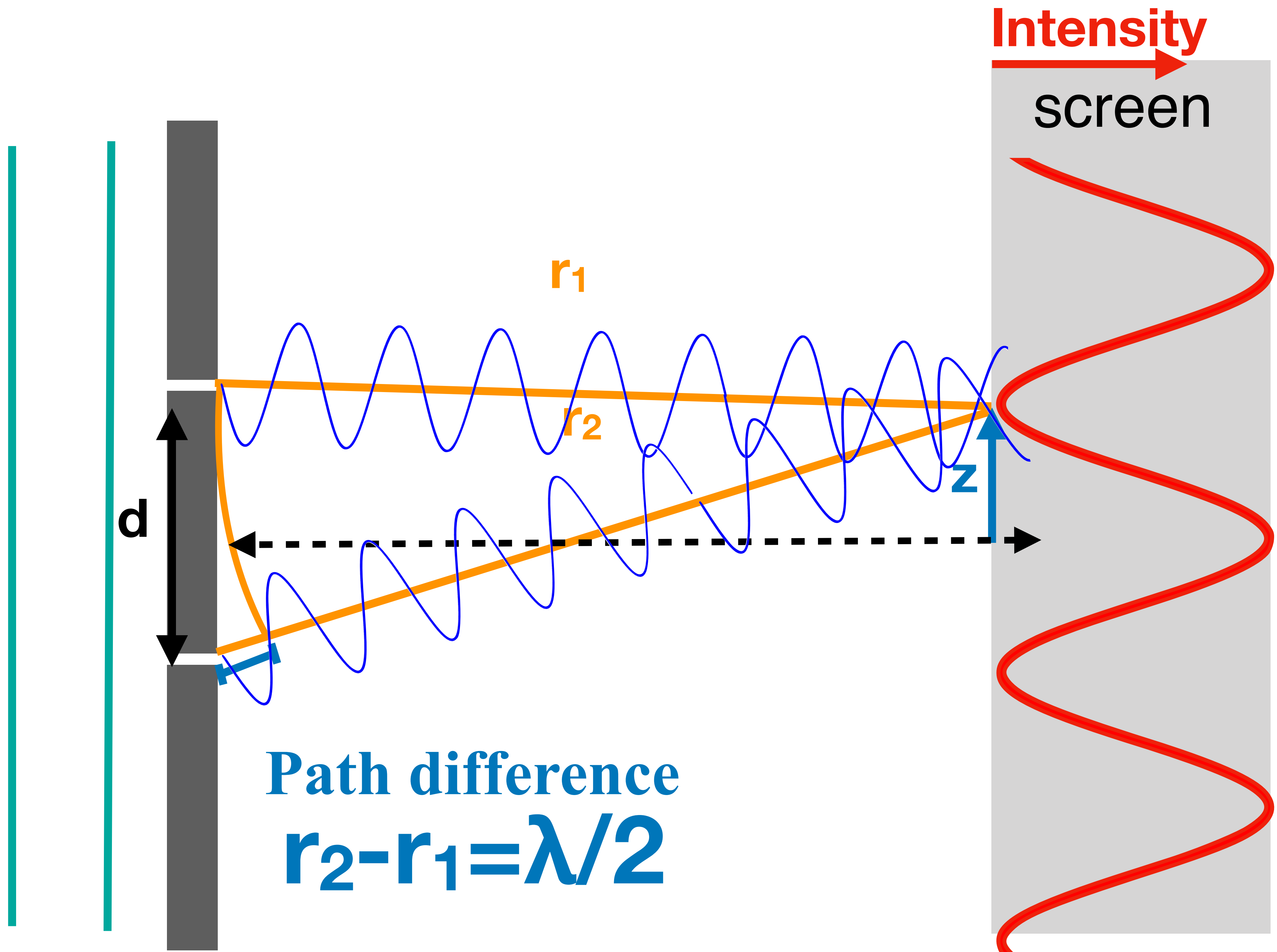
Diffraction and Interference



Diffraction and Interference



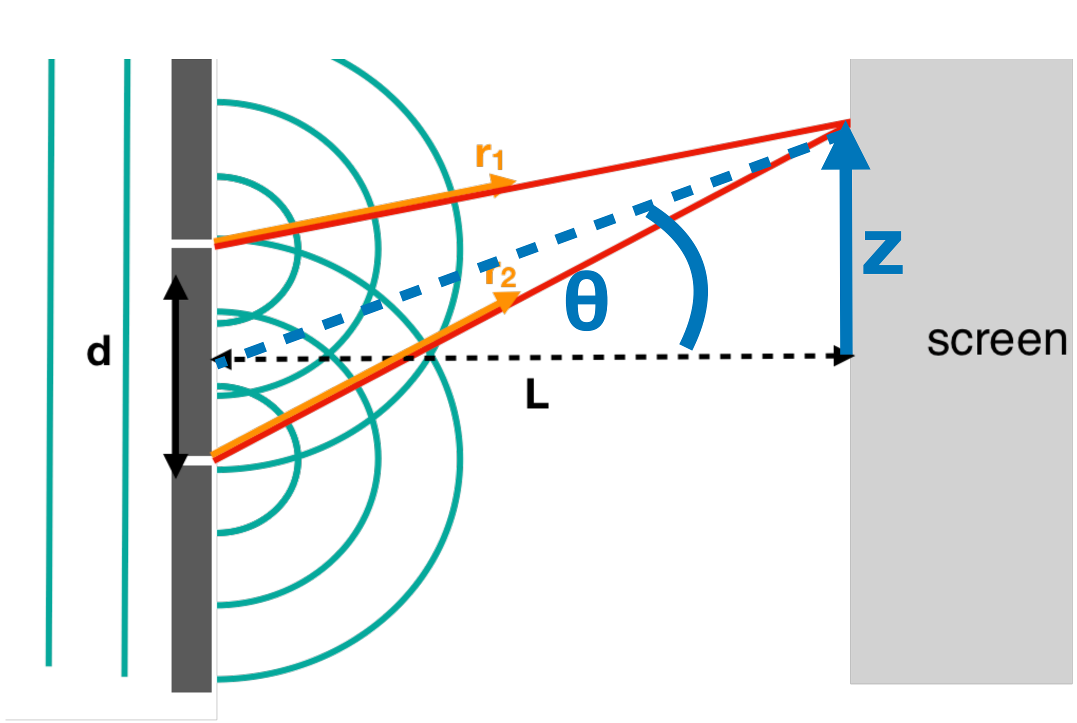
Diffraction and Interference

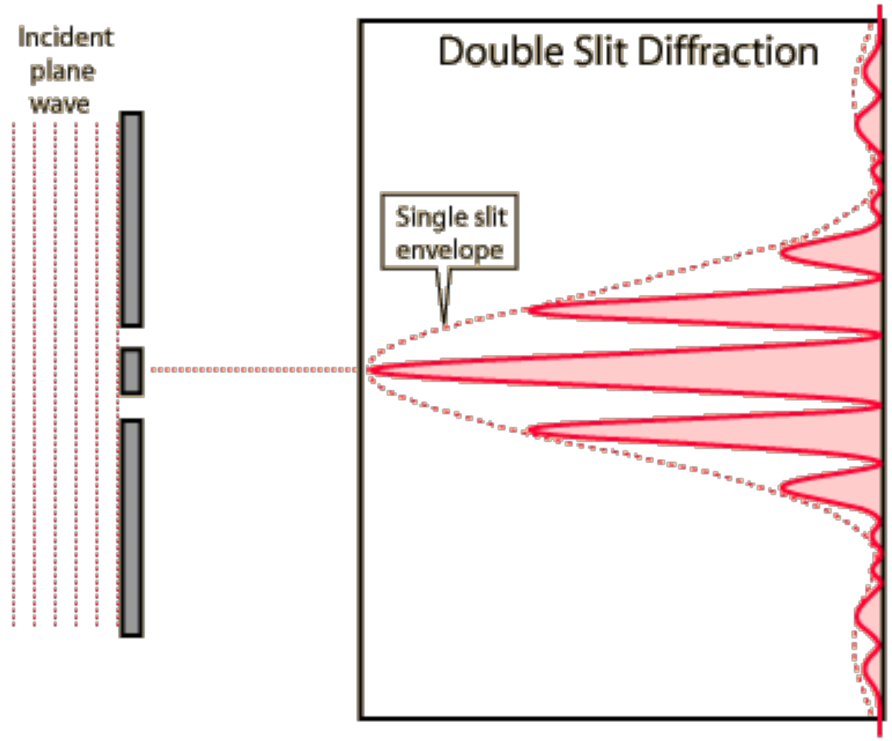


Path difference

$$r_2 - r_1 = \lambda/2$$

Diffraction and Interference





double slit **interference pattern**

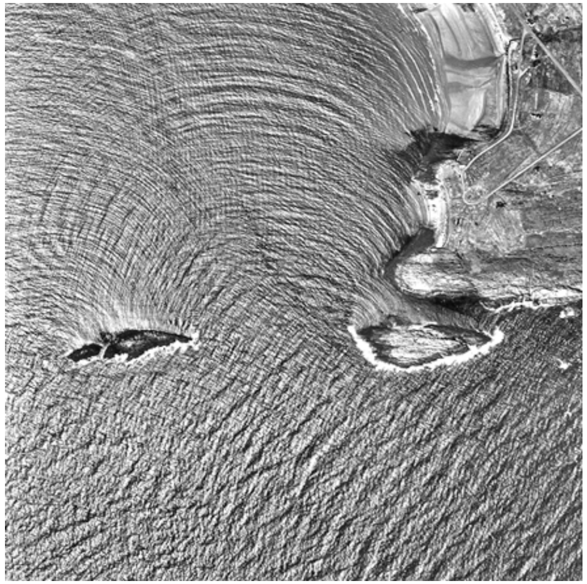
$$I(\theta) \approx I_0 \cos^2(\pi d \frac{\sin\theta}{\lambda}) \qquad (17)$$

Diffraction and Interference

Examples:

Colors reflected from CD

VLA Radio Astronomy

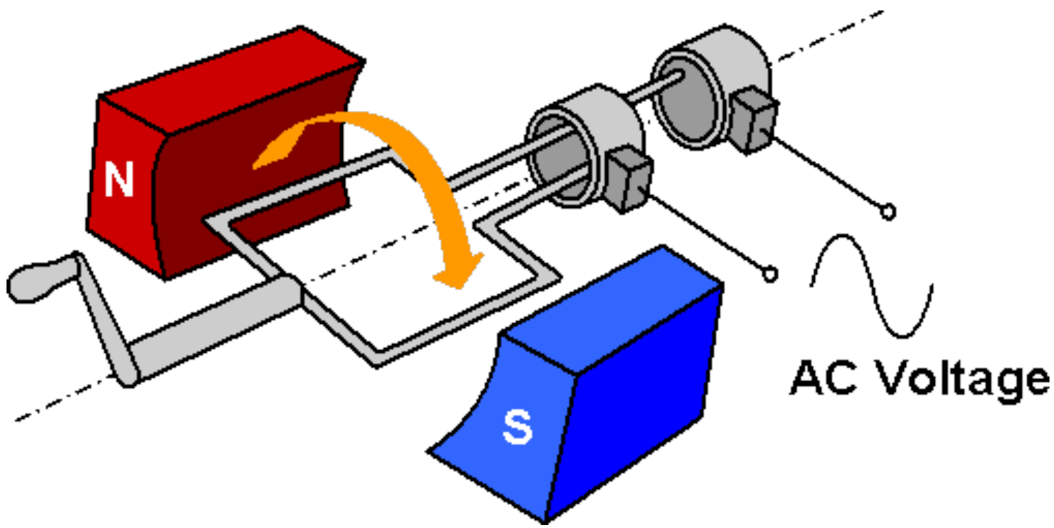


Water in bay

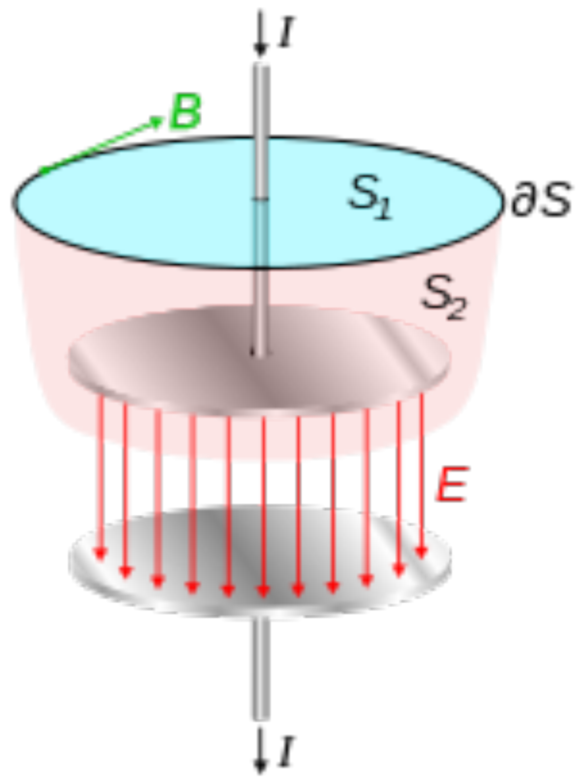
2.1.5) Electromagnetic waves

You will learn in Electro-magnetism lecture:

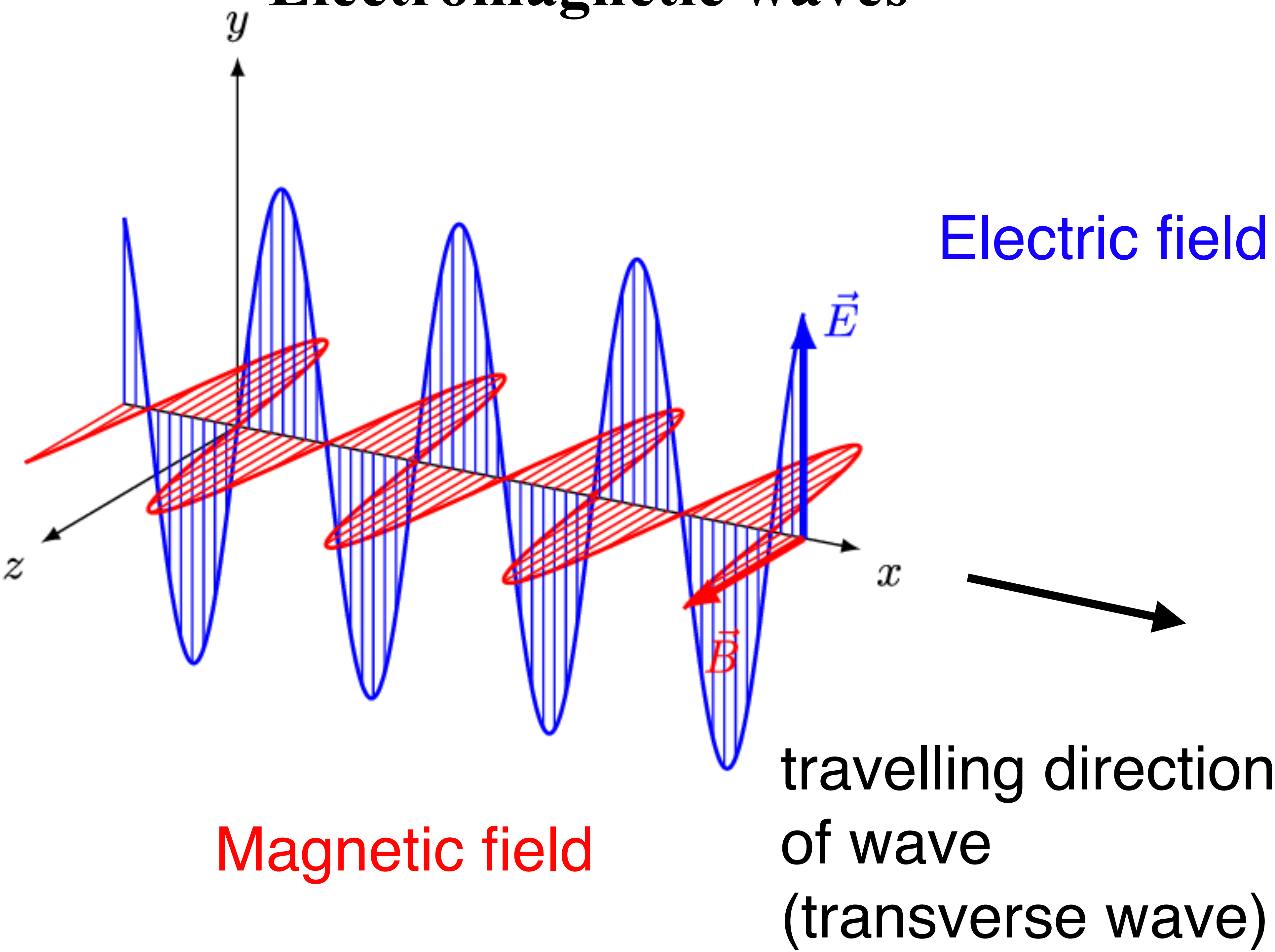
Changing **magnetic** field causes **electric** field (**induction**)



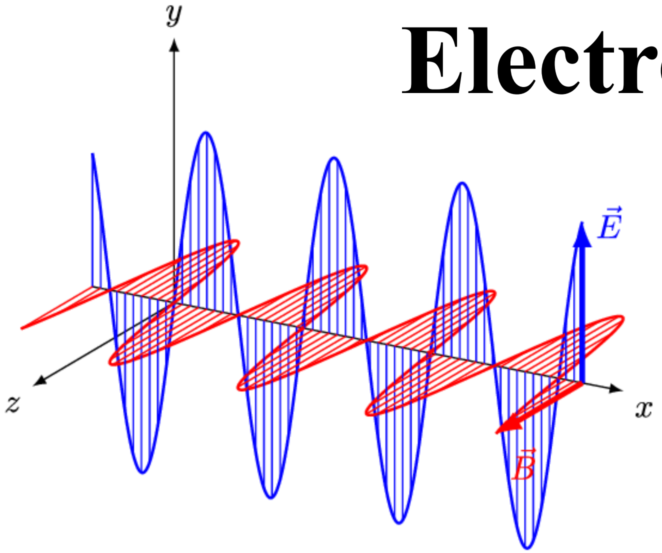
Changing **electric** field causes **magnetic** field



Electromagnetic waves



Electromagnetic waves



Electric field

Magnetic field

Electromagnetic wave equation:

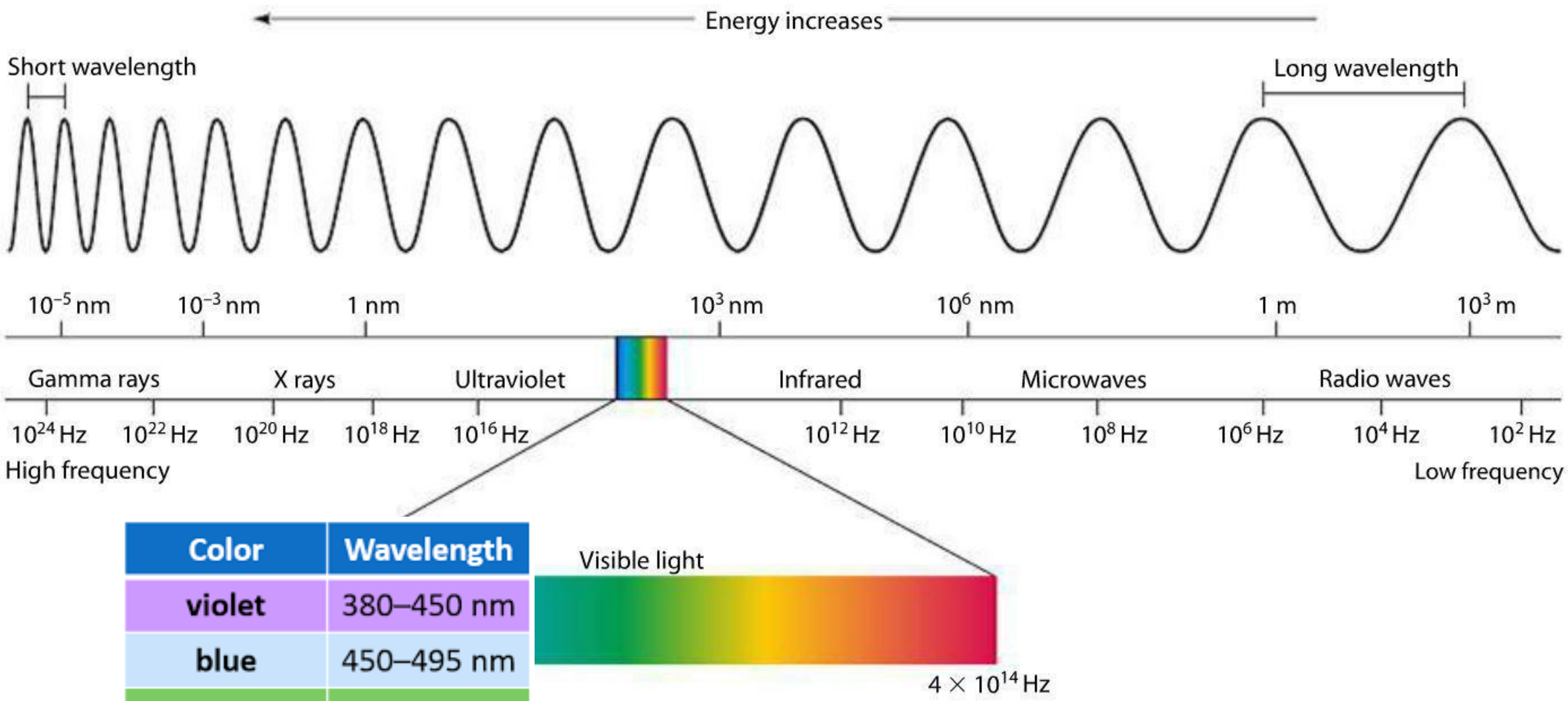
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\mathbf{E}(\mathbf{r},t)=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r},t) \quad (18)$$

Speed of light (vacuum) $c=\frac{1}{\sqrt{\mu_0\epsilon_0}}$ **(19)**

$$c = 29\,97\,92\,458\ m/s$$

Electromagnetic waves

$$\nu \lambda = c \qquad (10)$$



Color	Wavelength
violet	380–450 nm
blue	450–495 nm
green	495–570 nm
yellow	570–590 nm
orange	590–620 nm
red	620–750 nm