Phys106, II-Semester 2019/20, Tutorial 7, Fri 28.2.

Work in teams of three. Do "Stages" in the order below. When all teams finished a stage, make sure all students at your table understand the solution and agree on one by using the board.

Stage 1 Show that the de-Broglie wave-length of an object for which the kinetic energy equals the mean thermal kinetic energy (in one dimension) $E_{therm} = k_B T/2$ is $\lambda_{dB} = h/\sqrt{mk_B T}$ This is called the **thermal de-Broglie wave-length**.

Stage 2 Estimate the de-Broglie wave-length of the following objects. For that, google or guess their size, mass and typical non-zero velocity. When are wavelengths larger then the size? How do they compare to the typical spacing between one object and another one of the same kind? *Hint: In the cases where temperature comes into play, use the formula from stage 1.*

(i) An air molecule at ambient temperature.

(ii) A cat.

(iii) A virus.

(iv) An ultra cold atom in a Bose-Einstein condensate experiment. Assume this is a gas of Rubidium atoms at a temperature of $T = 10$ nK.

(v) A hypothetical candidate particle for dark matter that only has a mass of $m = 8 \times 10^{-23}$ eV, assume a velocity $v = 0.1c$. Express your answer in parsec (pc) 1 pc $= 3 \times 10^{16}$ m.

Stage 3 In the lecture we learnt the Heisenberg uncertainty principle that relates momentum uncertainty Δp and position uncertainty Δx. This is often used to also estimate the characteristic energy of a particle confined in a space Δx using $E \sim \Delta p^2/2m$.

(i) Use the relation above to estimate the characteristic energy of (i) an electron in an atom, (ii) a proton in a nucleus [ignoring relativity], (iii) an electron in a 10 angstrom nanostructure.

(ii) A measurement finds the position of an electron accurate to $\pm 5 \times 10^{-10} m$. Find the uncertainty in the momentum of the electron.

(iii) Find the uncertainty in the electron position 1 second later in the previous task.

Stage 4 For an image based on optics with waves of wavelength λ, the smallest spatial structure that can be resolved in an image has a size $d \approx \frac{\lambda}{2}$. This is called **diffraction limit**. What would be the diffraction limit for an electron microscope accelerating electrons through a voltage of 3 kV, 10 kV, 50 kV? What is the diffraction limit for light in the visible spectrum?

Images in this gallery of electron microscope pictures have a scale bar in the bottom right and the acceleration voltage in the bottom left corner:

https://www.fei.com/image-gallery/
Select one and compare the electron wavelength with the scale of the image.

The best resolutions achieved with electron microscopes are about 50 pm. Do you think this is due to the diffraction limit? For fun, also check out the other galleries of electron microscope pictures listed below:

(i) Gallery link (*google sizes for these ones*)

(ii) http://www.topdesignmag.com/25-amazing-electron-microscope-images/