Phys106, II-Semester 2019/20, Tutorial 5 solution

Stage 1 (i) Re-read the lecture notes regarding **beating** of two waves, that is section 2.3.1).

(ii) Now explore beating using this web applet . You can also hear the result for the example of acoustic waves. What does the beating effect sound like? What happens to the beating frequency if the two combined waves have a larger (smaller) frequency difference?

(iii) Revise the concept of group-velocity Eq. (53) and share with your table. You can now explore two moving combined waves with this web applet . How does it look if $v_g < v$, $v_g = v$, $v_g > v$, where v is the phase velocity? More control over the two constituent waves, like in the lecture, can be found in this app .

Solution:

(i) see lecture. The notes for week 5 were updated frequently. Please re-download the newest version for exam preparation.

(ii) The beating effect makes the volume of the sound oscillate (amplitude modulation). For larger (smaller) frequency difference between the two carriers, the beat frequency becomes smaller (larger)

(iii) For $v_g = v$ the fast oscillations within each wave group move in synch with the envelope. For $v_g < v$ the envelope lags behind the faster oscillations, for $v_g > v$ it moves faster. In the second app you can explicitly combine two sine waves as we did in the lecture. Try the following: You know the group-velocity is $v_g = \omega_1 - \omega_2/(k_1 - k_2)$. Use this to change the direction of the wavepacket. Keep $|k_1 - k_2| \ll k_1, k_2$ and $|\omega_1 - k\omega_2| \ll \omega_1, \omega_2$.

Stage 2 (i) Re-read the lecture notes regarding the **Fourier series** method to decompose any periodic function into cosines or sines, that is Eq. (43), (44), (46), (47). Make sure you all understand the gist of what this equation means (not yet why it works, see stage 4). Ask a TA otherwise.

(ii) Now explore this similarly to the pictures in the lecture using the online app on http://www.falstad.com/fourier/ . Select sequentially the triangle, sawtooth square function. Move the "Number of Terms" slider fully to the left, then add slowly term by term. The white dots appearing are the g_n (cosines), h_n (sines) coefficients from the lecture. What happens for larger n?. How can you tell if there are only cos or only sin? What happens if you switch $f(x)$ itself to be a cos or sin?
The app has a lot of advanced functionality, if interested please read its manual in detail. For example you can click into the space where the function is plotted to "draw" your own function and then calculate its decomposition into cosines and sines. Or you can click on the decomposition (dots) and manually increase or decrease the amount by which a certain sine or cosine is added.

(iii) Also check out https://www.geogebra.org/m/EYhBXfmK for some additional choice of functions.

Solution:

(i) see lecture

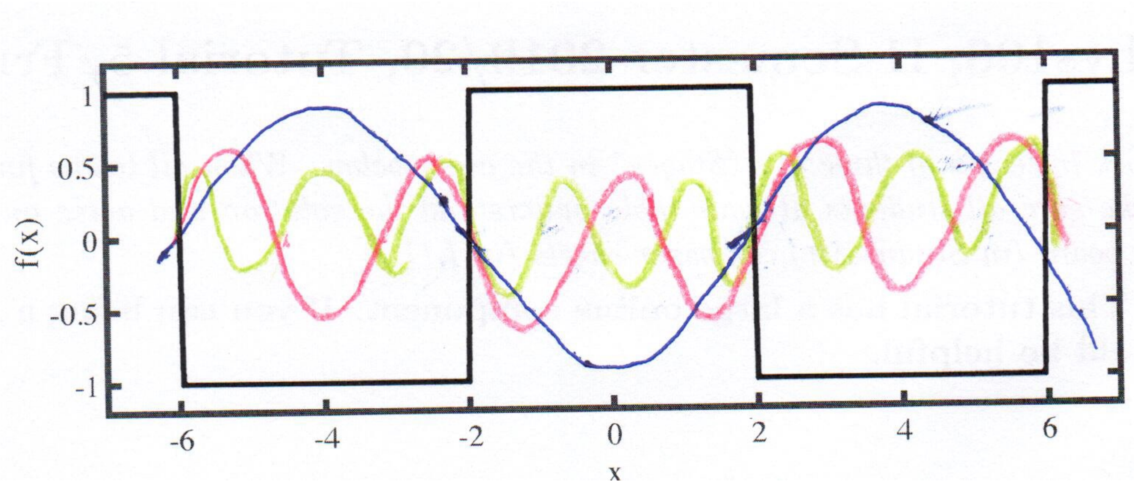
(ii) For larger n, typically the coefficients g_n and h_n become smaller. This means that the input functions $f(x)$ no longer contain significant extra detail on shorter and shorter length scales. For an odd input function, we get only sines, for an even one only cosines. If $f(x)$ itself was already *a sin or cos, we correctly get only a single non-zero coefficient g or h.*

Stage 3 Now redo yourself what you have seen in the apps as a team on the whiteboard (Air), sheet of paper (L1). Draw the square-wave function $f(x)$ shown below. You want to expand this as $f(x) = \sum_{n=0}^{\infty} g_n \cos(\lambda_n x)$ (Eq. 43), however we just "guess" the g_n and λ_n, instead of using (Eq. 44).

(i) Carefully draw the cosine wave that has most resemblance with $f(x)$.

(ii) Now draw the next cosine wave with a shorter wavelength that can improve the sum in Eq. (43) when added to the one in (ii).

(iii) Keep doing this until too difficult to draw. Also discuss in your team which waves would be unsuitable to add.



Solution:
TA PLEASE FIX FIGURE: The first "matching" cosine clearly is the one that has a positive or negative half-cycle in exactly the right intervals defined by the square wave (blue). The next matching cosine is the red one, which makes the function $\sum_{n=0,1}^{\infty} g_n \cos(\lambda_n x)$ rise faster at the edges of the squares and also correctly reduces the first cosine a little bit at the centre to make the overall result more square-shaped. The green one then does the same job again: Faster rise at the edge, remove excess pieces of earlier cosines. [Please re-check with this app (http://www.falstad.com/fourier/ .) after reading this text:]