Phys106, II-Semester 2019/20, Tutorial 4 solution

Stage 1

(i) How does an X-ray tube work? Which two components do the generated X-ray spectra have and why?

(ii) If you find a minimal X-ray wavelength of 0.04 nm, what was the acceleration voltage for the electrons in the tube?

(iii) Work with each other on the table through the lecture part regarding X-ray diffraction (XRD) to make sure you all understand how it works.

(iv) In an XRD analysis, a sample which we suspect to be partially crystalline is ground into powder and mounted as shown in Fig. 1. We illuminate it with a mono-chromatic (=single wavelength) X-ray source at $\lambda_{XR} = 0.10$ nm. We then rotate the sample and the X-ray detector as shown in (a) and record the indicated spectrum in (b) at certain scattering angles θ. Infer the lattice constant d of the crystals in the sample, ignoring the fact that multiple-Bragg planes might exist in the crystal (we consider only one). Why do we grind the sample into powder?

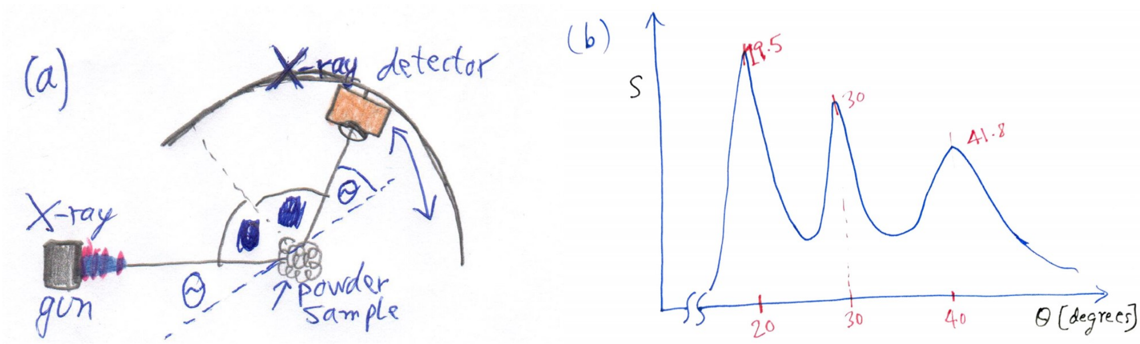

Figure 1: XRD scheme (a) setup, (b) recorded X-ray signal S.

Solution:

(i) The X-ray tube operates by emitting electrons from a heated cathode due to thermionic emission and accelerating them towards a high voltage anode. As the electrons hits the anode it is decelerated and the energy is released as X-rays. The two components that the generated X-ray spectra have are Bremsstrahlung, with continuous spectrum over a broad range of wavelengths, and characteristic narrow, large peaks. The first comes from the deceleration, the second from knocking out inner electrons from atoms.

(ii) $Voltage = \frac{hc}{e\,\lambda_{min}}$

$$Voltage = \frac{hc}{e\,(0.04 \times 10^{-9} m)} = 31\,kV$$

(iii) See lecture notes and book. To be discussed in group.

(iv) *Bragg crystal diffraction formula is given by*

$$2d\ sin(\theta) = n\lambda$$

In this question $\lambda = 1.0 \times 10^{-10}m$. *We can try various* n *and find that*

$d = \frac{n\lambda}{2sin(19.5^o)} = \frac{(n+1)\lambda}{2sin(30^o)}$ *correctly gives the first two peaks if* $n \approx 2$

Hence, $d = \frac{2\times1.0\times10^{-10}}{2sin(19.5^o)} = 3 \times 10^{-10}m$. *If we had assumed the first peak is* $n = 1$, *the other ones would be in the wrong place.*

Regarding the powder grinding: Most real samples are messy and have multiple small crystal "domains" which have different orientations. This would confuse the analysis, since some angles might not give peaks because all these domains are oriented wrongly. In the powder, we have so strongly Bragg plane directions that whenever the angle θ *relative to a plane allows Bragg peaks, there will be enough planes oriented with the correct direction to see the peak.*

Stage 2 Discuss the following questions:

(i) Why is Compton-scattering from electrons best seen with X-rays?

(ii) Why do Compton-scattering spectra also show an unshifted part (at the same wavelengths as the incoming X-ray)?

(iii) Why can a photon not produce an electron-positron pair in free space?

Solution:

(i) *Because the electron Compton wavelength is in the X-ray range. The clearest way to measure a wavelength shift is to compare it with the incoming wavelength. If we would use much larger wavelength light, like UV, the wavelength shift would be only a tiny fraction of the incoming one. Additionally, other processes such as photo effect become more likely. Soft* γ*-rays would probably also be fine, but then again the shift might be so large compared with the incoming wavelength that we get confused.*

(ii) *The X-rays can not only scatter off the outermost electrons (which are quite weakly attached to the nucleus so we can consider them free), but also off innermost electrons of the atom. These are very close to the nucleus so they are firmly attached to the atom. This means that the X-ray is effectively scattering off the entire atom. We thus have to replace* m_e *in the Compton scattering formula by* M, *the mass of that atom including its nucleus. This is a few* 1000 *times heavier than an electron, hence X-rays that scatter of the inner electrons have almost no wavelength shift at all.*

(iii) *See lecture notes, book and assignment 4.*

Stage 3 The top-quark is an exotic elementary particle akin to up-quarks which are an important part of the proton, but much heavier with a rest mass of $m_t = 3.07402 \times 10^{-25}$ kg (compare the mass of a proton/electron of $m_p = 1.672 \times 10^{-27}$ kg). For which frequency of γ-ray photon could the photon annihilate in top-anti-top pair production (in the presence of a heavy nucleus)? Suppose you want to create this γ-ray akin to the X-ray tube by first accelerating an electron between a cathode and an anode, what voltage do you need between these?

Solution:

$$\nu = \frac{2m_t c^2}{h} = 8.34 \times 10^{25}\, Hz$$

The voltage has to supply sufficient energy to the electron to produce such a gamma ray. Thus $e\,Voltage = h\nu$. *Inserting the numbers we find:* $Voltage = \frac{h\nu}{e} = 345 \times 10^{9}$ *Volt. Already because of this (and other reasons), the X-ray tube alike idea is not going to work.*