Phys106, II-Semester 2018/19, Tutorial 2 solution

Stage 1 (i) For the following electro-magnetic waves, calculate the missing variable (wave-length λ or frequency ν): $\lambda = 5$ m, $\lambda = 200$ nm, $\nu = 2 \times 10^{14}$ Hz, $\nu = 1 \times 10^{24}$ Hz. What name do waves from that part of the electro-magnetic spectrum have? Discuss where they are used/ where they occur in nature.

(ii) For the following waves, calculate the phase velocity and guess which type of wave it might be: $\lambda = 5$ mm and $\nu = 1$ MHz, $\lambda = 10$ cm and $\nu = 3300$ Hz, $\lambda = 500$ km and $\nu = 600$ Hz (based on the speeds of waves given in the lecture or the internet).

Solution:

(i)

- $\lambda = 5m$: $\nu = 6 \times 10^7 Hz$, *Radio wave.*
- $\lambda = 200nm$: $\nu = 1.5 \times 10^{15} Hz$, *Ultraviolet (UV) .*
- $\nu = 2 \times 10^{14} Hz$: $\lambda = 1.5 \mu m$, *Infrared.*
- $\nu = 10^{24} Hz$: $\lambda = 0.3 fm$, *Gamma-ray.*

(ii)

- $\lambda = 5mm$ *and* $\nu = 1MHz$: $v = 5 \times 10^3$ *m/s, speed of sound in some metals like brass, steel.*
- $\lambda = 10cm$ *and* $\nu = 3300$ *Hz:* $v = 330$ *m/s, sound waves in air.*
- $\lambda = 500km$ *and* $\nu = 600$ *Hz:* $v = 3 \times 10^8$ *m/s, electro-magnetic waves.*

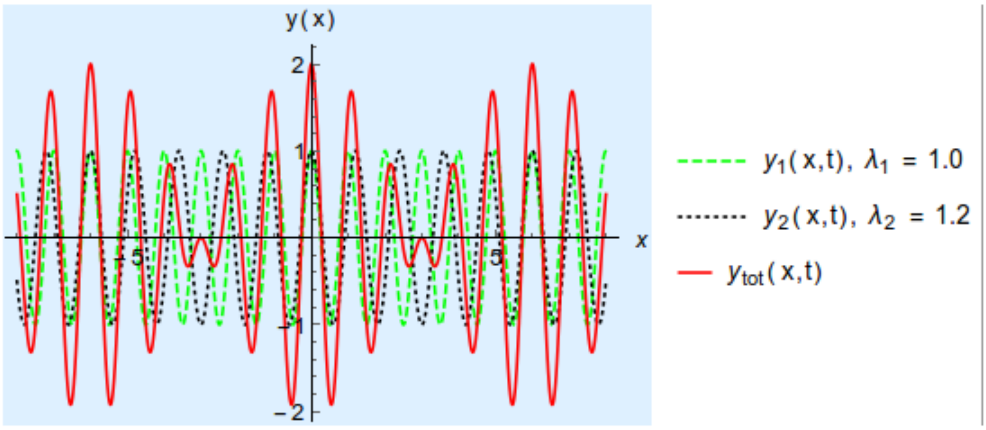
Stage 2 *(i) Consider the following functional shape:* $y_{tot}(x,t) = y_1(x,t) + y_2(x,t)$ *with* $y_1(x,t) = A\cos\left(\frac{2\pi}{\lambda_1}(x - Vt)\right)$ *and* $y_2(x,t) = A\cos\left(\frac{2\pi}{\lambda_2}(x - Vt)\right)$*, with almost but not quite equal wavelengths* $\lambda_1 \approx \lambda_2$*. Is this a solution to the wave equation for velocity* V*?*

(ii) Make a drawing of $y_1(x,t)$ *and* $y_2(x,t)$ *at* $t = 0$ *for some* $\lambda_1 \approx \lambda_2$ *of your choice. Make sure to draw it for* many *wavelengths. Using only your drawing "add" them together to form* $y_{tot}(x,t)$*. How does the result look? Does the pattern have a name?*

(iii) Now lets do the same with math, at $t = 0$ *using the trigonometric identity:* $\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$*. Compare with your drawing.*

Solution: (i) Yes, due to the superposition principle.

(ii)



This is called a "beating pattern".
(iii) $y_{tot}(x,t) = 2A\cos\left[(\frac{\pi}{\lambda_1} + \frac{\pi}{\lambda_2})x\right]\cos\left[(\frac{\pi}{\lambda_1} - \frac{\pi}{\lambda_2})x\right]$. *If* $\lambda_1 \approx \lambda_2 \approx \lambda$, *then the whole can be written as* $y_{tot}(x,t) = 2A\cos\left(\frac{2\pi}{\lambda}x\right)\cos\left[\frac{(k_1-k_2)}{2}x\right]$. *The cos term has a very small wavenumber, which gives the slow, enveloping oscillations of the beating pattern.*

Stage 3 *Do experiments with rope waves. Either ties a string onto a chair (or hold it in the air) or use the following online app:* https://phet.colorado.edu/sims/html/wave-on-a-string/latest/waveonastring_en.html *. Make contact with week3 of lectures notes, and try to generate: travelling waves, standing waves, and wave pulses. Note for the app: Fixed end reflects the wave with phase-shift, loose end reflects without phase shift, no end does not reflect.*

Stage 4 *(i) In the lecture we discussed double slit interference as arising from the superposition of waves coming from the two different slits. We can also treat a single slit as an infinitely dense collection of single point "emitters" as sketched in the figures below. On your table, discuss qualitatively how you would expect based on these pictures that waves passing through this single slit behave when they reach the far right side. How does this differ from a stream of particles? Does the effect depend on wave-length? Did you experience this in your lives already?*

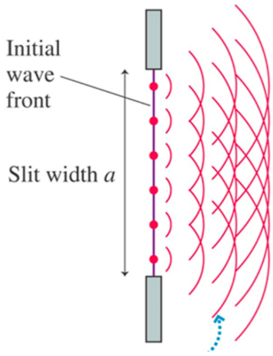


Fig. A

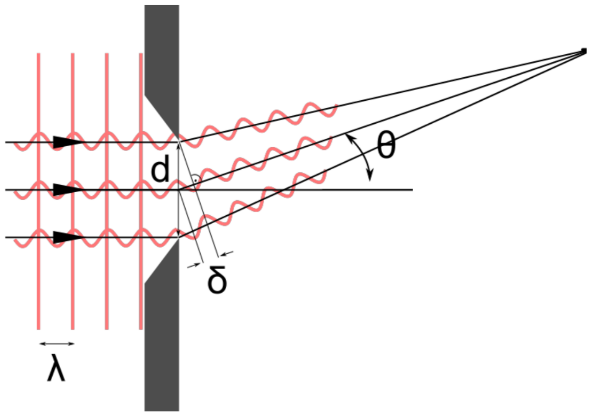


Fig. B

Solution:

The waves will diffract around the corner. We have to consider if waves coming from different locations within the slit constructively or destructively interfere in a certain direction. As long as their path difference is much less than a wavelength, they constructively interfere. After passing through a slit smaller than a wavelength, the wave thus can go in any direction. If the slit is much larger than the wavelength, most directions other than straight forward destructively interfere.

For a stream of particles there is no diffraction, there will be only one central spot on the other side of the single slit.
Experience: the sound waves audible to the human ear are in the range 17mm to 17m. This enables sound waves to diffract through the edges or corners of a door. When a music is played with the door slightly opened, it is possible to hear sound from any direction near the door.This is because sound waves are of wavelength comparable to the width or aperture near the door enabling them to diffract and spread out. Since diffraction can cause larger turns for large wavelengths, you hear the low sounds of some music better around corners of a house than the high pitch.