

# Phys106, II-Semester 2019/20 Assignment 5

Instructor: Sebastian Wüster

*Hint: For all these questions (and later ones) you may use the math software `mathematica`, available from CC. The little overhead in familiarising yourself with it now, will pay off manyfold later, regardless of your major.*

1. (i) Add the two waves  $y_1(x, t) = \sin(k_1x - \omega_1t + \varphi_1)$  and  $y_2(x, t) = \sin(k_2x - \omega_2t + \varphi_2)$  to form  $y_{tot}(x, t) = y_1(x, t) + y_2(x, t)$  as a product of sine and cosine, similarly to what was done in the lecture (*you may want to look at: [this collection](#)*).
- (ii) Make a computer script that can plot  $y_1(x, t)$ ,  $y_2(x, t)$  and  $y_{tot}(x, t)$  and test your calculation is right. For this, use parameters e.g.  $k_1 = 1$ ,  $k_2 = 0.9$ ,  $\omega_1 = 1.0$ ,  $\omega_2 = 0.8$ ,  $\varphi_1 = 0$ ,  $\varphi_2 = \pi/2$ . Plot  $y_{tot}(x, t)$  for couple of values of  $t$ .
- (iii) Now fix  $t$  and plot for some different values of  $\varphi_1 = 0$ ,  $\varphi_2 = 0.1, 0.2, 0.3, \dots$ . What do you see?

Solution:

(i) Using the formula from week 5 lecture,

$$\sin(a) + \sin(b) = 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)$$

yields,

$$y_{tot}(x, t) = 2 \sin\left(\frac{(k_1 + k_2)x - (\omega_1 + \omega_2)t + (\varphi_1 + \varphi_2)}{2}\right) \times \cos\left(\frac{(k_1 - k_2)x - (\omega_1 - \omega_2)t + (\varphi_1 - \varphi_2)}{2}\right)$$

(ii) In `Mathematica` We will first define  $y_1(x, t)$ ,  $y_2(x, t)$  and  $y_{tot}(x, t)$ :

```
y1[x_] := Sin[k1 x - ω1 t + φ1]
y2[x_] := Sin[k2 x - ω2 t + φ2]
ytot[x_] := y1[x] + y2[x];
```

and set the values of parameters,

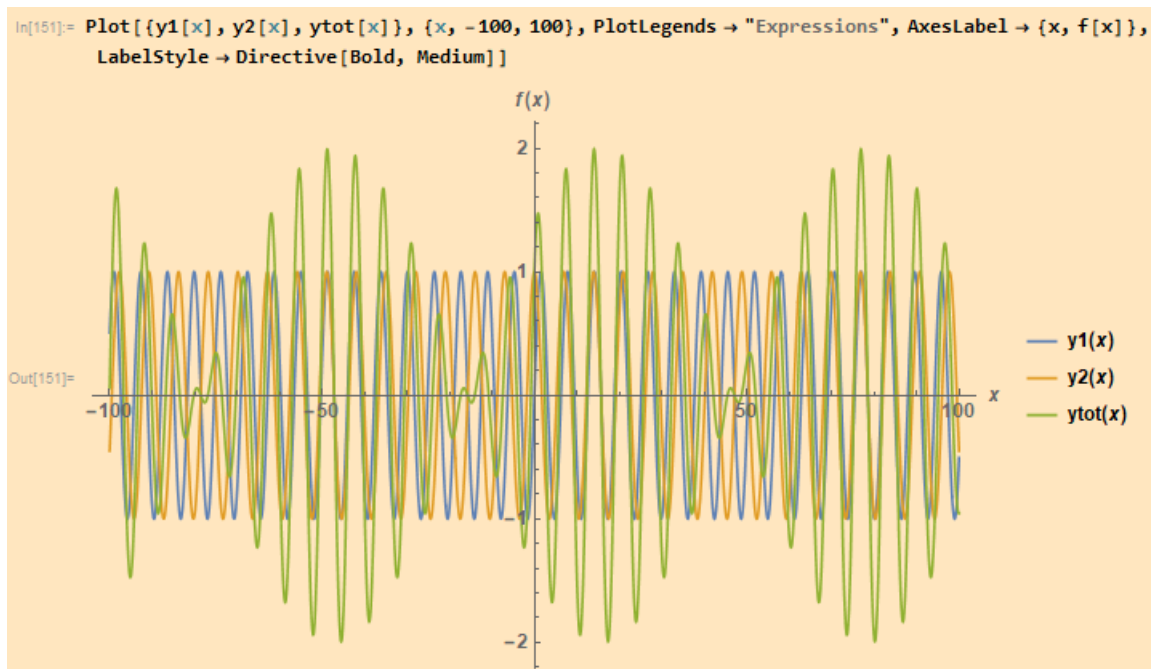
```

In[143]:= k1 := 1;
          k2 := 0.9;

In[139]:= ω1 := 1;
          ω2 := 0.8;
          φ1 := 0;
          φ2 := π / 2;

```

We plot the three functions at  $t = 0$  using plotting tool "Plot[]":



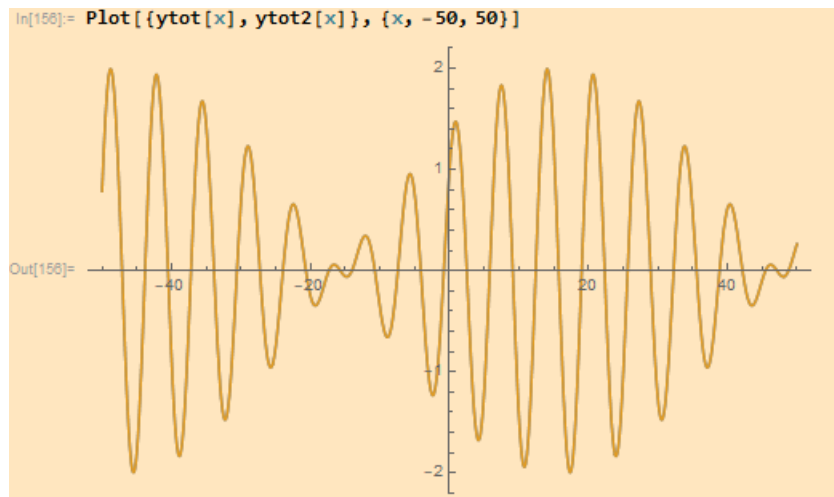
To test the calculation in (i), plot and compare the calculated  $y_{tot}(x, t)$  with mathematica plot. Let us call the result of (i)  $y_{tot2}(x, t)$ :

```

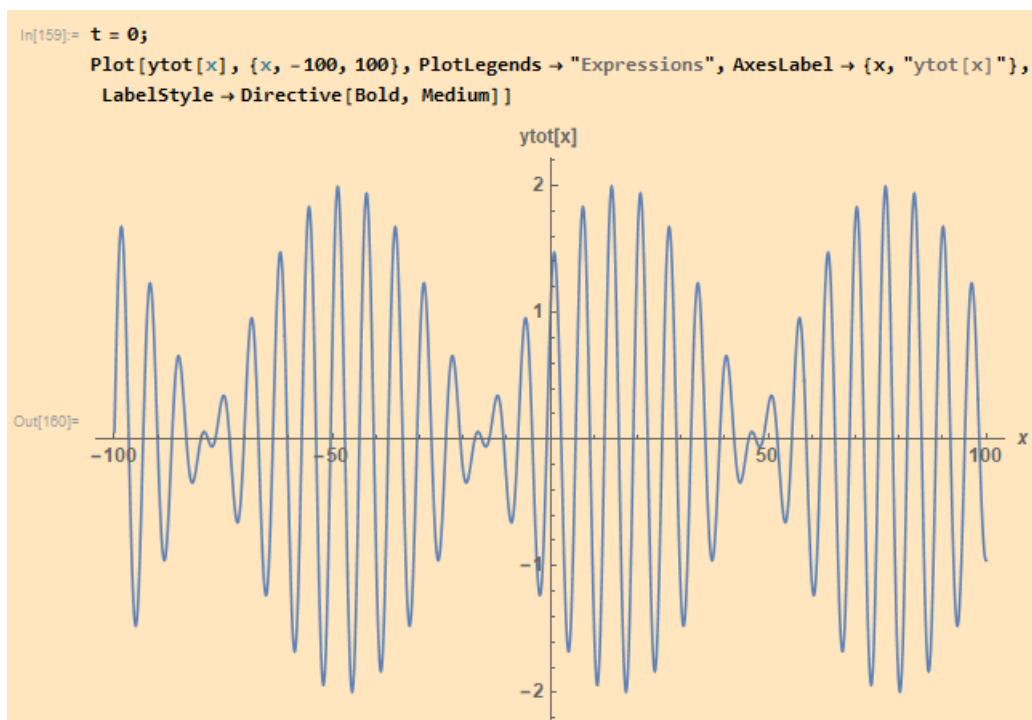
ytot2[x_] := 2 Cos[(k1 x - ω1 t + φ1 - (k2 x - ω2 t + φ2)) / 2] Sin[(k1 x - ω1 t + φ1 + (k2 x - ω2 t + φ2)) / 2]

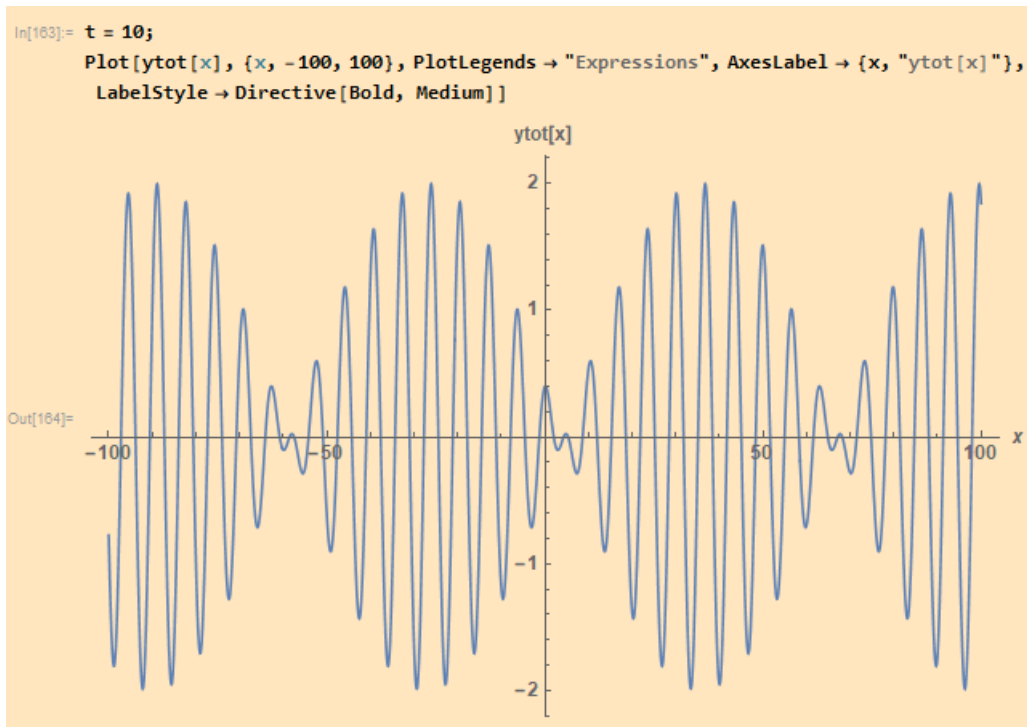
```

and plot simultaneously  $y_{tot}(x, t)$  and  $y_{tot2}(x, t)$ ,

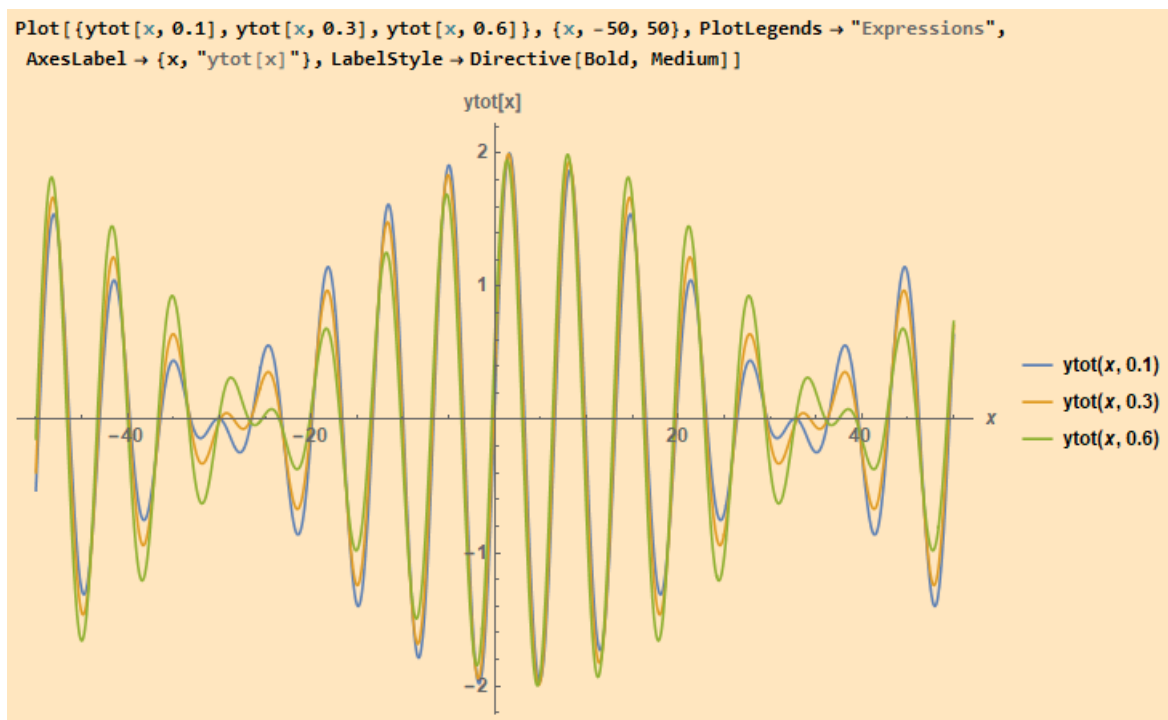


Clearly, the two graphs overlap implying that the calculation in (i) is right.  
 Plotting  $y_{tot}(x, t)$  at two different times:  $t = 0$  and  $t = 10$ :





(iii) We now fix  $t = 0$ . Plots for  $\varphi_2 = 0.1, 0.3, 0.6$  are below. We see that different choices of phases  $\varphi$  are moving the positions of the wave packets around.



- Download the script `fourier_decomposition_gaussian_v1.m` (Fourier Demo gaussian) from the webpage. Install matlab from CC at <https://www.iiserb.ac.in/cc/download> on your computer, or use the computers at *CC Lab 1* or *CC Lab 2*. Run the script by typing its name (without `.m`). The bottom shows you a Gaussian wavepacket in black. We now sequentially express this using cosines as in Eq. (42) of the lecture, by pressing ENTER. The violet line shows you the latest added cosine and the red-dashed line the sum of all cosines added so far. The violet dots at the top show you “how much” of a given cosine is added. Once a certain cosine no longer significantly contributes, violet color changes to blue.

Now you can change the width of the Gaussian wavepacket by changing the parameter  $d$  in line 18 slightly up and down. What changes? Discuss.

*Solution:* Increase in the width of the Gaussian wavepacket in position space results in decrease in width of the plot in momentum ( $k$ ) space. Whereas when we decrease the width of the Gaussian wavepacket in position space the spread in  $k$  space is increased in accordance with the uncertainty principle.

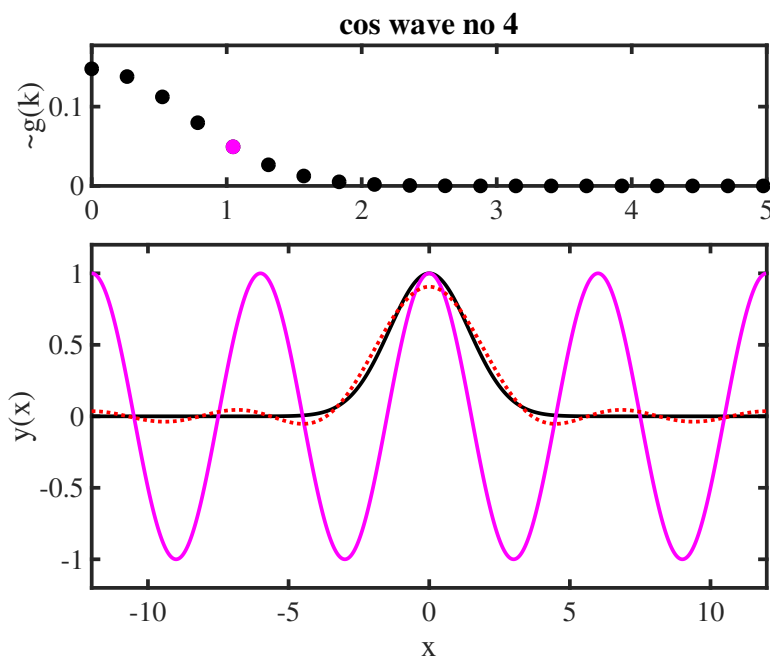


Figure 1: For  $d=2$ .

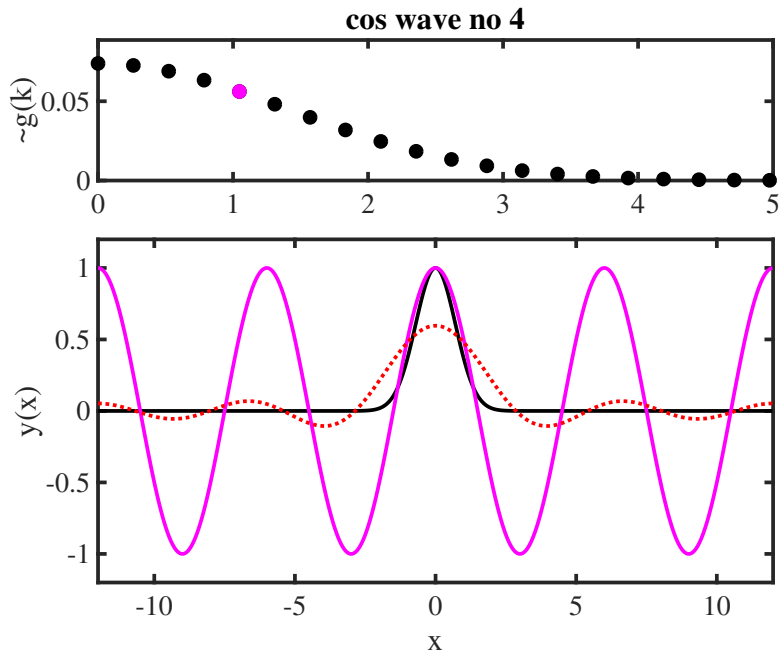


Figure 2: For  $d=1$ .

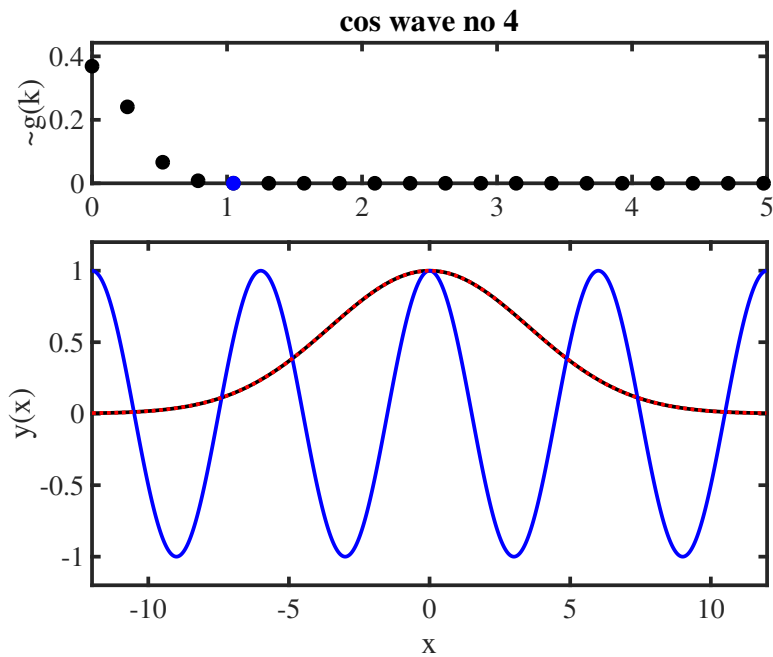


Figure 3: For  $d=5$ .

3. *Await week 6 for this, if we haven't covered it yet:* Find the de-Broglie wave-length of the objects below (treat the all non-relativistically). Discuss your results and expectations. Compare with the sizes of the objects.

- An electron with velocity 3m/s.
- An electron with velocity 2000m/s.
- A proton with velocity 0.04c.
- A plane with mass  $m = 200$  t and velocity 800km/h.
- A bacterium with mass  $m = 4 \times 10^{-12}$  g and velocity  $30\mu\text{m/s}$ .

*Solution: Mass of electron =  $9.1 \times 10^{-31}$ kg. To calculate de-Broglie wavelength we use the formula:*

$$\lambda = \frac{h}{mv}$$

*where,  $h = 6.626 \times 10^{-34} \text{ m}^2 \text{ kg/s}$ . Hence, for electron with  $v = 3 \text{ m/s}$ ,*

$$\lambda_{v=3}^e = 2.425 \times 10^{-4} \text{ m}.$$

*For electron with velocity 2000 m/s,*

$$\lambda_{v=2000}^e = 3.637 \times 10^{-7} \text{ m}.$$

*For proton having mass =  $1.6726 \times 10^{-27} \text{ kg}$  and velocity =  $0.04c$  de-Broglie wavelength will be,*

$$\lambda_{v=0.04c}^p = 6.0616 \times 10^{-11} \text{ m}.$$

*For plane of mass =  $2 \times 10^5 \text{ kg}$ ;  $v = 222.222 \text{ m/s}$ ,*

$$\lambda^{plane} = 1.491 \times 10^{-41} \text{ m}.$$

*For bacterium, mass =  $4 \times 10^{-15} \text{ kg}$ ;  $v = 3 \times 10^{-5} \text{ m/s}$ ,*

$$\lambda^{bact} = 5.522 \times 10^{-15} \text{ m}.$$

*Typical order of magnitude of sizes:*

$$\begin{aligned} \text{electron : } & \text{no size/size} = 0 & (1) \\ \text{proton : } & 10^{-15} \text{m} \\ \text{plane : } & 40 \text{m} \\ \text{bacterium : } & 10^{-6} \text{m} \end{aligned}$$

*The de-Broglie wavelength in the proton and electron cases is much larger than their size. So wave character might be important. For larger objects (bacteria, planes), wavelength are much shorter than object sizes, so typically wave character will be difficult (here impossible) to see.*