Phys106, II-Semester 2019/20, Assignment 3

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Hint: For all these questions (and later ones) you may use the math software **mathematica***, available from CC. The little overhead in familiarising yourself with it now, will pay off manyfold later, regardless of your major.*

1. Write the Rayleigh Jeans law in terms of wavelength, i.e. we want $u(\lambda)d\lambda$, the total energy within a wavelength interval $d\lambda$. Take care in converting the differential $d\nu$.

2. Ultraviolet light of wavelength 200 nm and intensity 3.00 W/m^2 is directed at a Potassium surface. (a) Find the maximum kinetic energy of the photoelectrons (For the workfunction (ϕ) of Potassium see the book "Concept of Modern Physics by Arthur Beiser") (b) if 0.5 percent of the incident photons produce a photoelectron, how many electrons are emitted per second if the surface has an area of 4 cm^2. *Recall 1 Watt = 1J/s, i.e. energy per unit time.*

3. Consider the light emitted by a 35 W room lamp. Assume this is all yellow light with $\lambda = 600$ nm (in reality it is a mix of colors). How many photons per second are there? Why would quantum physics frequently be not that important in determining what happens due to this light?

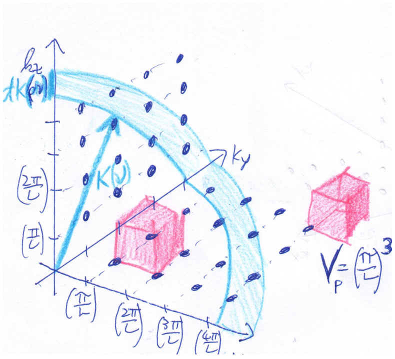

Figure 1: (for question 4) Sketch of reciprocal space. Big dots indicate allowed 3D wave-vectors. Red shade indicates the volume effectively taken by one of these dots.

4. Derive Eq. (20) of the lecture, that the number of standing waves within a cavity that have a frequency between ν and $\nu + d\nu$ is

$$G(\nu)d\nu = \frac{8\pi\nu^2}{c^3}d\nu \tag{1}$$

   Proceed as follows:

   (i) Confirm that condition (16) for a standing wave can be re-written in terms of wave number as $k = \frac{\pi}{L}n$ for $n = 1, 2, 3, \cdots$.

(ii) 3D waves are described by a wave vector $\mathbf{k} = [k_x, k_y, k_z]^T$. The relation to frequency is $2\pi\nu/|\mathbf{k}| = c$.

(iii) Now for 3D standing waves that fit into a cubic 3D cavity of volume $V = L^3$, the condition is $k_j = \frac{\pi}{L} n_j$ for $n_j = 1, 2, 3, \cdots$ and $j \in \{x, y, z\}$.

(iv) That means, that in the space of all vectors $\mathbf{k}$, and allowed standing wave must always lie on one of the dots in Fig. 1. This space is called reciprocal space.

(v) To infer $G(\nu)d\nu$, we have to now "count" the number of dots within a spherical shell of radius $k(\nu)$ and thickness $dk(d\nu)$. We use that each dot effectively occupies a volume $\left(\frac{\pi}{L}\right)^3$. Also take into account that each wave can have two different polarisations (directions of the E-field vector within the wave), so we have to count it twice.

(vi) Divide by the total volume of the cavity to get the number of waves *per unit volume.*

(vii) Use all these steps to show Eq. 1 above.