Phys106, II-Semester 2019/20, Assignment 2

Instructor: Sebastian Wüster

Hint: For all these questions (and later ones) you may use the math software `mathematica`*, available from CC. The little overhead in familiarising yourself with it now, will pay off manyfold later, regardless of your major.*

1. A function f is called periodic with period T if $f(t+T) = f(t)$. The time average of a such a function is given by

$$\overline{f(t)} = \frac{1}{T}\int_0^T f(t)dt. \tag{1}$$

Calculate $\overline{\cos(\omega t)}$, $\overline{\sin(\omega t)}$, $\overline{\cos^2(\omega t)}$, $\overline{\sin^2(\omega t)}$. What average would you get when averaging over a larger number of periods nT?

2. Do the missing steps in the lecture for the derivation of the double slit interference pattern:

   (i) Start with Fig. 2 in week 3, using geometry, express r_1 and r_2 through z, L, and the angle θ of the vectors connecting the slits and location z on the screen. These angles are slightly different, make the approximation that they are equal, as shown in the attached sketch Fig. 1.

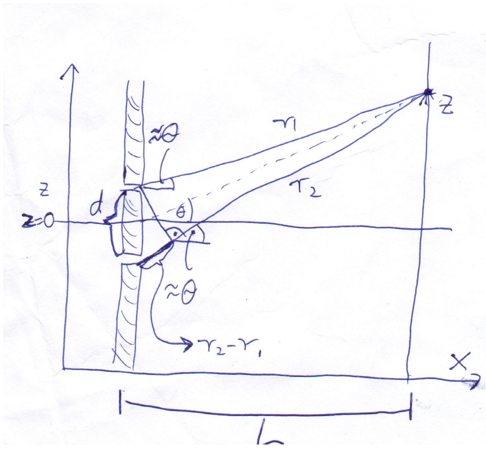

Abbildung 1: Sketch of variables for double slit geometry.

   (ii) Also approximate the $r_{1,2}$ dependence of the prefactors of y_1 and y_2 (Fig. 2) as equal $r_1 \approx r_2 \approx L$. Do not do this approximation for $r_{1,2}$ within the argument of the wave (cos) in Fig. 2.

   (iii) Within $y(z,t)$, split the space and time dependence in the trigonometric functions into the form $f(t)g(x)$ using the trigonometric identity $\sin a + \sin b = 2\sin(\frac{a+b}{2})\cos(\frac{a-b}{2})$.

(iv) Now calculate the intensity $I(z,t) = |y(z,t)|^2$, and perform the long time average over this using your results from question one. Simply define here $I_0 = 2A^2/L^2$

3. The following wave equation is called "sine-Gordon equation":

$$\frac{\partial^2}{\partial t^2} f(x,t) - \frac{\partial^2}{\partial x^2} f(x,t) + \sin\left[f(x,t)\right] = 0. \tag{2}$$

Determine if the superposition principle holds for this equation and why or why not.