Week 7

PHY 106 Quantum Physics
Instructor: Sebastian Wüster, IISER Bhopal, 2020

3) Atomic Physics and Quantum Mechanics

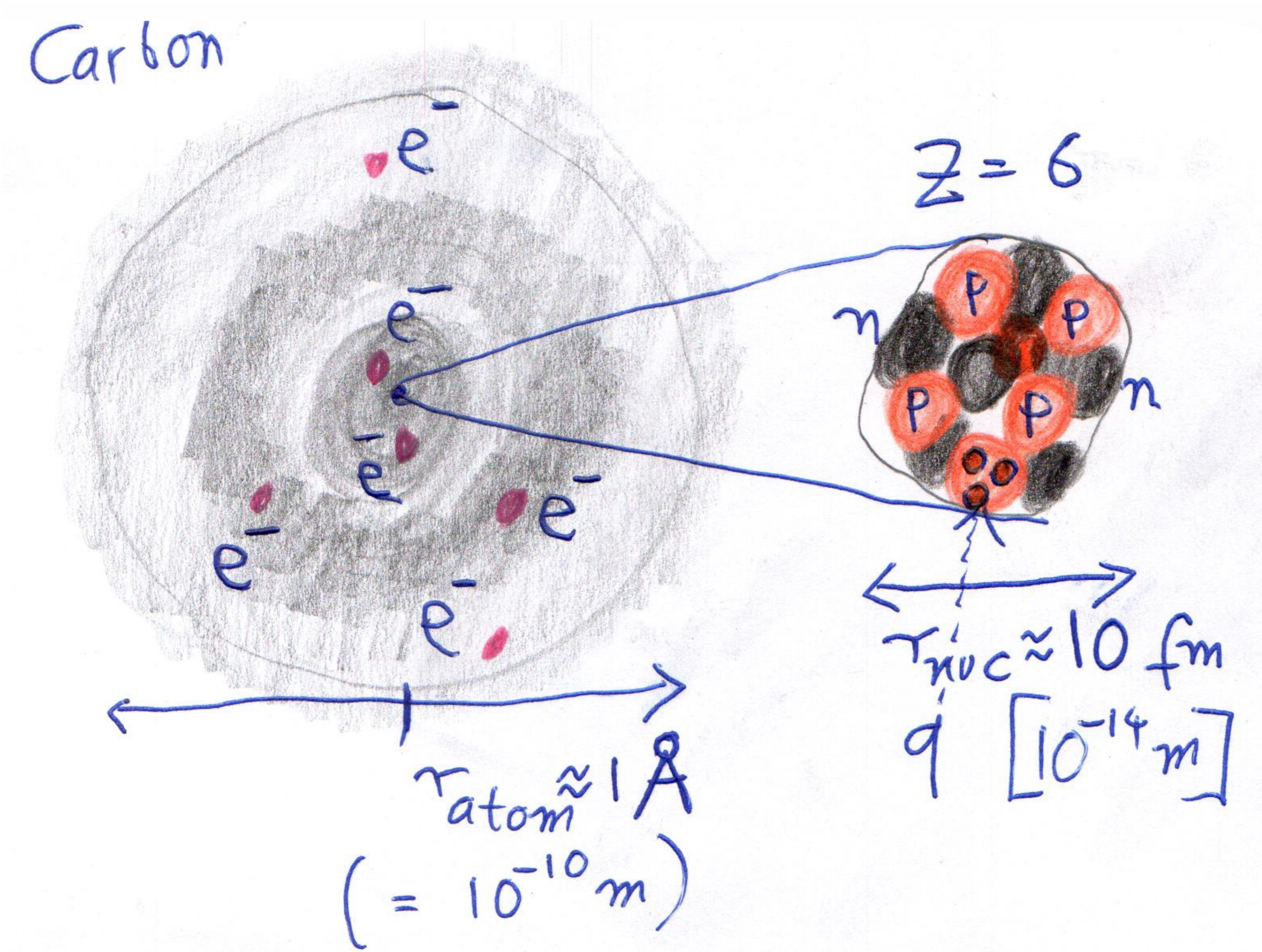
3.1) Structure of the atom

We now know particles are matter-waves….

Examples after Eq. (60) matter-wavelengths tiny…

Matches atom sizes, let’s revisit the atom….

Structure of the atom Nowadays we know:



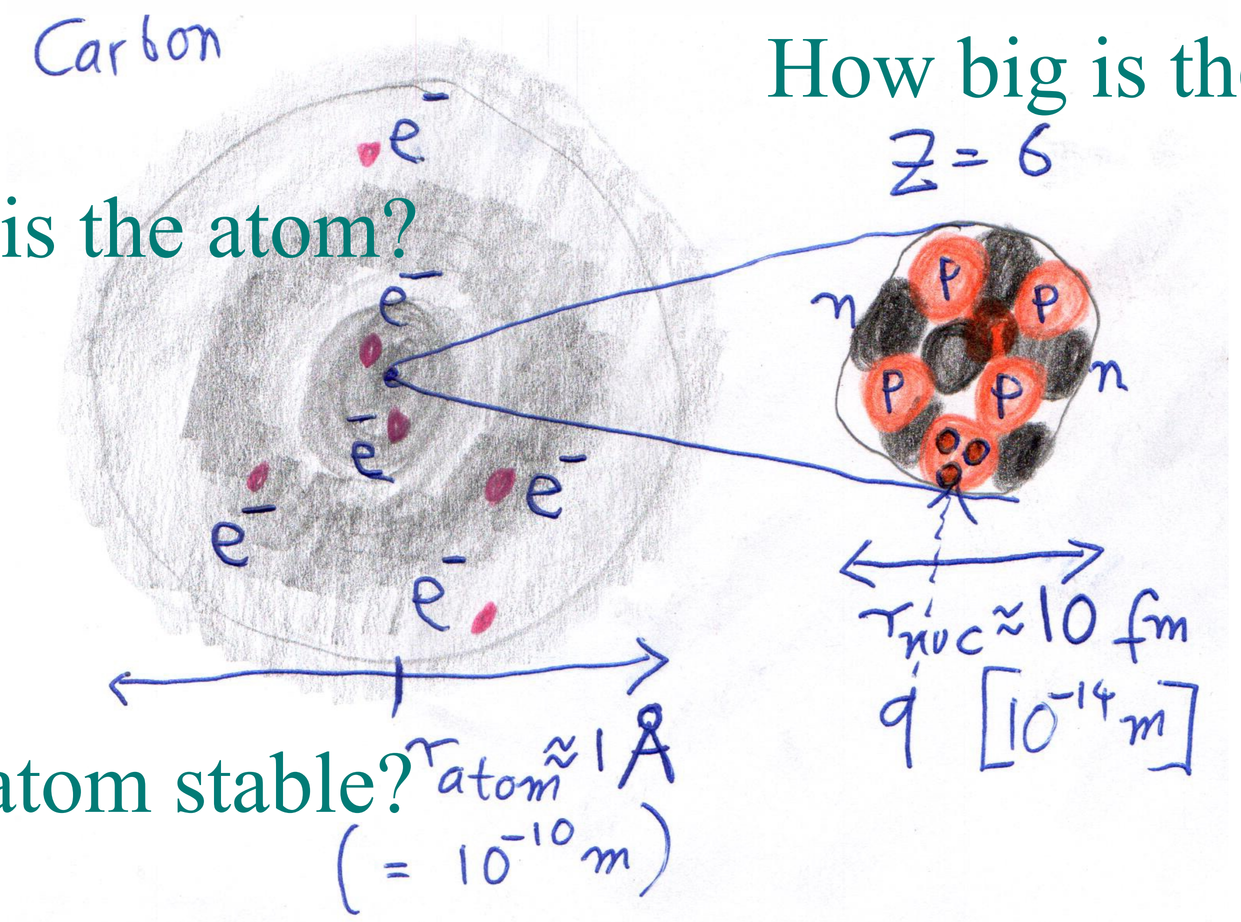
How did science arrive at this picture?

Structure of the atom

How are the pos. and neg. charges distributed?

How big is the nucleus?

How big is the atom?



Why is the atom stable?

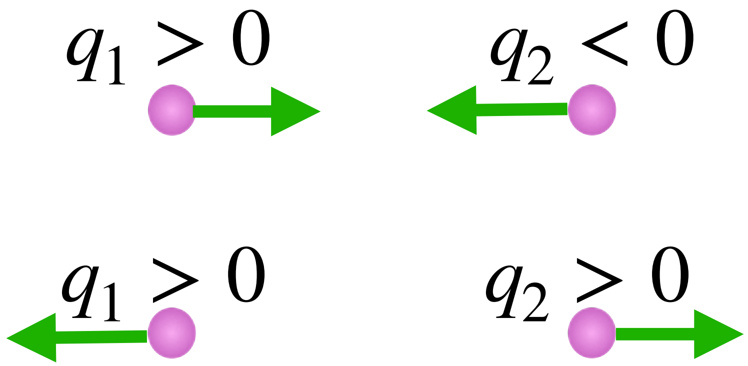
How did science arrive at this picture?

How did science answer these questions?

Excursion/reminder: Coulomb force

Two slow electric charges feel a force (electro-static)

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \quad \textbf{(67)}$$



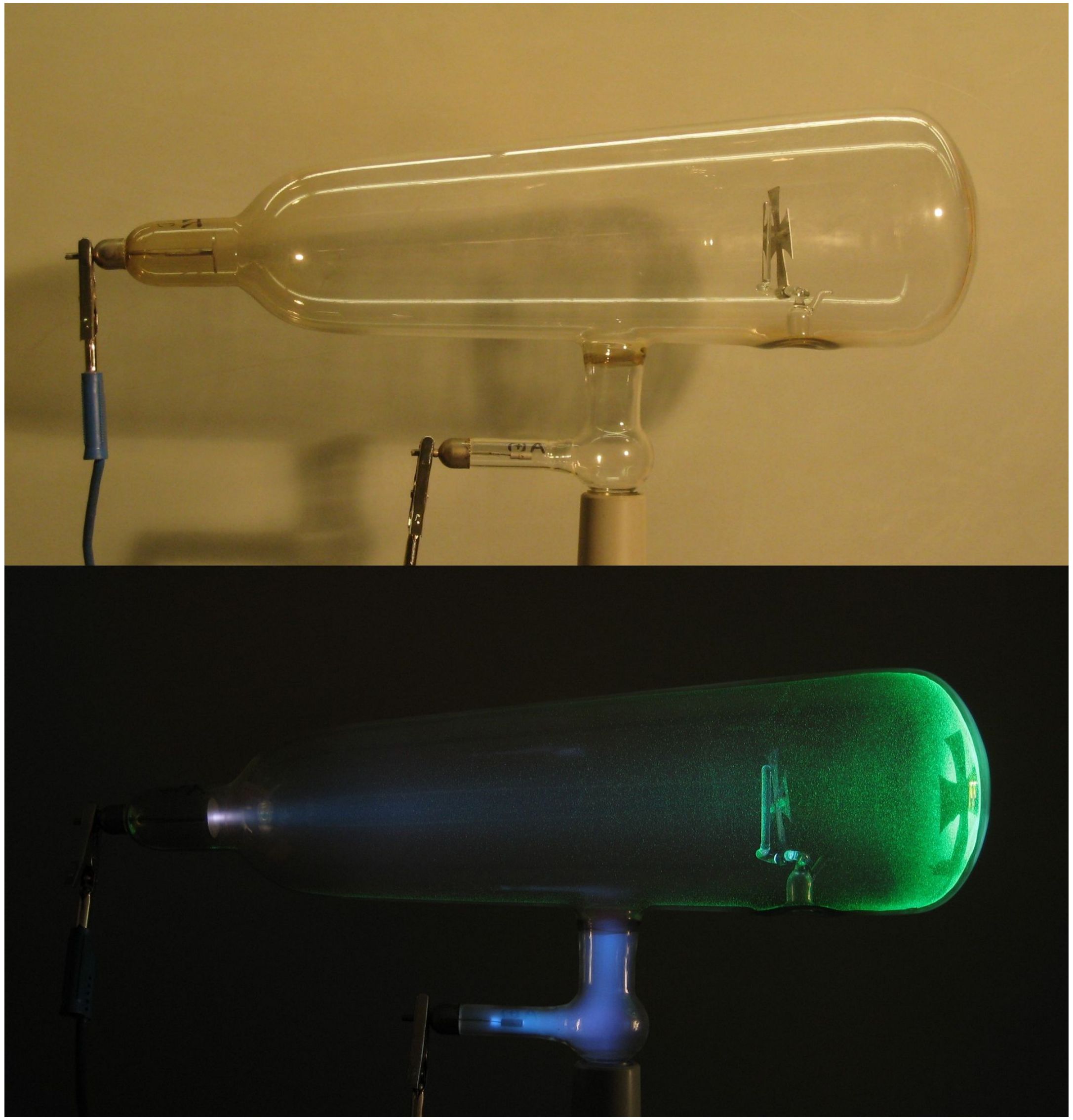
draw

- The force is attractive between opposite sign charges, else repulsive

- Here $\epsilon_0 = 8.854 \times 10^{-12} C^2/N/m^2$ is the **vacuum permittivity**. It just sets the strength of electro-static forces.

3.1.1) Thomson's model of the atom (1898)

Crookes tube for making cathode rays

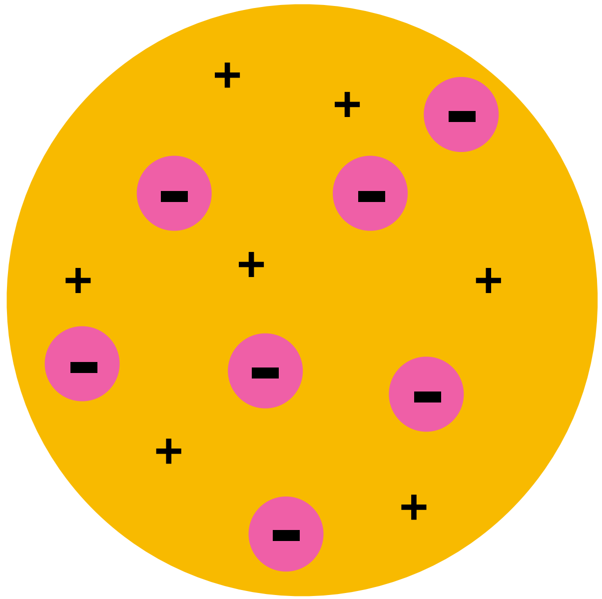


Discovery of the electron (1897, Thomson)

3.1.1) Thomson's model of the atom (1898)

Atoms contain electrons but are neutral.
What with the positive charge required?

Thomsons proposal:

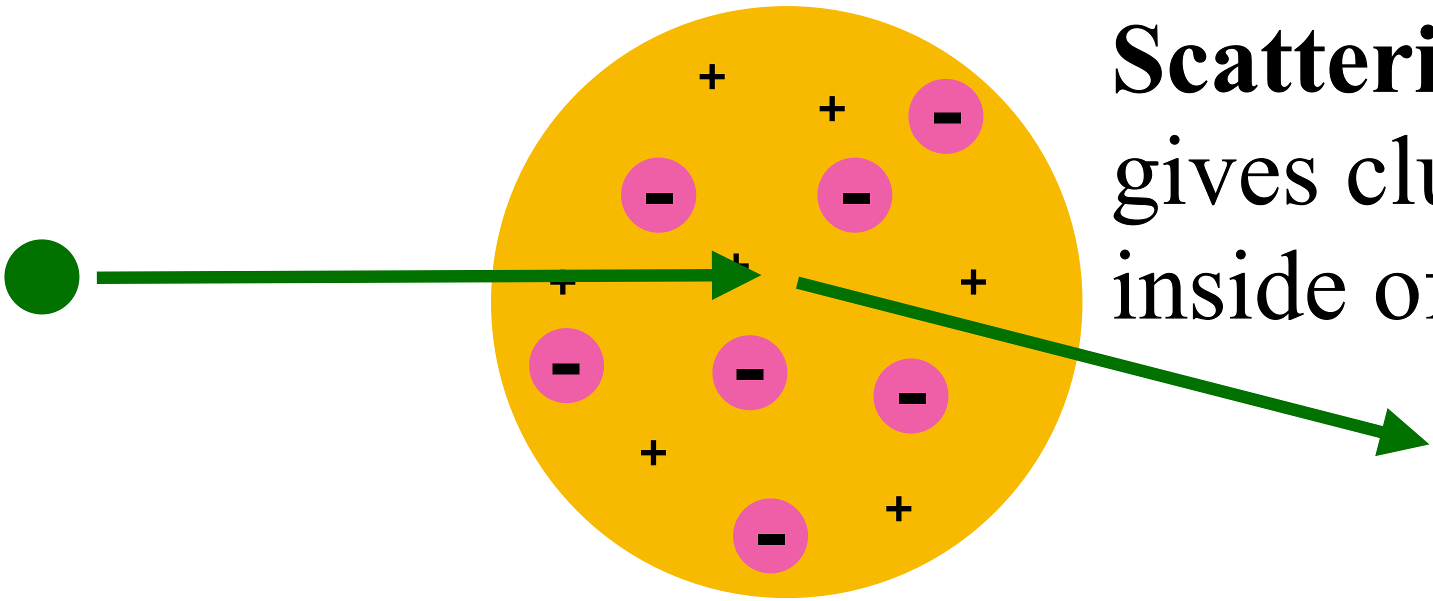


Electrons are embedded in positively charged background like raisins in a cake

3.1.2) Rutherfords scattering experiment

How can we find out what is inside something we can't see?

- break it?
- send a probe in/through!!

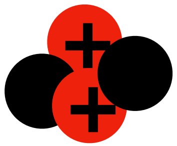


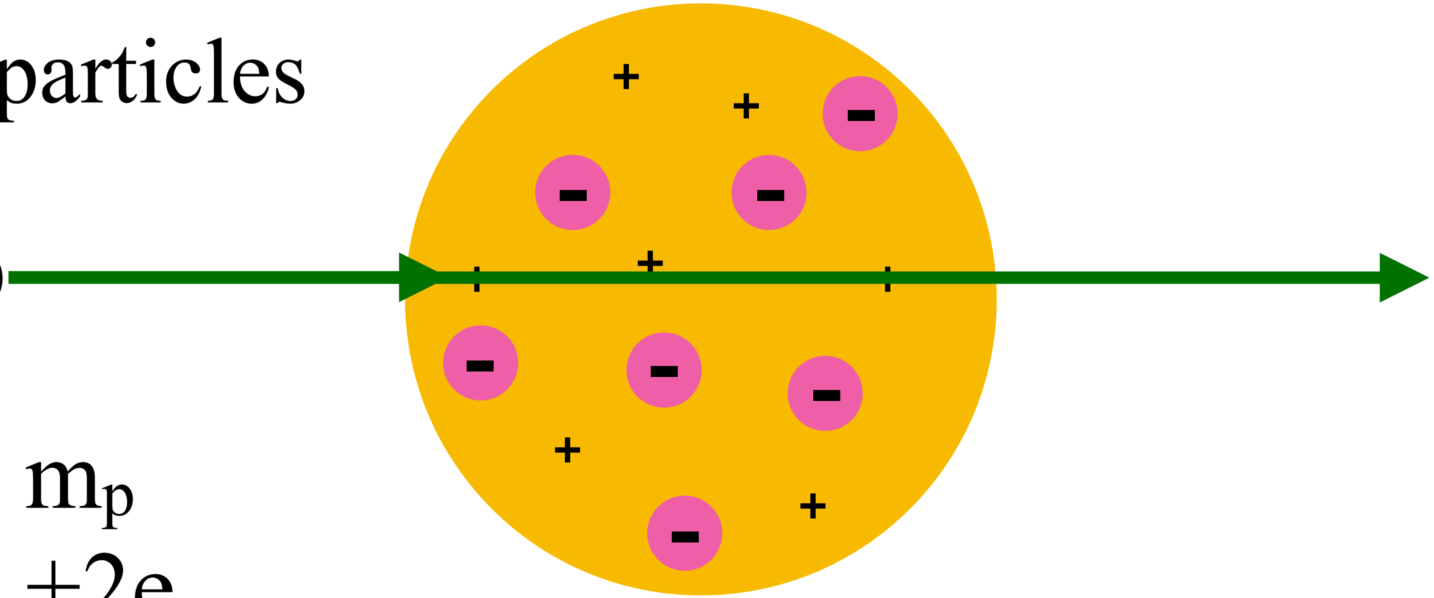
Scattering of probe gives clues about inside of atom

What probe to take?

Rutherfords scattering experiment

Probe: alpha-particles





Mass: 4 m_p

Charge: +2e

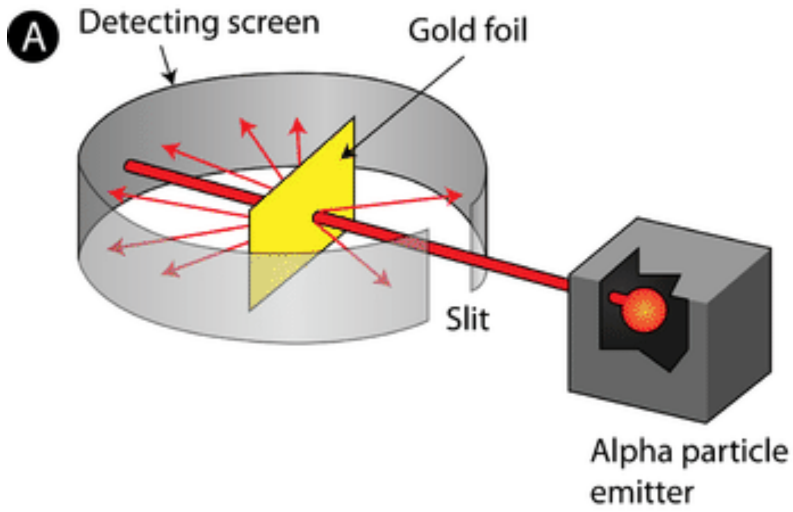
Can calculate electric field inside Thomson atom using **classical** physics **see tutorial 8**

- \+ charge too widely distributed for strong field
- electrons (- charge) too light to stop alpha-particle

We expect **almost no deflection** of alpha-particle

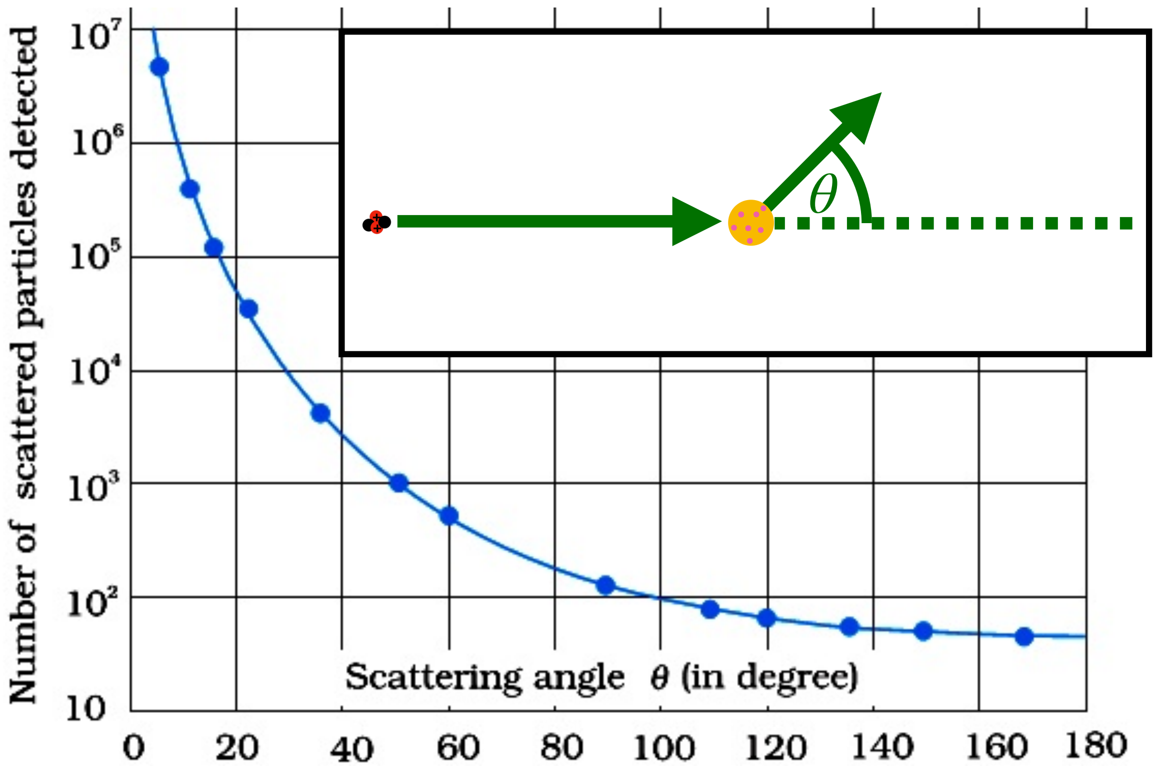
Rutherfords scattering experiment

Geiger-Marsden experiment



Results:

- Note log-scale
- Thus most α-particles go straight
- However **unexpectedly** many have **very large angles**

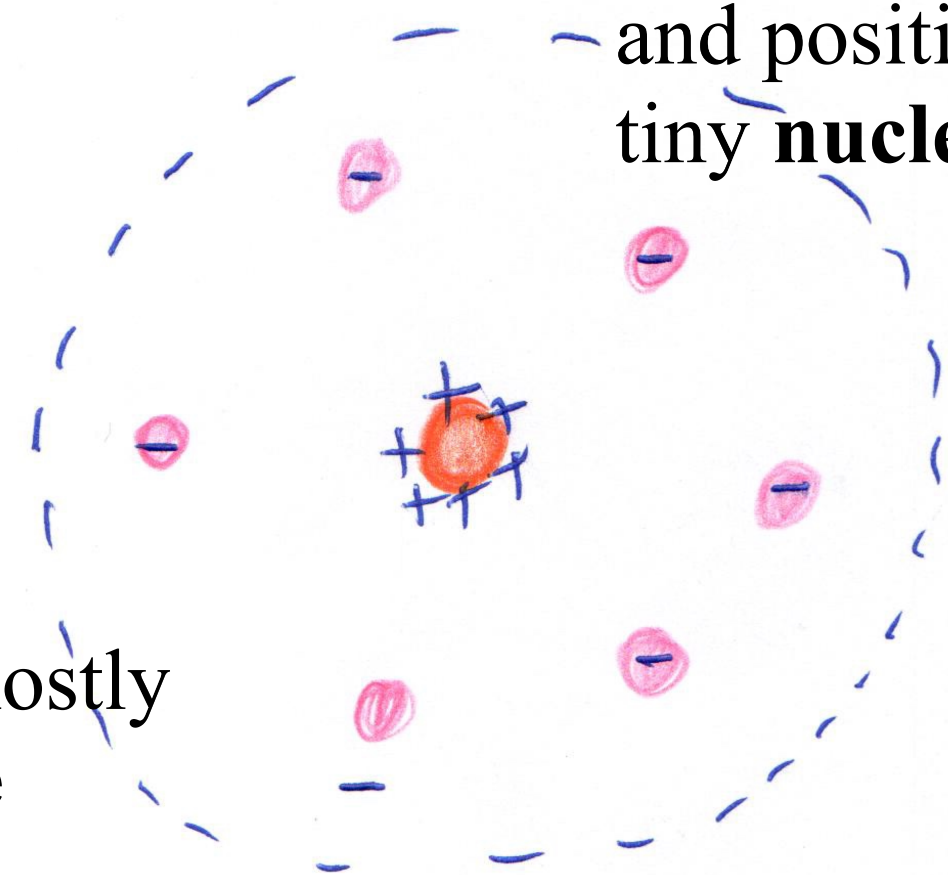


3.1.3) Rutherford's model of the atom (1909)

How do we explain this?

Rutherford model:

Concentrate most **mass** and positive **charge** in a tiny **nucleus**

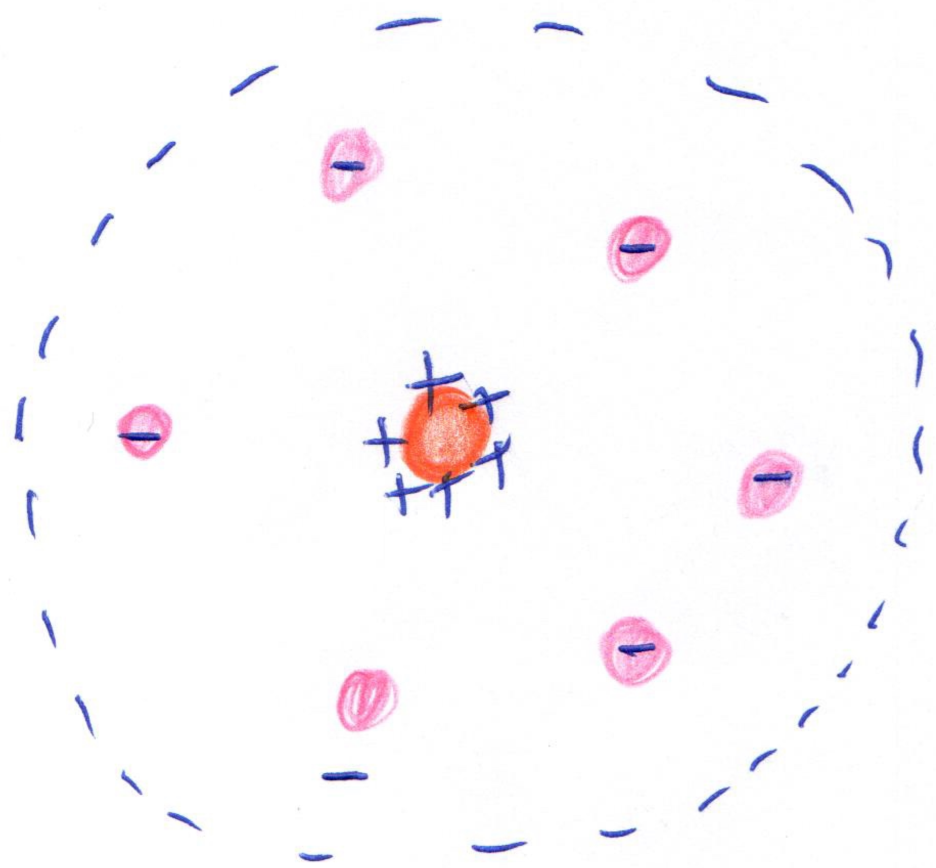


Atom thus mostly **empty space**

Rutherford's model of the atom (1909)

What to expect now for scattering ??:

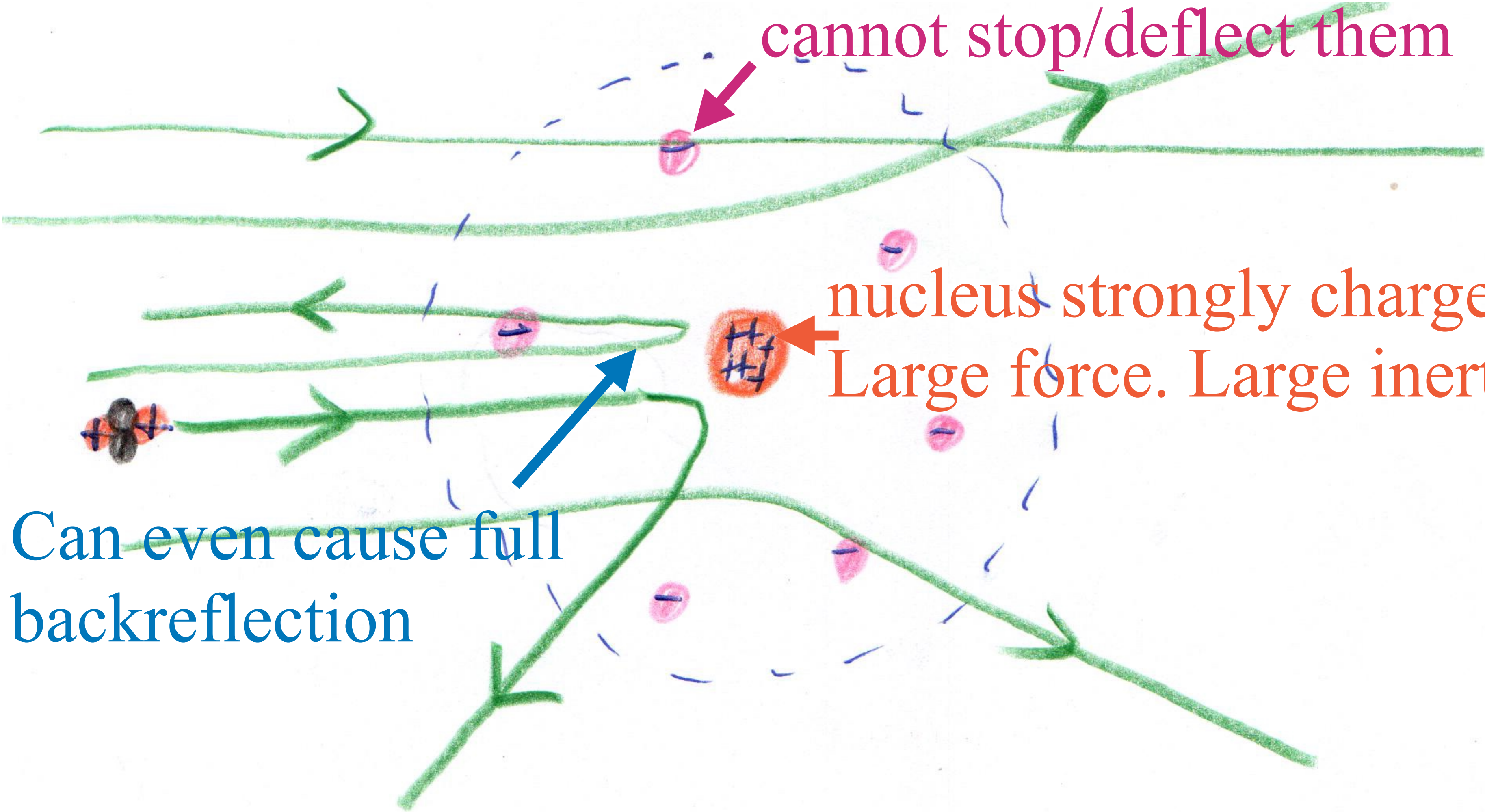
Rutherford model:



Rutherford's model of the atom (1909)

What to expect now for scattering:

Rutherford model:



Rutherford's model of the atom (1909)

Calculations (Beiser chapter 4 appendix):

Rutherford scattering formula:

$$N(\theta) = \frac{N_{inc} n d Z^2 e^4}{(8\pi\epsilon_0)^2 r^2 KE^2 \sin^4(\theta/2)} \qquad (68)$$

number of alpha particles hitting a unit area

scattering angle

- Definition of all other quantities, see Beiser book
- Let's boil it down to the main point….

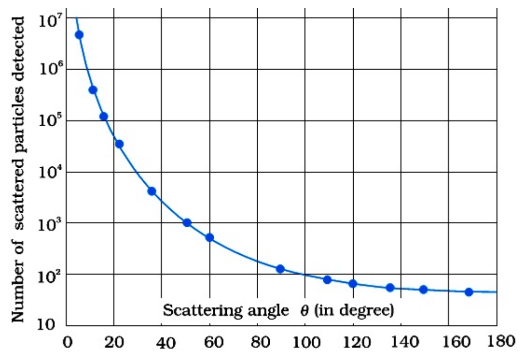
Rutherford's model of the atom (1909)

Rutherford scattering formula (II):

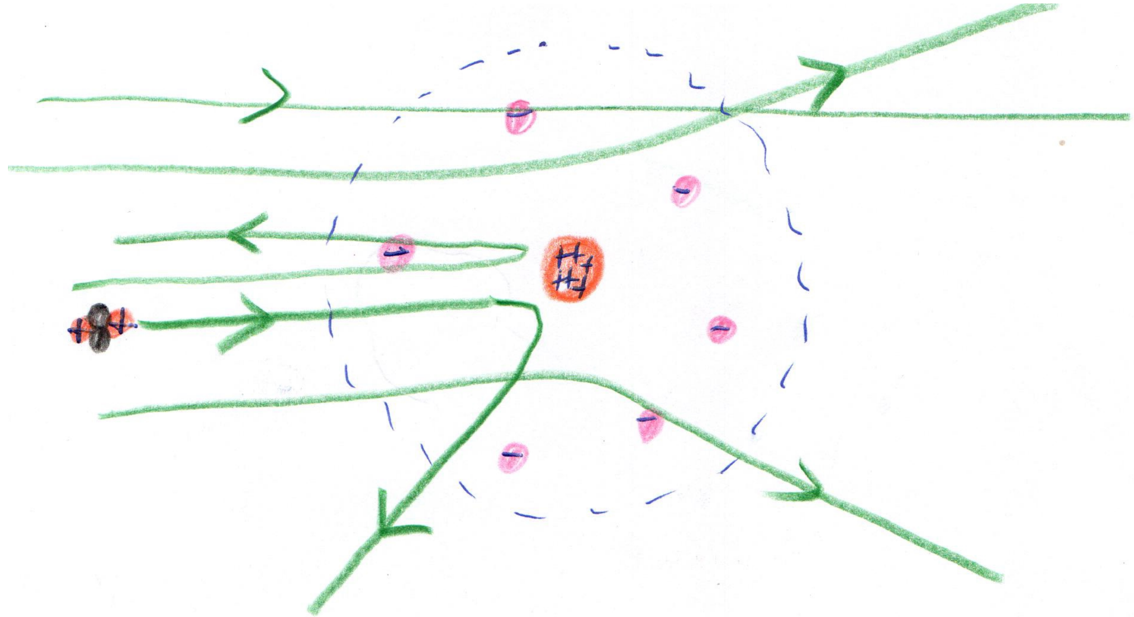
$$N(\theta) = \frac{const}{\sin^4(\theta/2)} \qquad \text{(68b)}$$

- $\sin(\theta/2)$ dependence decides the relative likelihood of large vs. small angle deflections

- Agrees with experimental results



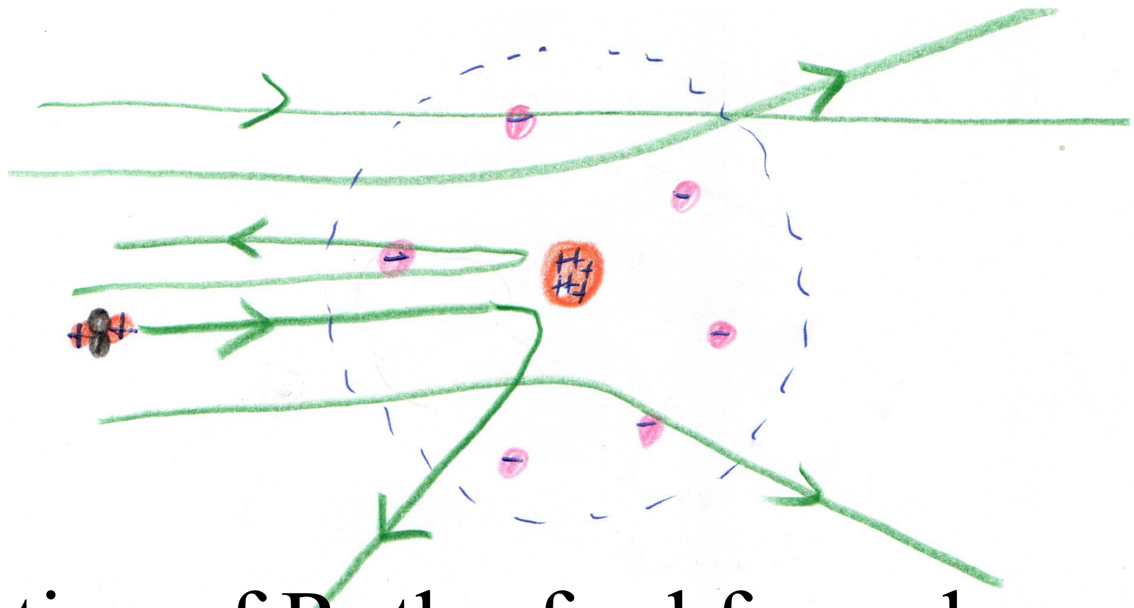
3.1.4) The nucleus of the atom



Calculation of Rutherford formula assumes the α-particle can come arbitrarily close to the nucleus

[based on Coulomb force Eq. (67), also applies only **outside** charges!!]

The nucleus of the atom



Calculation of Rutherford formula assumes the α-particle can come arbitrarily close to the nucleus

thus we infer a **tiny nuclear radius**

$$r_{nuc} < 1 \times 10^{-14} m \quad (69) \qquad r_{atom} \; 1 \times 10^{-10} m$$

that contains **almost all of the mass** of the atom

The nucleus of the atom

The two features above imply that nucleii are very(!) dense.

Example (carbon nucleus r=2.5 fm):

$$\rho_{nuc} = \frac{\textbf{mass}}{Volume} = \frac{m}{\frac{4}{3}\pi r^3} = 3 \times 10^{17}\ \textbf{kg/m}^3$$

Compare this with lump of lead (**Pb**):

$$\rho_{lead} = 1.1 \times 10^{4}\ \textbf{kg/m}^3$$

A needle-head filled with nuclear matter would weigh **1 million tons**!!!!

Example: Neutron Stars

There are actually macroscopic objects that have nuclear density, called Neutron Stars.

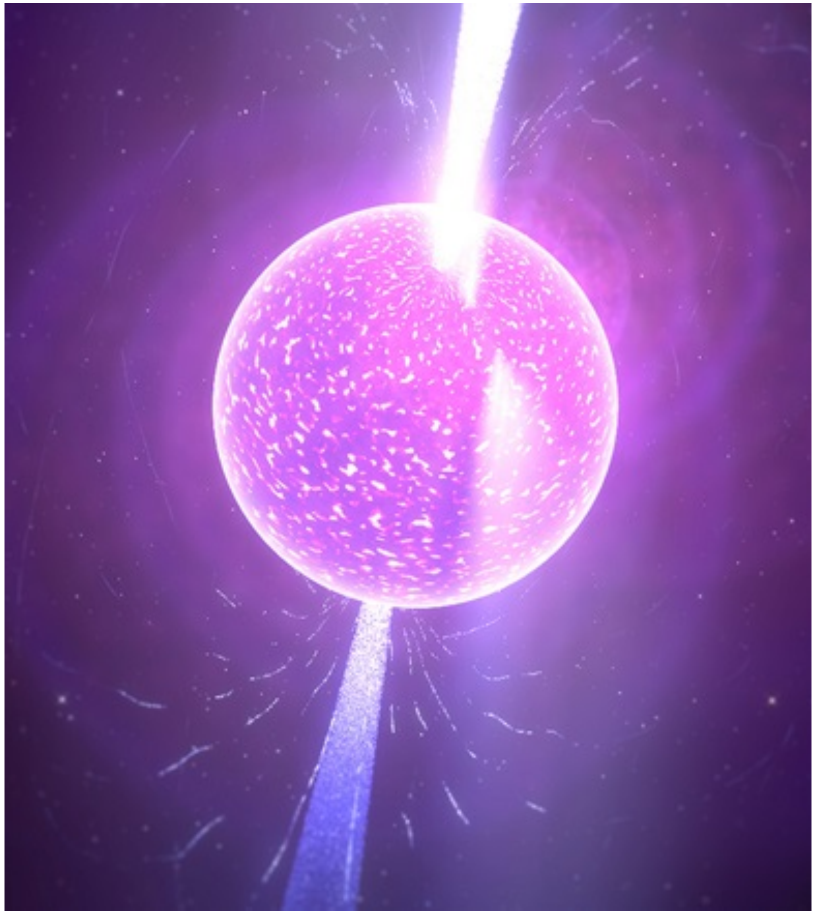
They have a radius of 10 or 20 km!!!

They consist entirely of nuclear matter.

They have a mass M

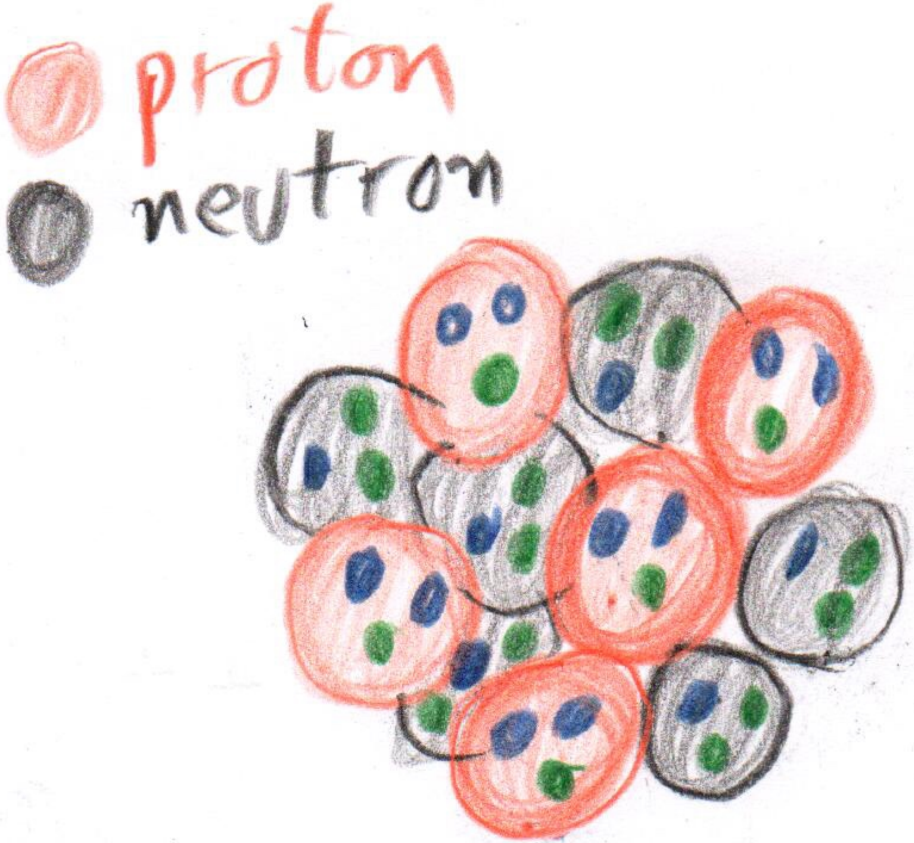
$1.5\ M_{\odot} < M < 3\ M_{\odot}$

($M_{\odot} = 2 \times 10^{30}$ **kg** is the mass of our sun)



picture: (c) Kevin Mc Gill

Bonus example: Inside of a nucleus?



- Nucleii are made of **neutrons** and **protons**

  electric charge: q=0 q=+e>0

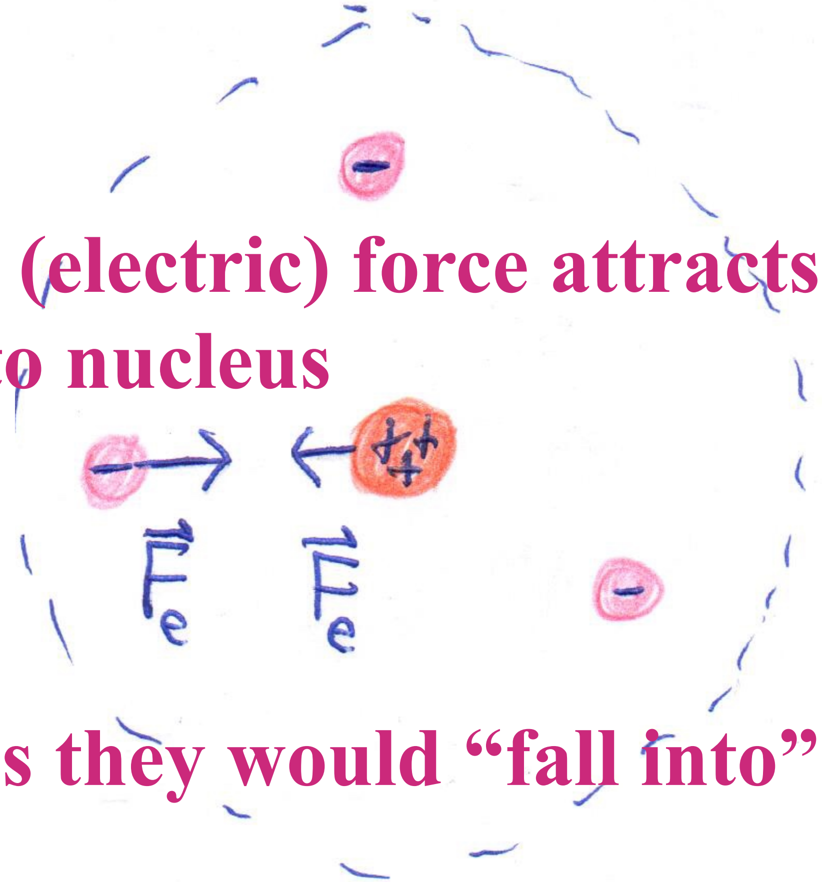
- These are made of **up-quarks** and **down-quarks**

  q=2/3 e q=-1/3 e

3.1.5) Electron orbits

Now let's turn from the nucleus to the electrons…..

Coulomb (electric) force attracts electron to nucleus

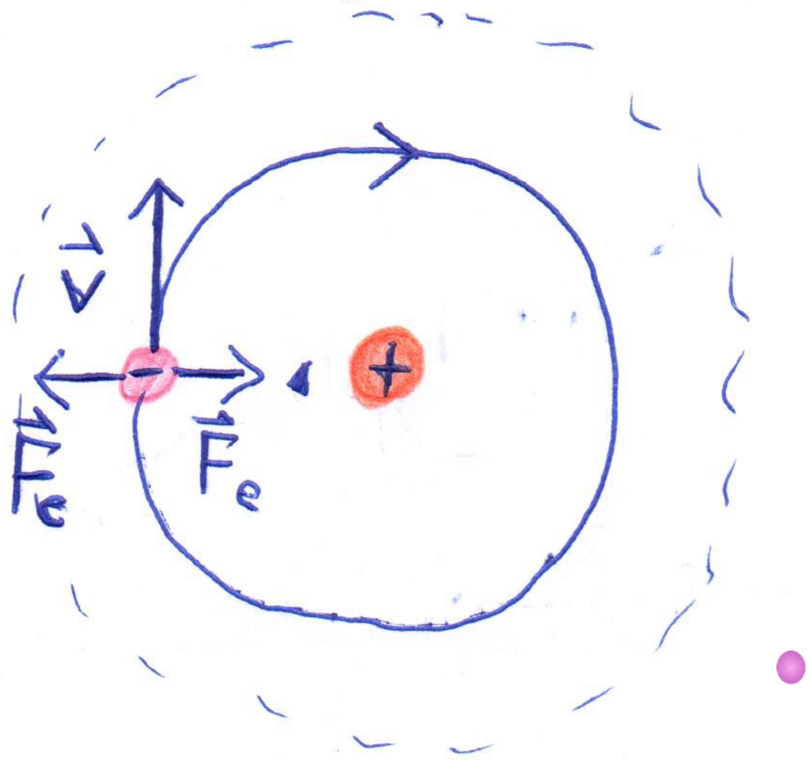


This means they would "fall into" the nucleus

If they are not embedded in anything (Thomson model), what keeps electrons in place ??…..

Electron orbits

Could be like in a miniature solar system….



Force laws same structure!

Electric force

$$|F_e| = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

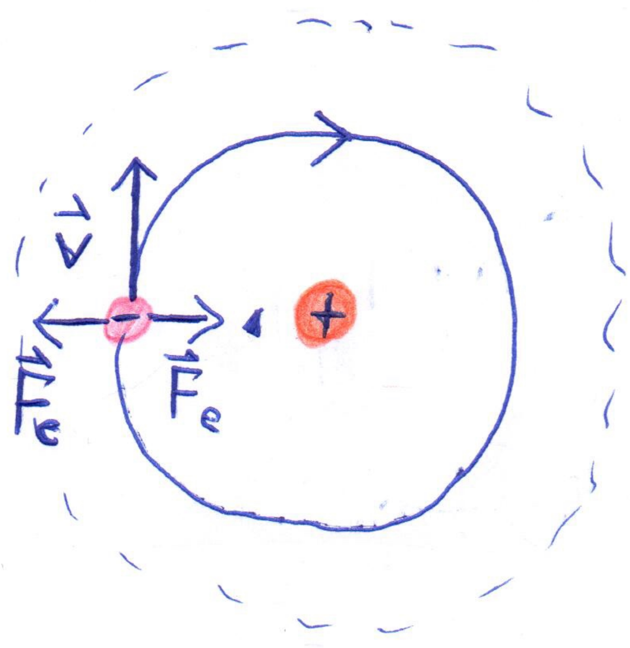
Gravitational force

$$|F_g| = G\frac{M_1 M_2}{r^2}$$

Centrifugal force $\mathbf{F_c}$ of **orbital motion** could keep electron away from nucleus, like planets away from the sun.

Electron orbits

Lets calculate how this would work for a **Hydrogen** atom (nuclear charge e)



Force from nucleus on electron $$F_e = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

Centrifugal force (see PHY101) $$F_c = \frac{mv^2}{r}$$

Equal in stable orbit $$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

Required **velocity** of electron for **circular orbit** of radius r

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}} \quad (70)$$

Electron orbits

Total energy is: $E = E_{kin} + E_{pot}$

$$E = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r}$$

Energy of **electron orbiting** proton at distance **r**

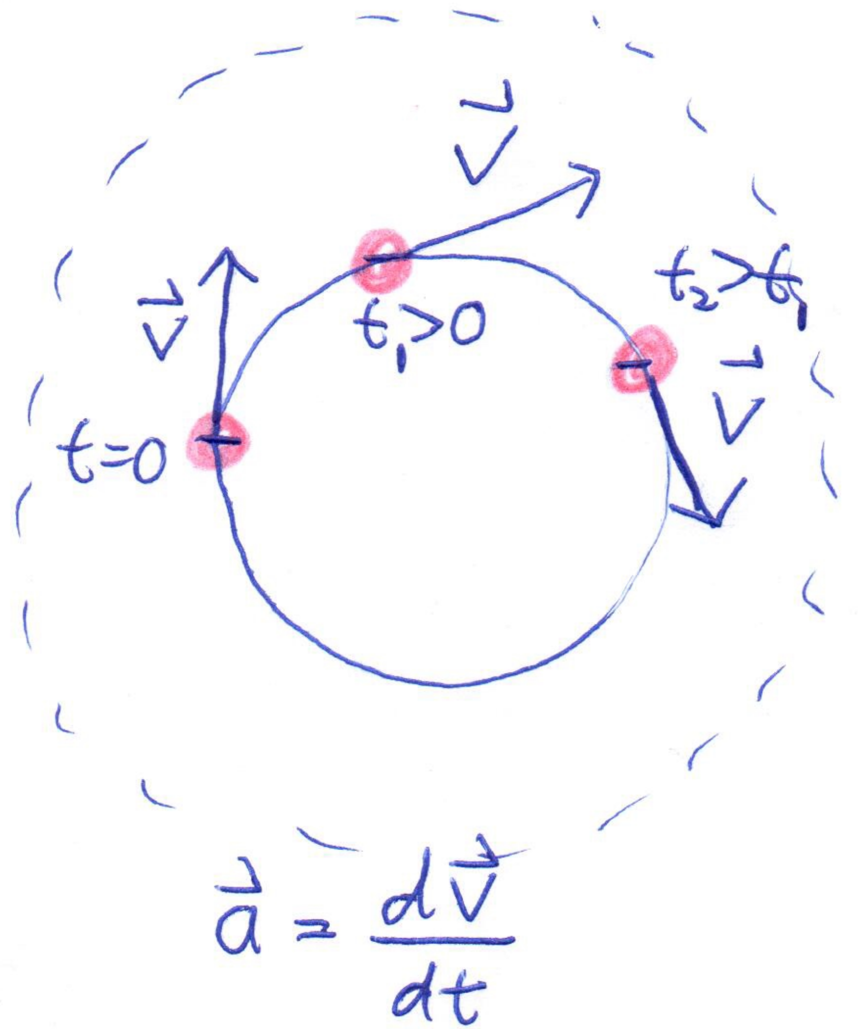
$$E = -\frac{e^2}{8\pi\epsilon_0 r} \quad (71)$$

- In this **classical** calculation, **any radius** and hence **any** (negative) **energy** is allowed

Electron orbits

Circular orbit means electron is **always** being **accelerated**

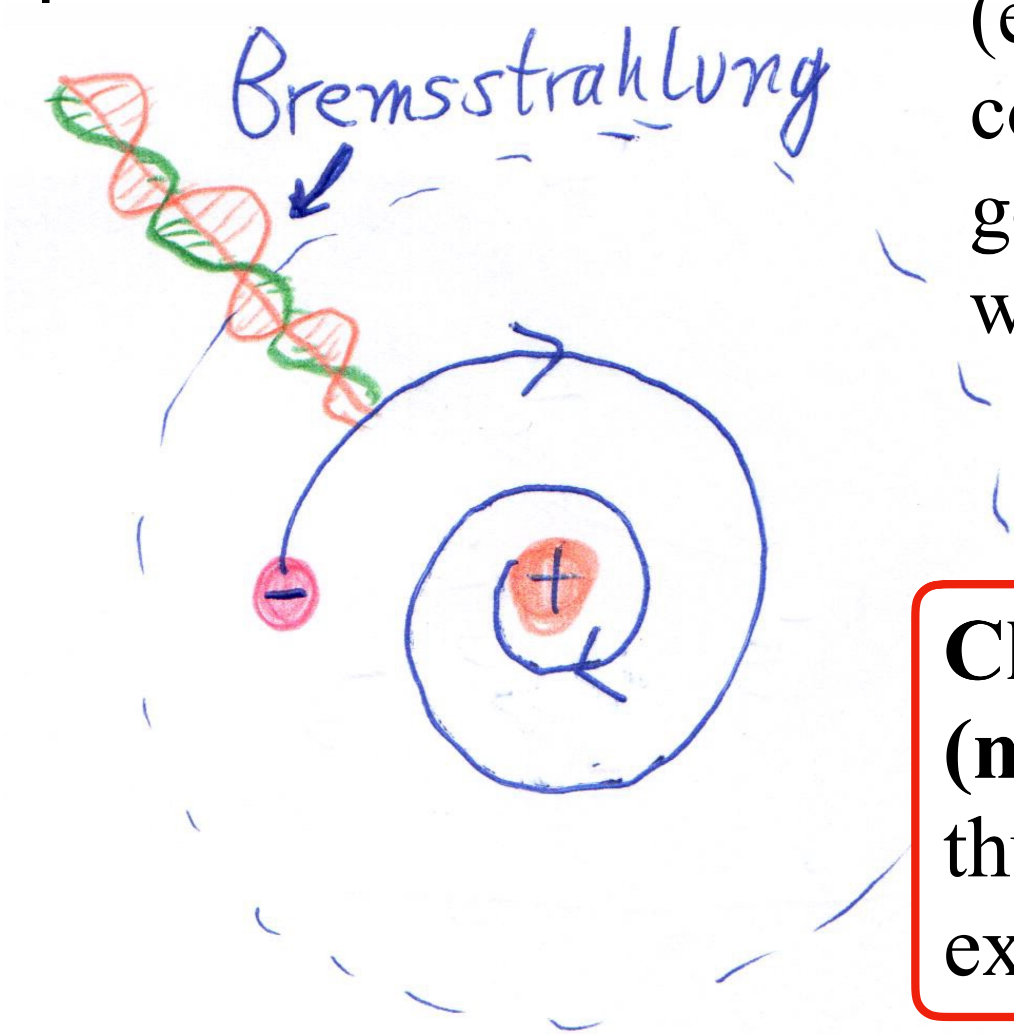
$$\overrightarrow{a} = \frac{d}{dt}\overrightarrow{v}$$



Accelerated charges emit radiation
(see Bremsstrahlung, section 2.2.4)
Thus the electron would loose energy

Decaying electron orbits

Since it looses energy, the electron would spiral into the nucleus



(energy of radiation has to come from electron so E goes down. From Eq. (71) we see that r thus reduces)

Classical physics (mechanics+electro-mag) thus fails to explain the existence of stable atoms

3.1.6) Atomic spectra

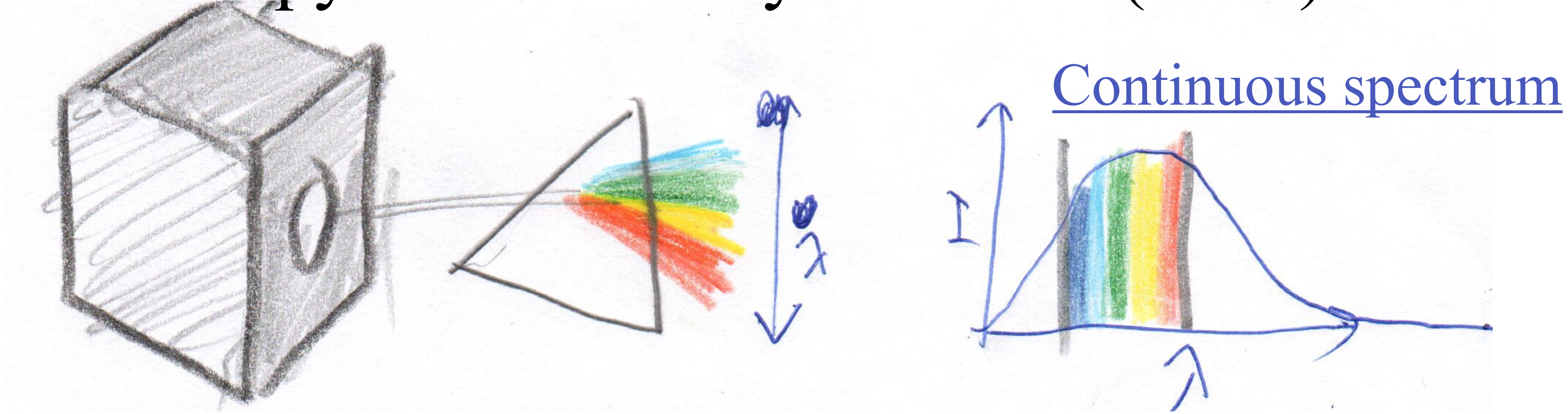
There is another observation that the planetary orbit model for the electron cannot account for…

In black-body radiation (2.2.1) the purpose of the "black-body" concept was only to remove all dependence on material.

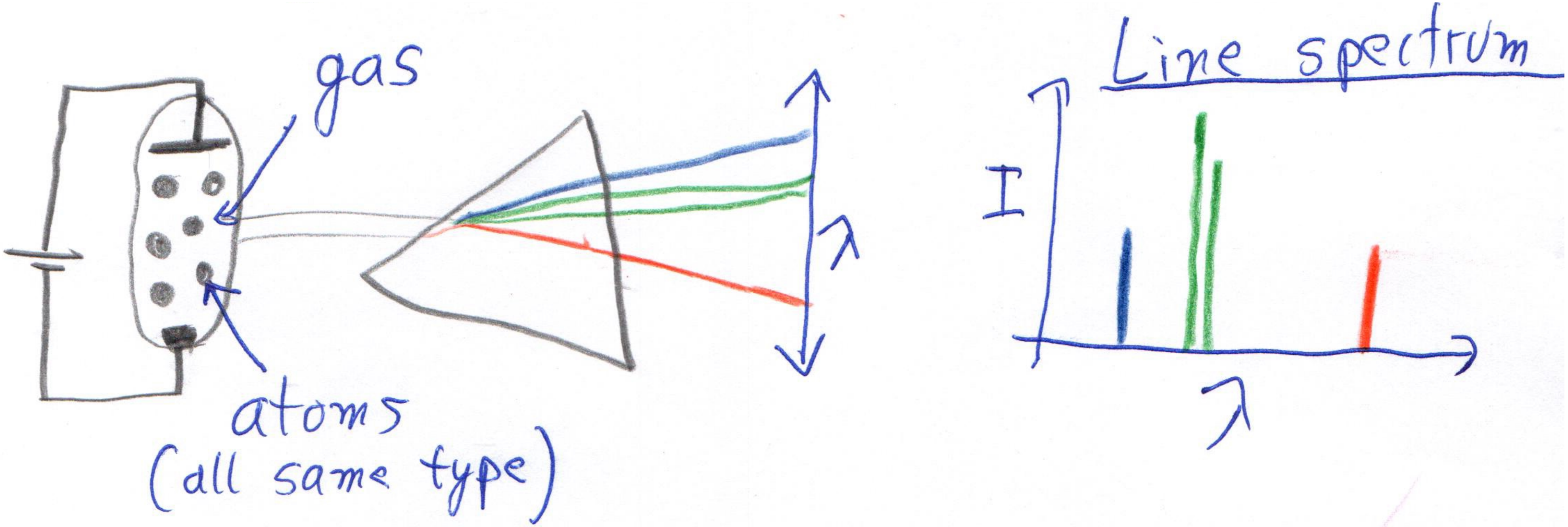
In contrast, if we directly look at emission of a certain specific **atom**, spectra look very different:

Atomic spectra

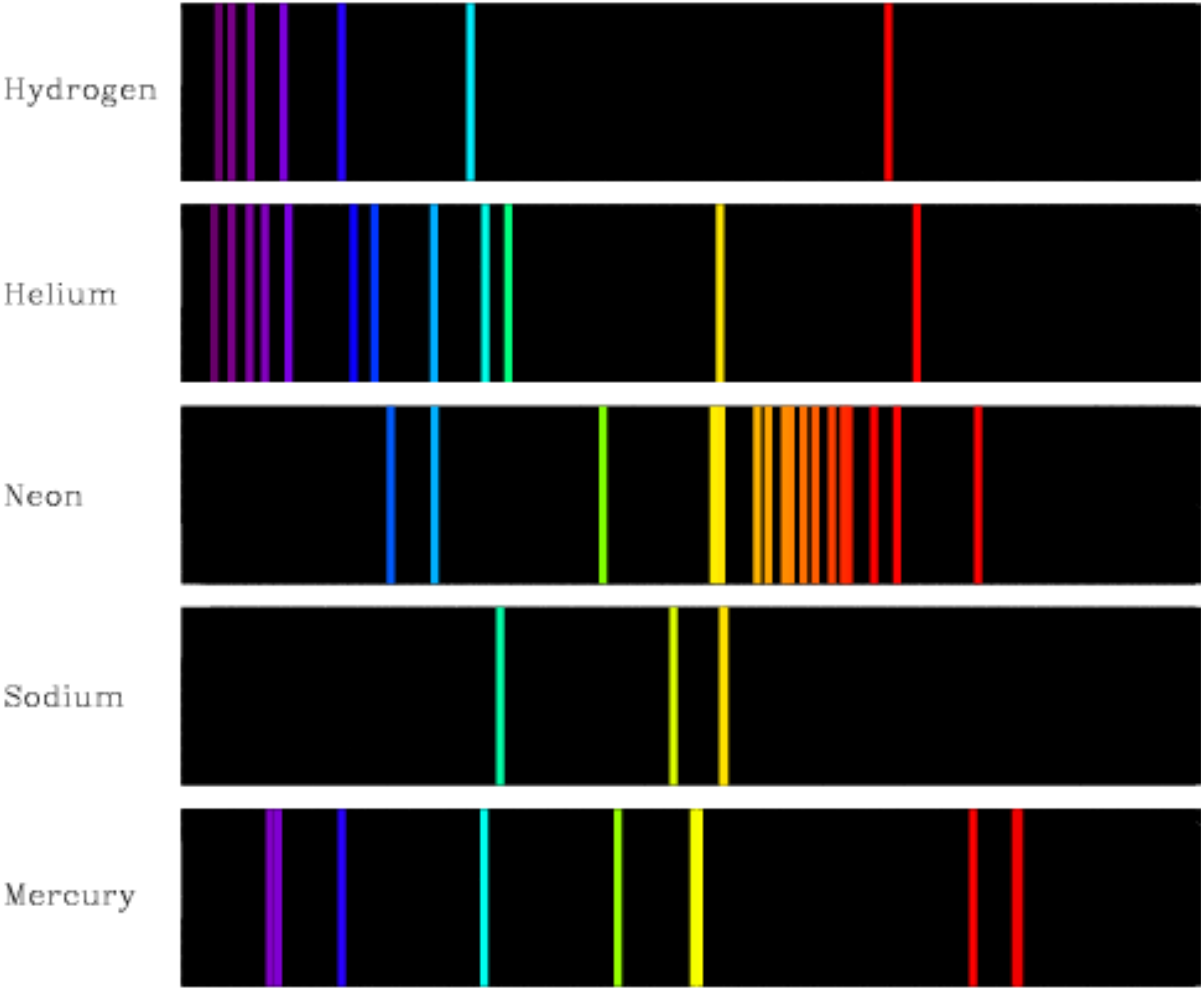
Spectroscopy of black-body-radiation (2.2.1)



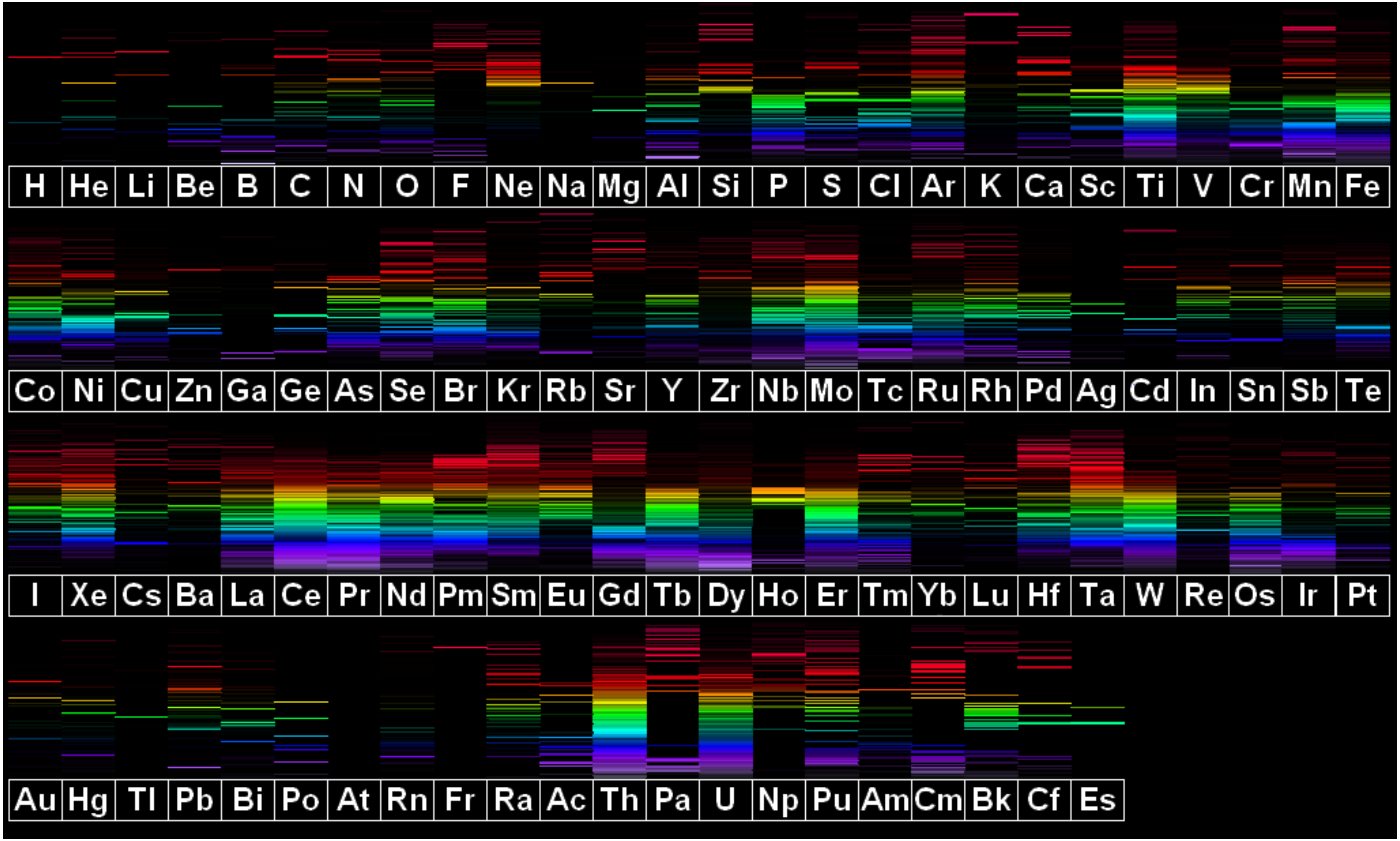
Spectroscopy of emission from a mono-atomic gas



Atomic spectral lines are different for each atom



All atomic spectral lines of the period table

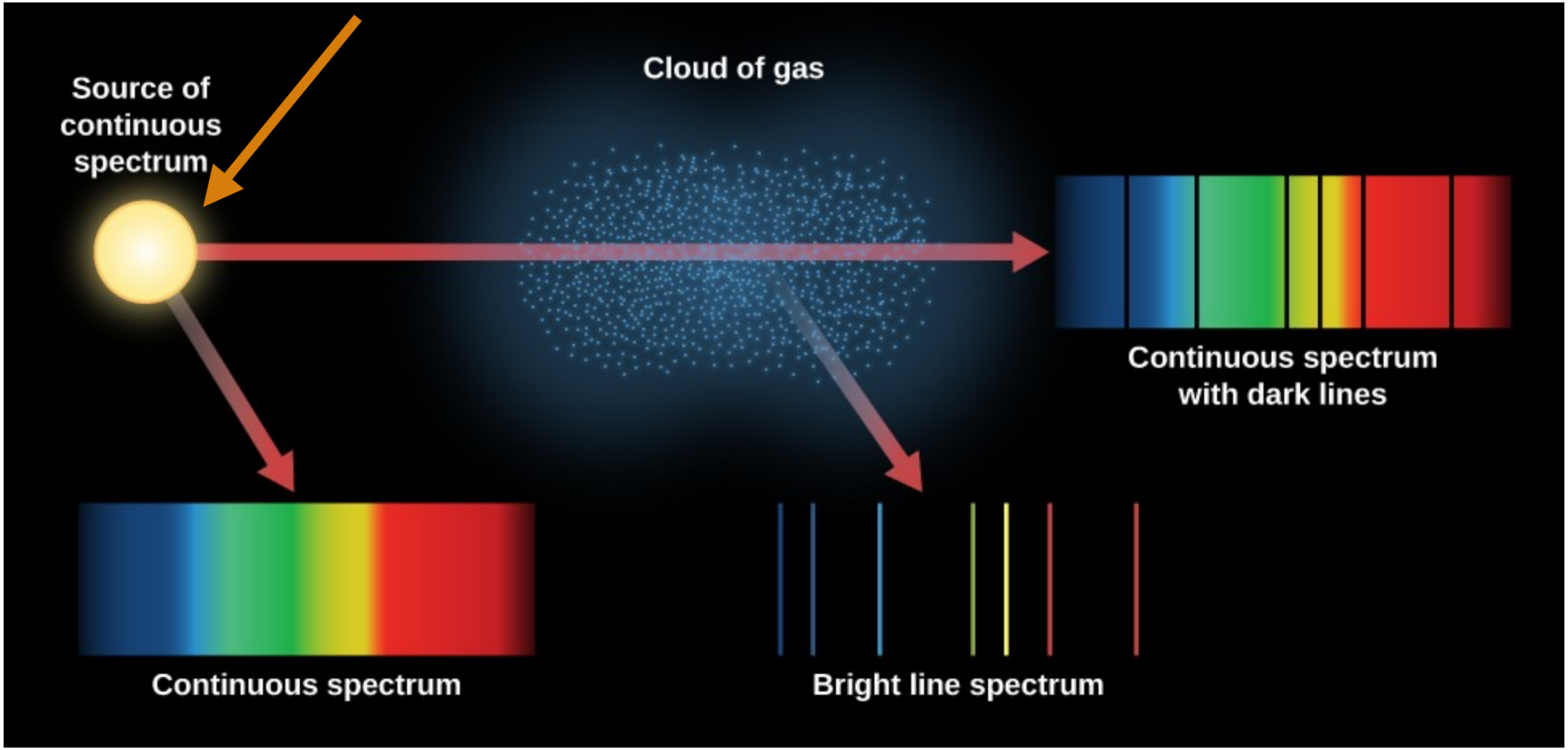


http://www.umop.net/spctelem.htm

Atomic spectra Emission versus absorption lines

Also when atoms absorb, this causes specific lines

Star = blackbody= continuum emission



Atomic spectra

- Details of spectral lines (e.g. widths) depend on external fields, temperature, pressure of the gas!

- Can use this to learn a lot, even about a remote star

Atomic spectra

Hydrogen has one of the simplest spectra, we find spectral lines follow:

Spectral series formula: $$\frac{1}{\lambda} = R\left(\frac{1}{a^2} - \frac{1}{n^2}\right) \qquad (72)$$

- λ is wavelength of line, a constant integer.
- n integer>a.
- R is the **Rydberg constant** $R = 1.097 \times 10^7\ \mathbf{m}^{-1}$

Atomic spectra

Spectral lines imply photons (energy quanta) of specific energies only

Thus atoms might only have specific energy states

But (i) electrons are matter waves (week 6)
(ii) confined matter waves might only have discrete energies (2.4.3)

Thus: Let's build a model of the atom based on the matter-wave concept

3.1.7) Bohr's model of the atom (1913)

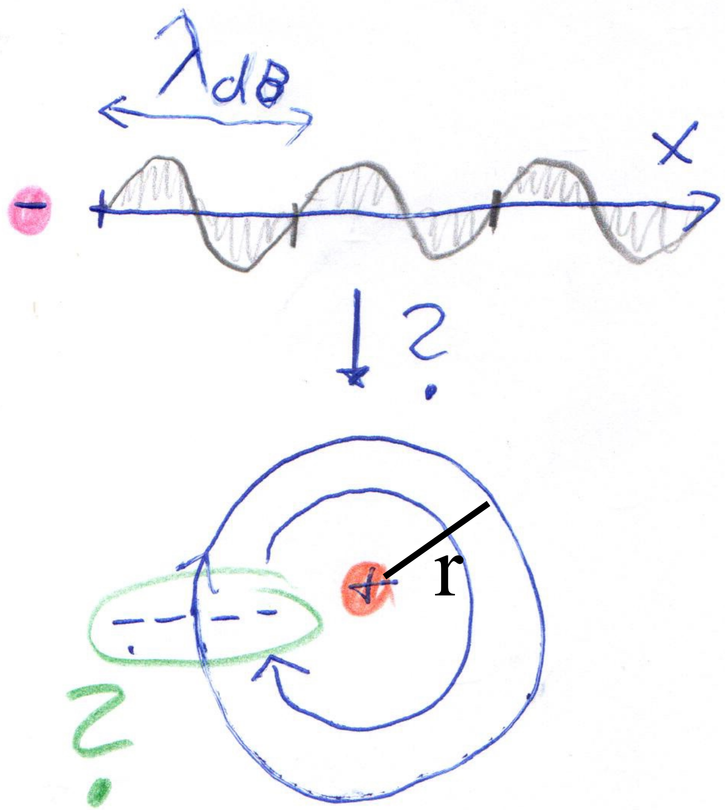
What happens if we try to wrap a wave into an orbit?

We require (n integer)

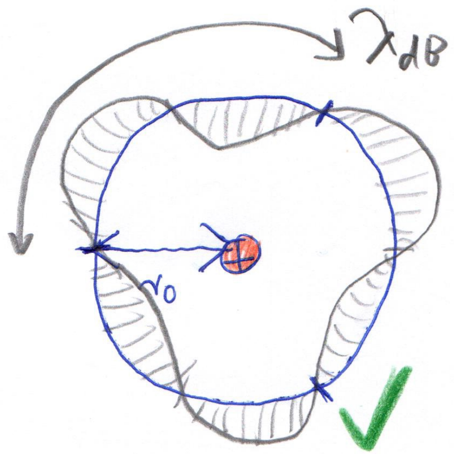
$$n\lambda_{dB} = (2\pi r_n) \qquad (73)$$

electron de-Broglie wave-length

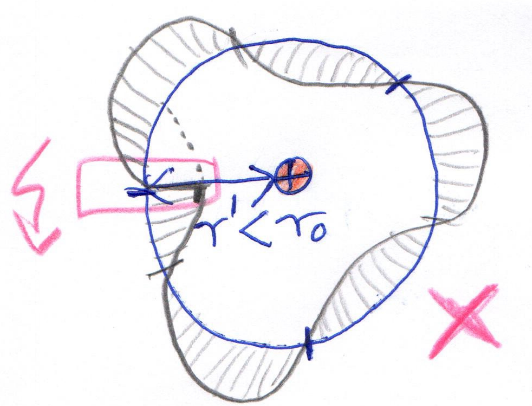
orbit circumference



Periodic boundary condition, wave has to match itself!!!!



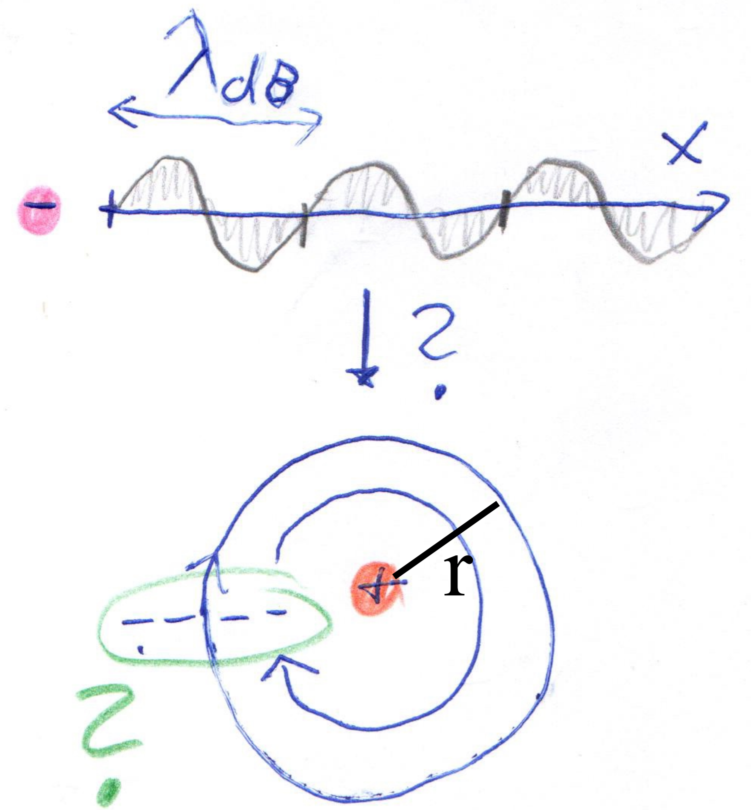
This works



This doesn't

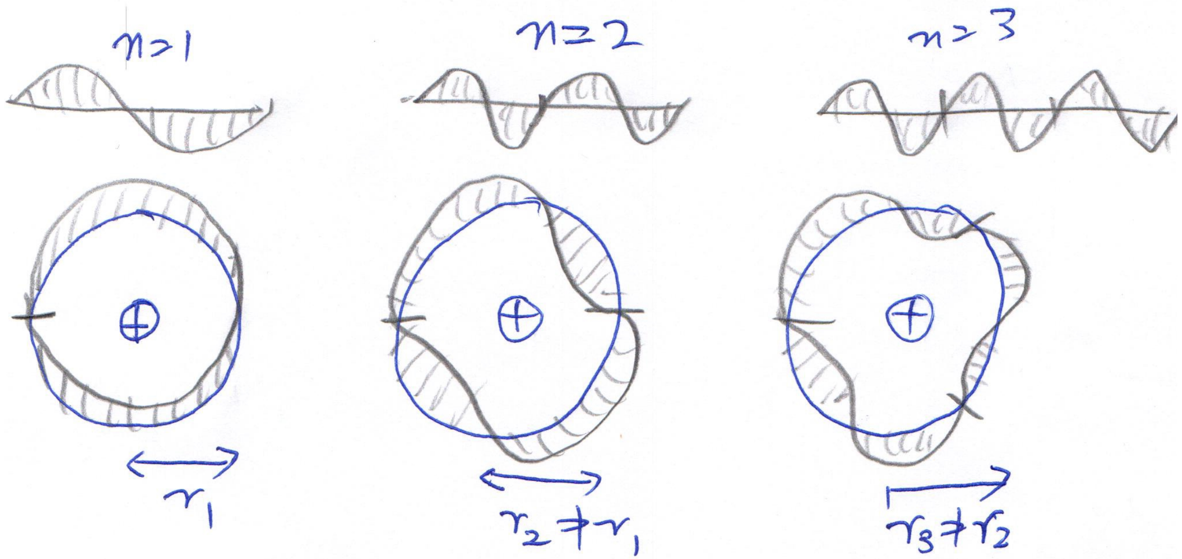
3.1.7) Bohr's model of the atom (1913)

What happens if we try to wrap a wave into an orbit?



Periodic boundary condition, wave has to match itself!!!!

For different integers n, we get different types of orbiting electron waves:



Size of good orbit depends on n

3.1.8) Energy levels and spectra

Let's see what these picture give us when we do the math….

We get electron wave length from Eq. (70):

$$\lambda_{dB} = \frac{h}{p} = \frac{h}{mv} = \frac{h}{e}\sqrt{\frac{4\pi\epsilon_0 r_n}{m}} \overset{!}{=} (2\pi r_n)/n$$

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 m r}}$$

Solve for r_n

Orbital radii in Bohr's atom:

$$r_n = \frac{h^2\epsilon_0}{\pi m e^2} n^2 = a_0 n^2 \quad (74)$$

Energy levels and spectra

Orbital radii in Bohr's atom: $$r_n = \frac{h^2\epsilon_0}{\pi m e^2} n^2 = a_0 n^2$$ (74)

- n >0 is an integer, called the **quantum number** of the orbit
- a_0 is the radius of the innermost orbit, called **Bohr radius** $$a_0 = 5.292 \times 10^{-11}\ \textbf{m}$$ (74b)

Also get electron energy from Eq. (71)

Hydrogen electron energies $$E_n = -\frac{me^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n^2}\right)$$ (75)

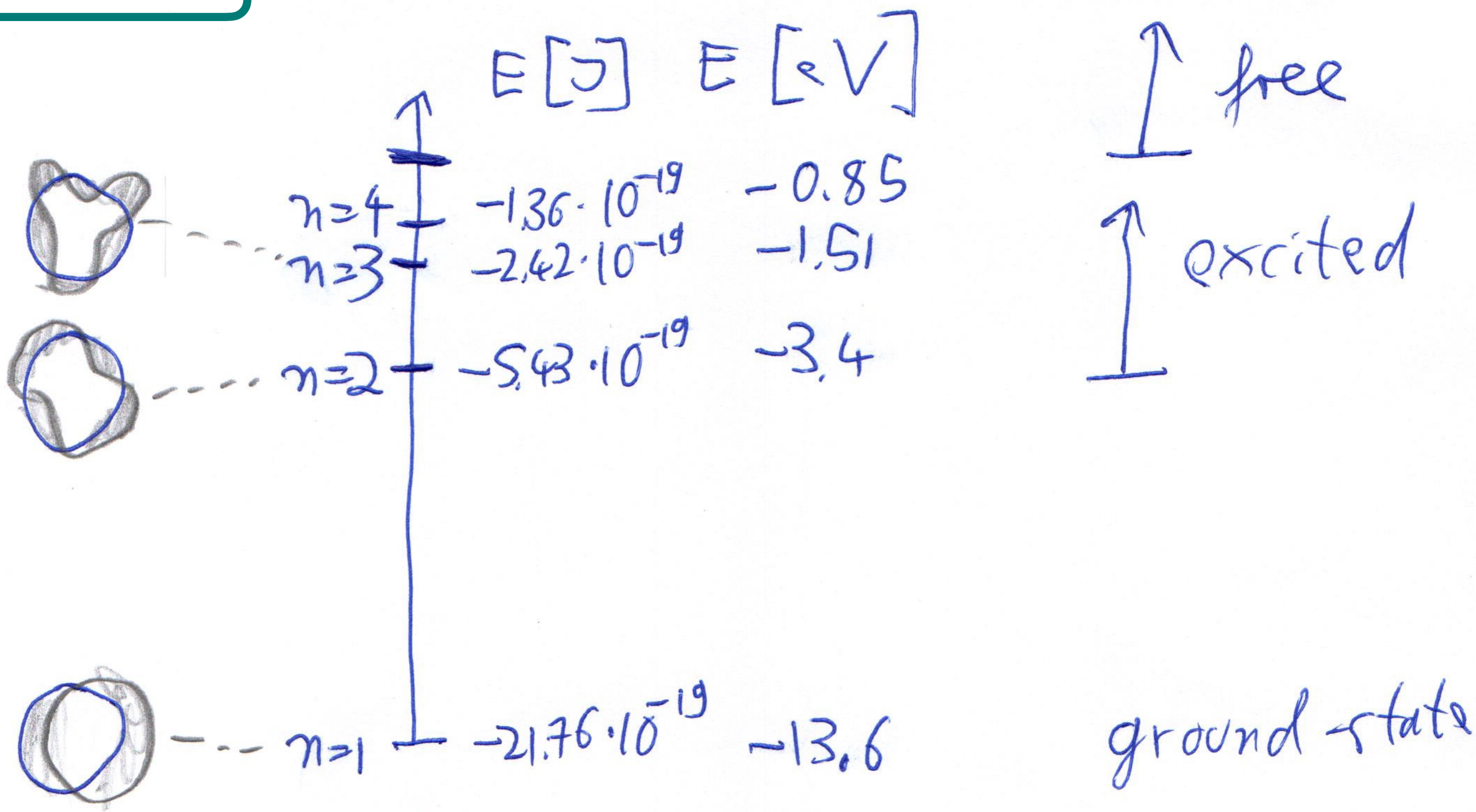
Q: Seen $1/n^2$ somewhere?

Energy levels and spectra

Hydrogen electron energies

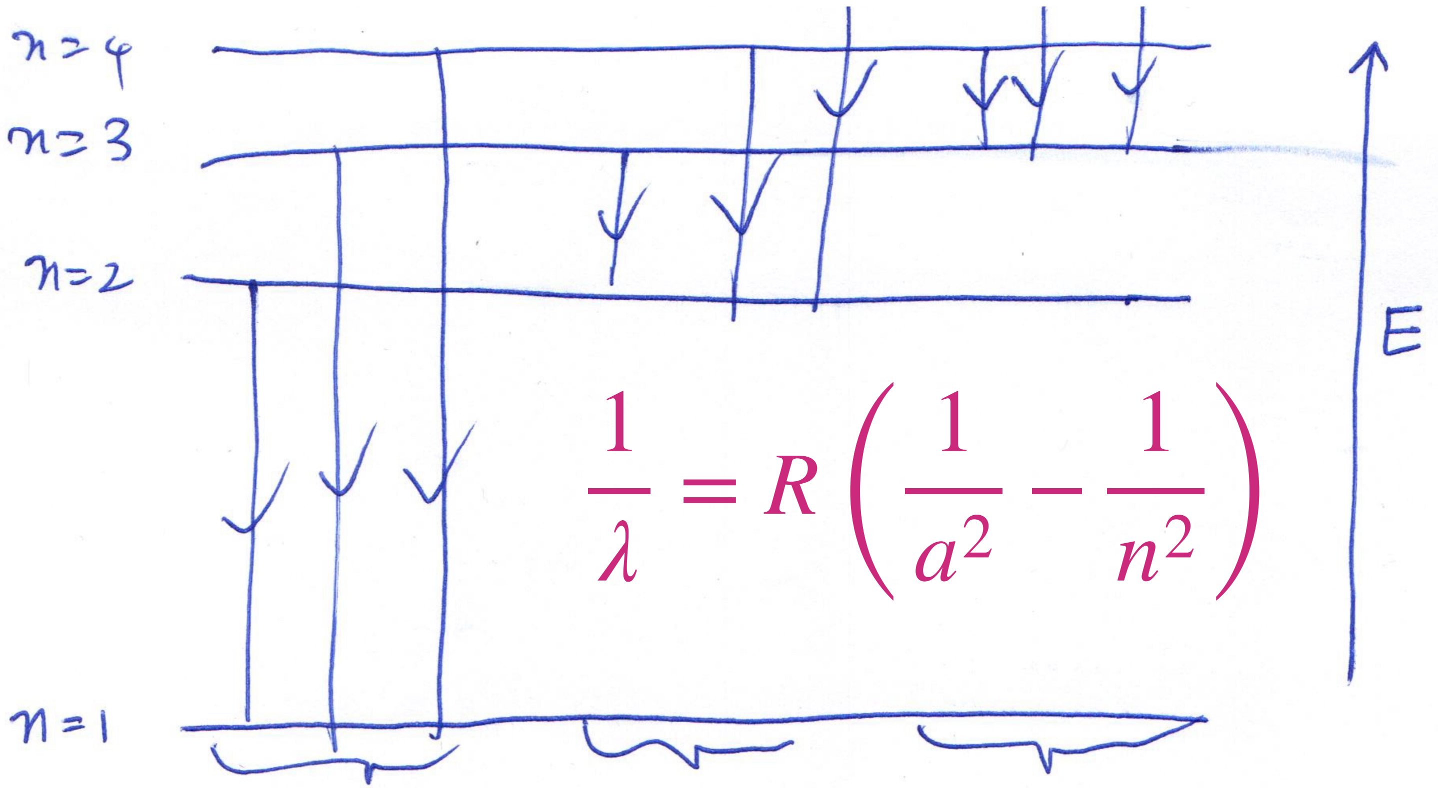
$$E_n = -\frac{me^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n^2}\right) \qquad (75)$$

- these are called **energy levels**
- pre-factor = 13.6 eV



Energy levels and spectra

Now photons in emission line spectra, must have gotten their energy from **transitions** between these **levels** → spectral series (Eq. 72)



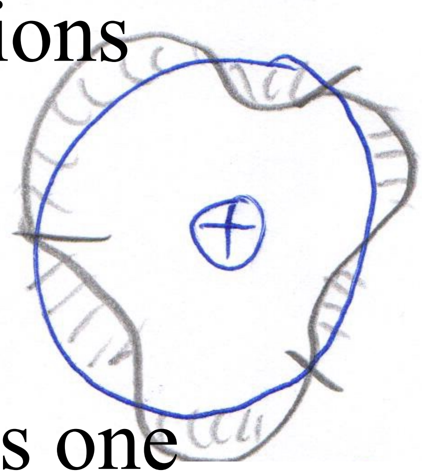
$$\frac{1}{\lambda} = R\left(\frac{1}{a^2} - \frac{1}{n^2}\right)$$

series: a=1 *Lyman* | a=2 *Balmer* | a=3 *Paschen*

Summary Bohr's atom model

Successfully describes **spectral lines of Hydrogen**

Matter waves of electrons can only form certain discrete energy standing waves. Transitions between these cause spectral lines.



Successfully predicts **stable atom**:

The lowest energy state is n=1. Thus this one must be stable. Cannot jump to lower state, so no radiation/photon can be emitted.

Sadly it miserably fails for spectral lines of larger atoms. We need sth. better (week 8)

3.1.9) Correspondence principle

Bohr's model implies turning away from classical physics. But classical works for large things…?

See book: For very large n~400, classical model of radiation emission and quantum again agree. For low n, huge deviations!!!

This is the case in general:

Quantum physics tends to agree with classical physics when the **quantisation** becomes negligible

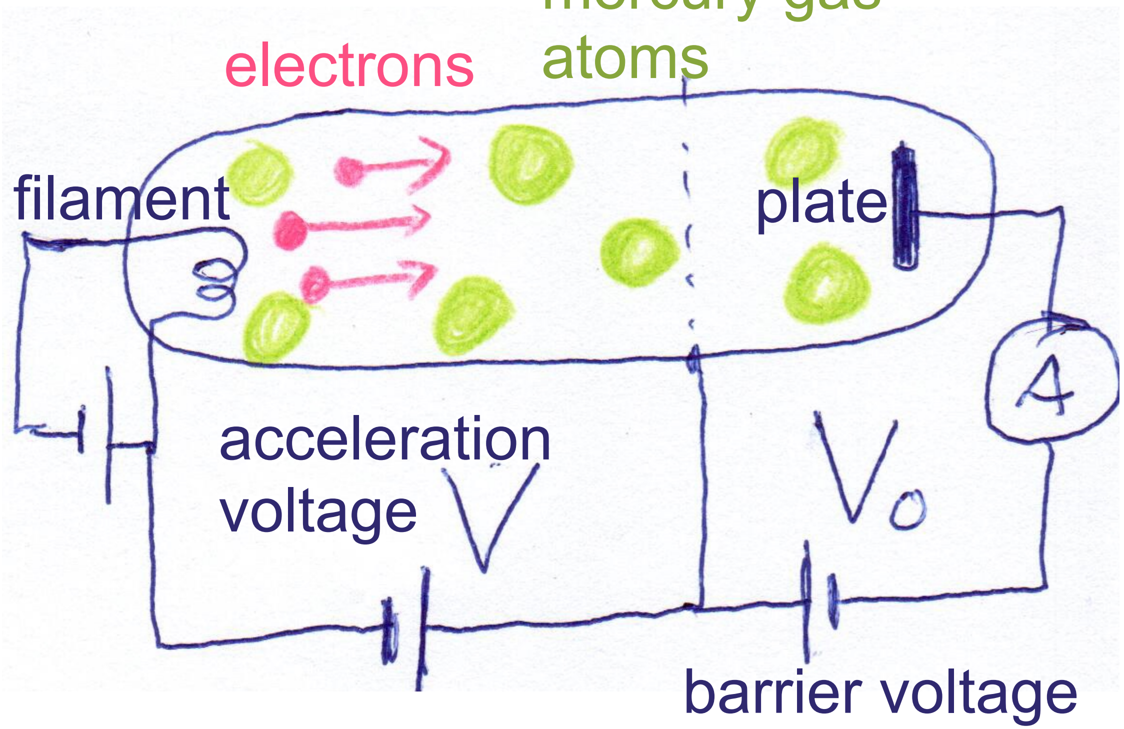
E.g. $r_{400} \approx r_{401}$ in Eq. (74).

3.1.10) Atomic absorption and emission

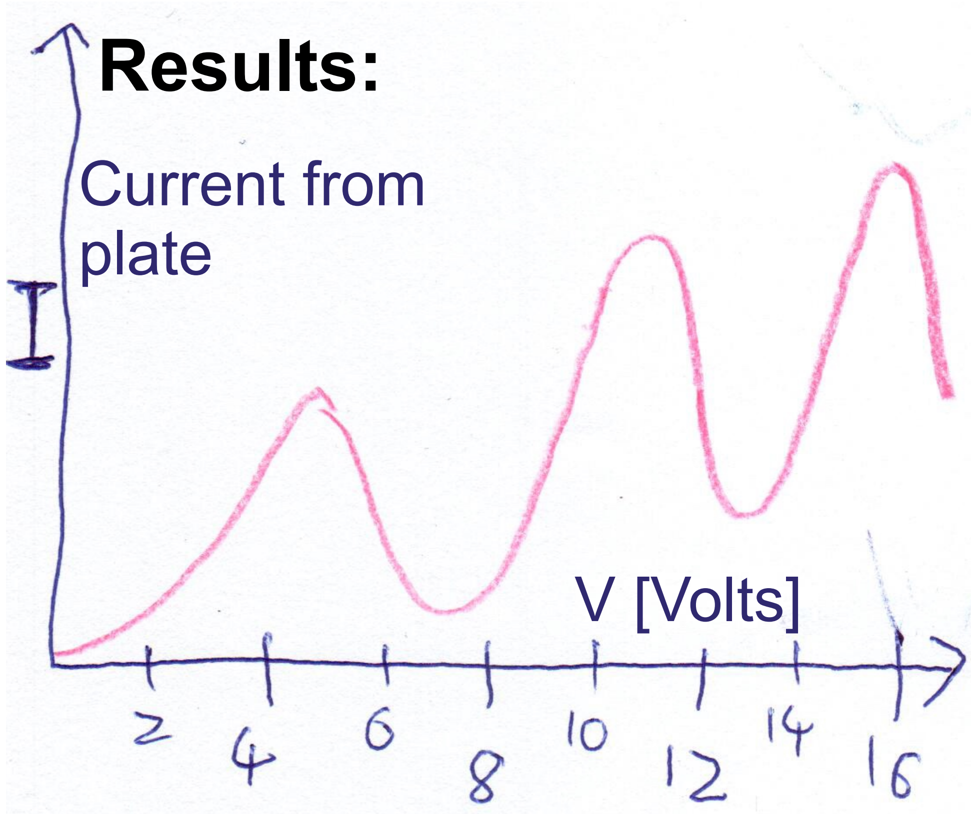
We want independent confirmation of electronic energy levels, not using photons…

Franck Hertz experiment (1914): Inelastic collisions (free electrons with atoms)

Apparatus:

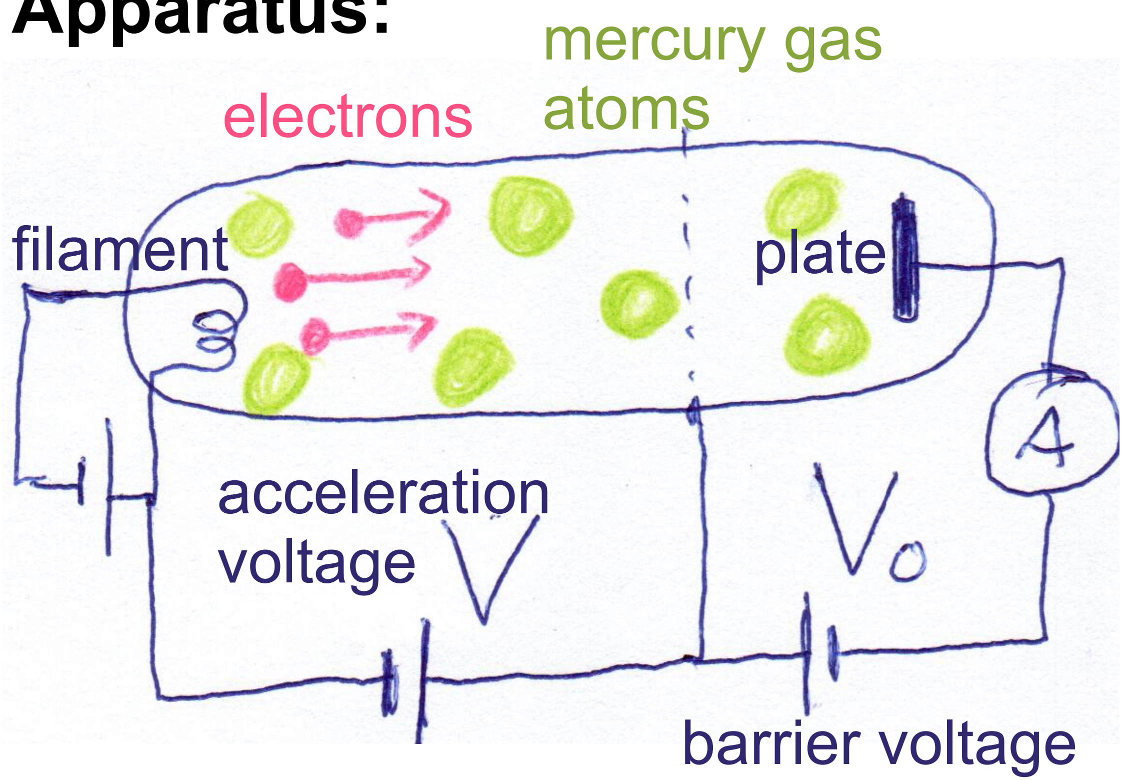


Results:

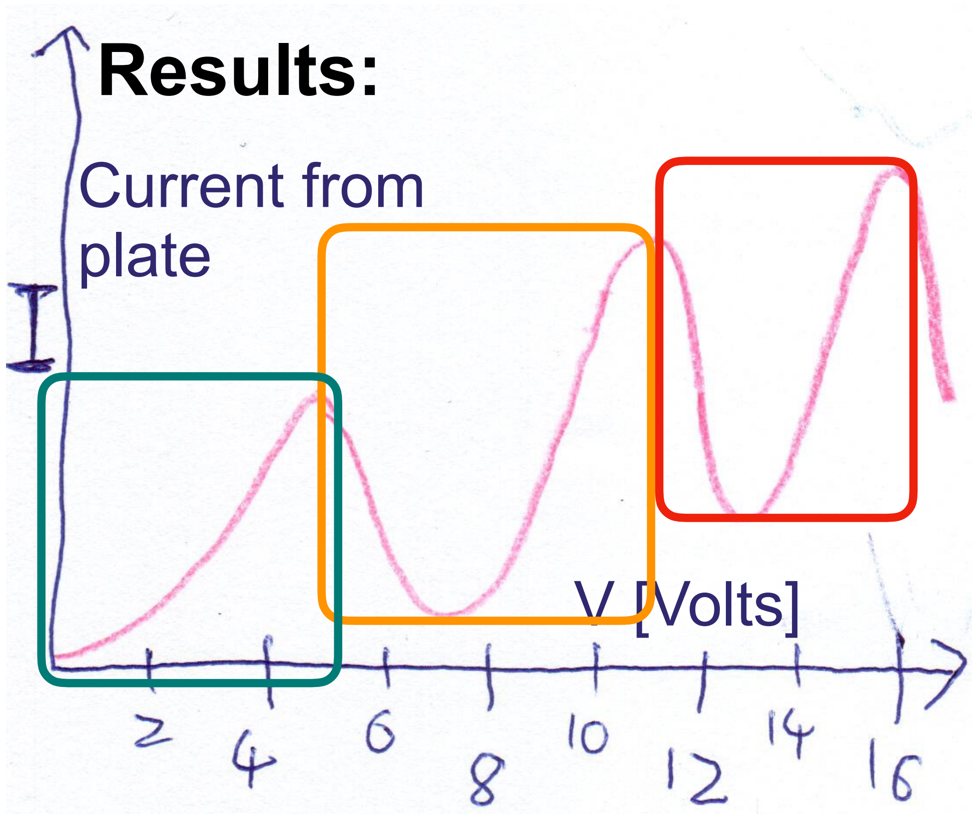


Atomic absorption and emission

Apparatus:

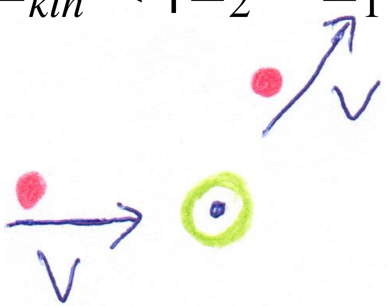


Results:



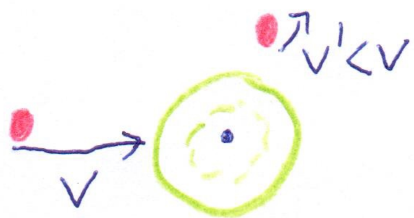
Elastic collisions

$$E_{kin} < |E_2 - E_1|$$

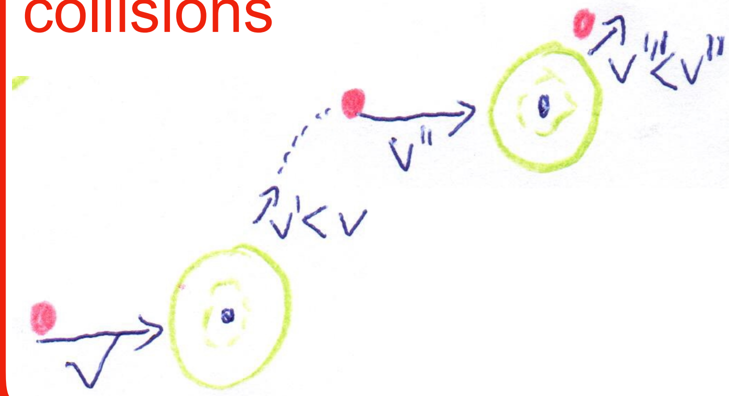


One in-elastic collision

$$|E_2 - E_1| < E_{kin}$$



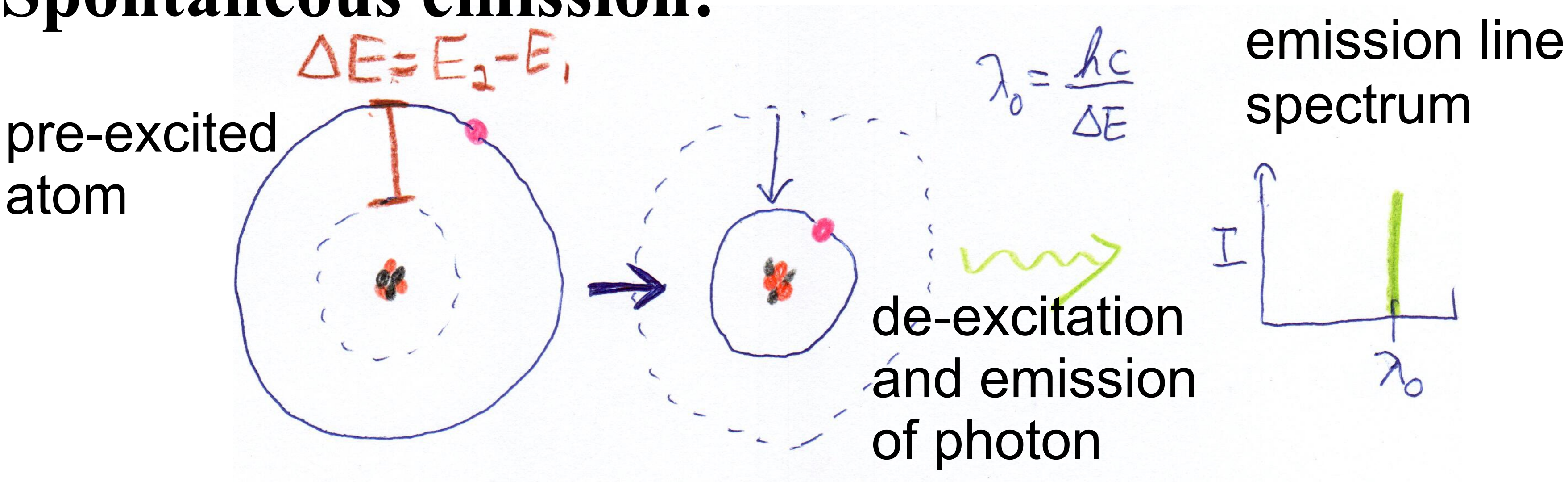
Two sequential in-elastic collisions



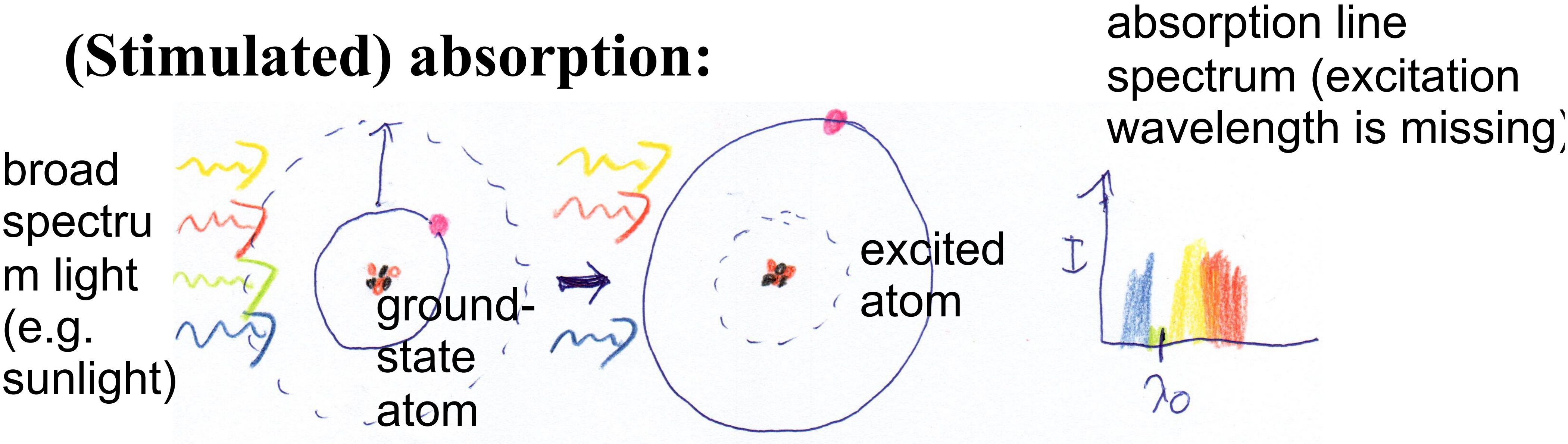
Atomic absorption and emission

Saw three processes for atoms to absorb or emit energy:

Spontaneous emission:

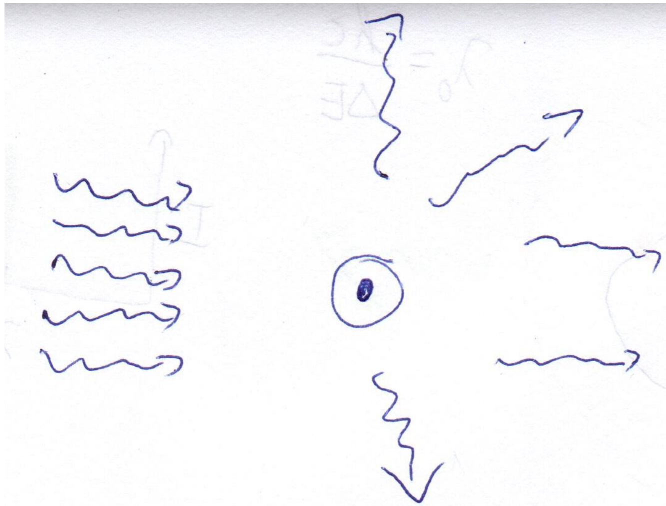


(Stimulated) absorption:



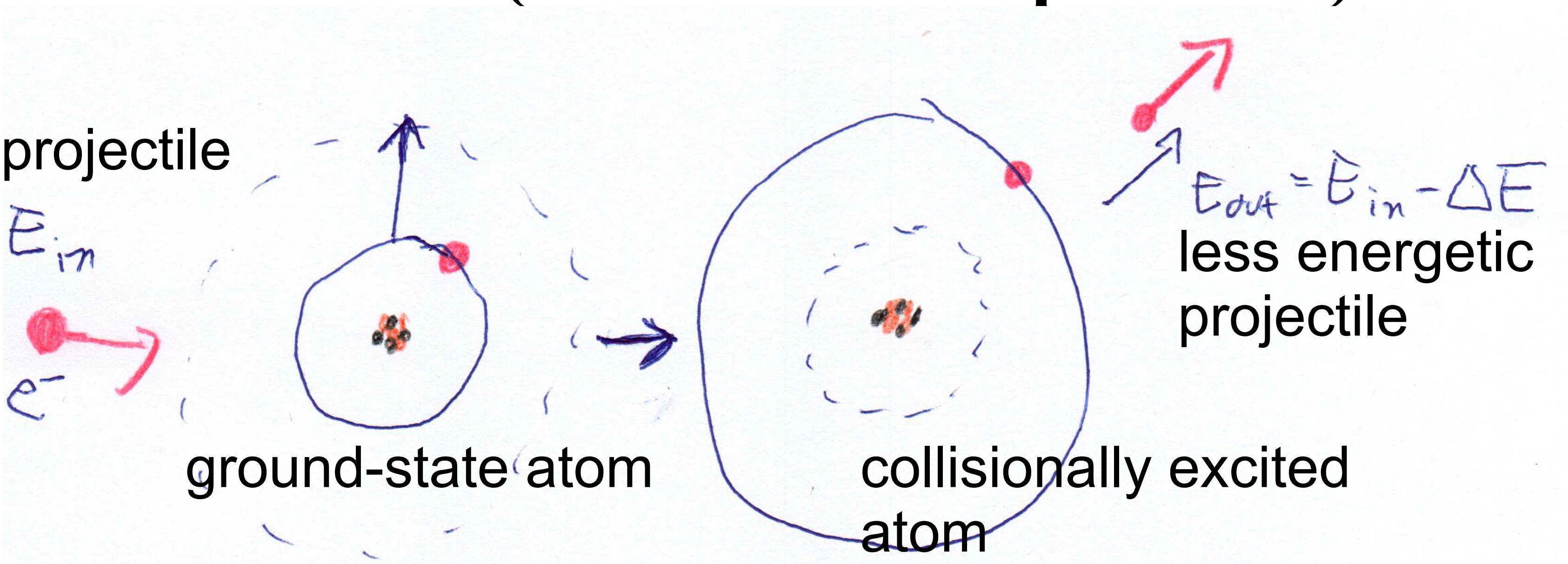
Atomic absorption and emission

Q: Why does emission following absorption not put missing light “back in”???



A: re-emission is in ***all*** *directions, thus typically out of the beam*

Collisional excitation (Frank Hertz experiment)

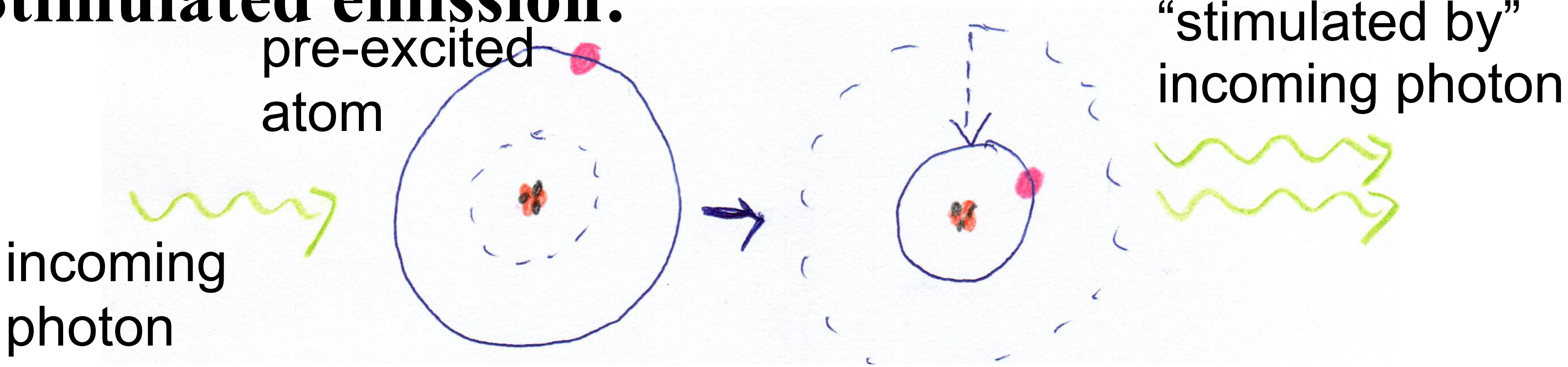


Example: Lasers

Now we understood atoms, let’s use then as tool for technology

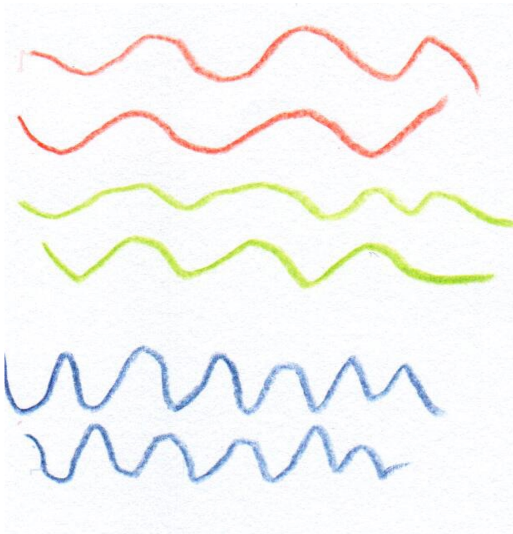
Third way atoms can interact with photons:

Stimulated emission:

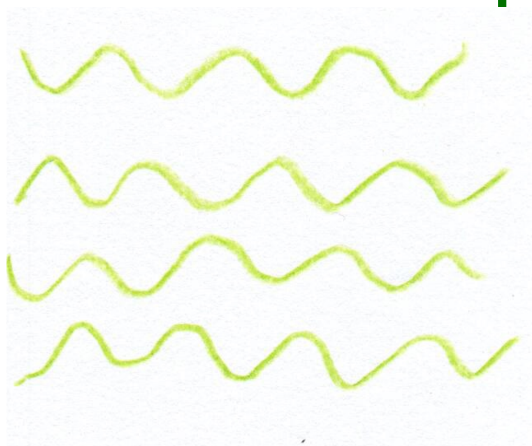


Emitted photon **copy** of incoming one = “coherence”

incoherent sunlight. Many wavelength, random phase relations



incoherent monochromatic light. ~one wavelength, random phase relations

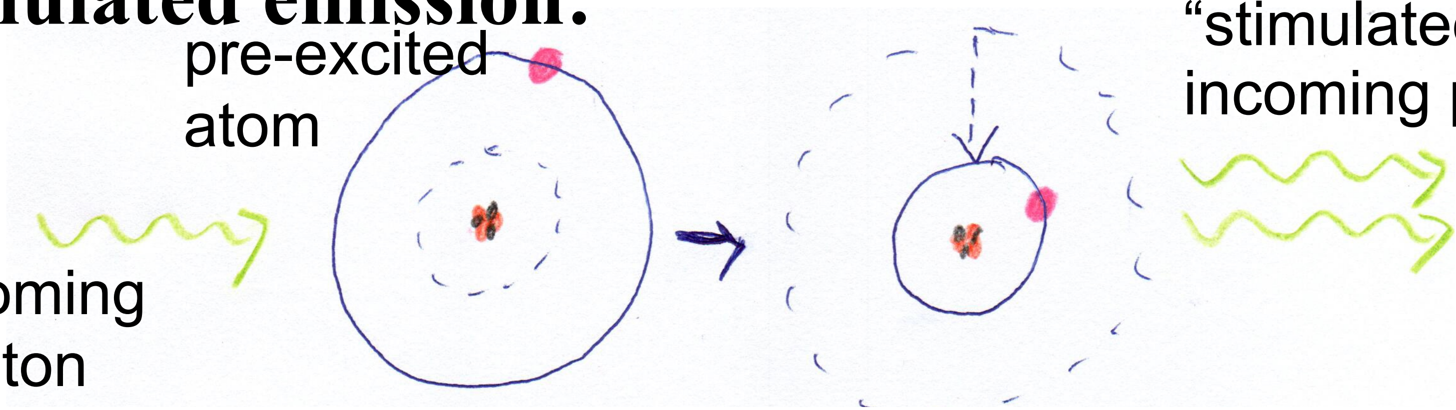


Example: Lasers

Now we understood atoms, let’s use then as tool for technology

Third way atoms can interact with photons:

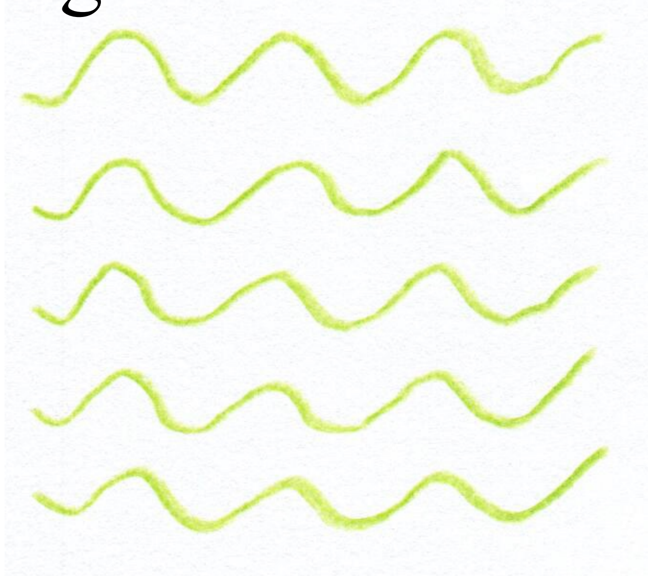
Stimulated emission:



Emitted photon **copy** of incoming one = “coherence”

COHERENT monochromatic light.

~one wavelength, all in phase

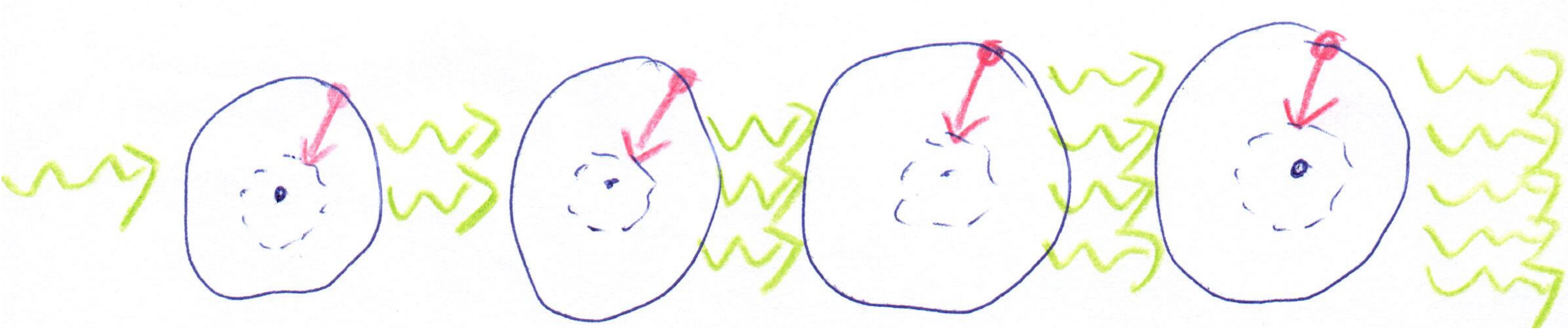


This is the result of stimulated emission.

Lasers

L.A.S.E.R.: **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation

If we have most atoms in excited states (population inversion):

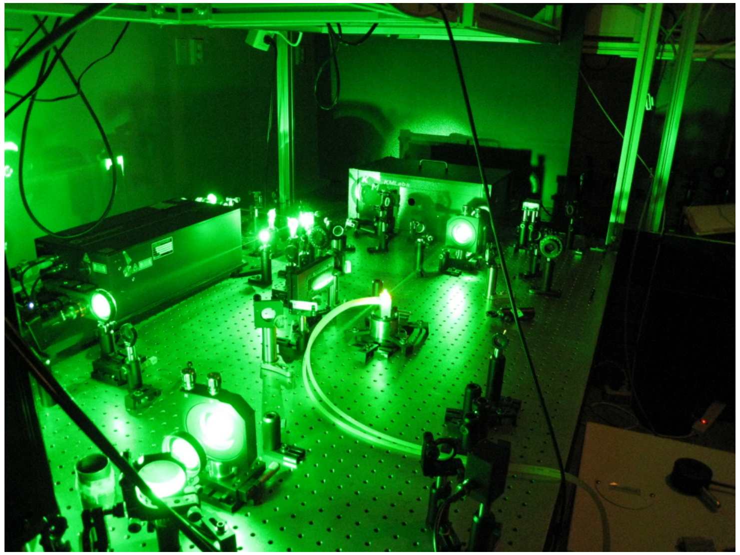


Incoming light gets **coherently amplified** to huge intensities.

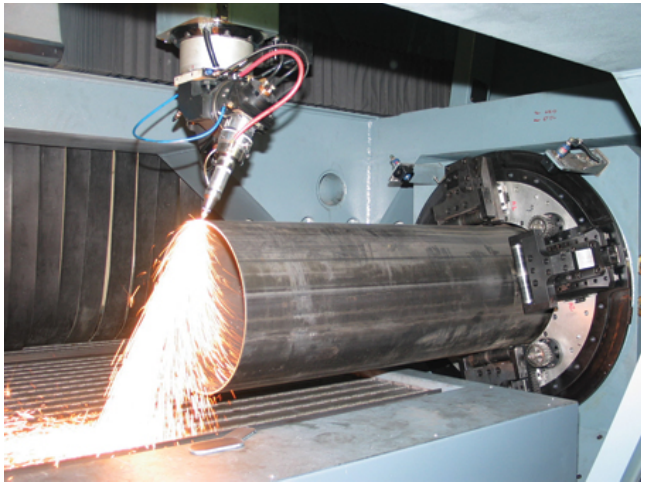
Lasers for…

Lecture, Power P=1 mW

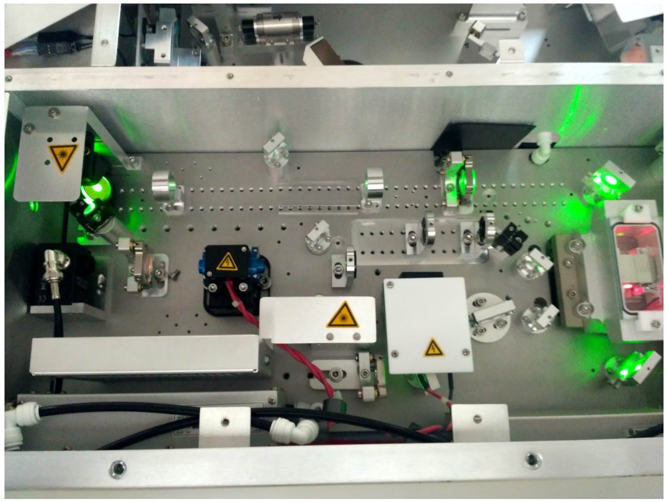
Quantum physics, P=1 W



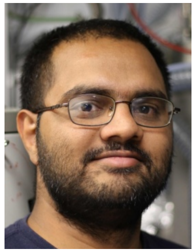
Industrial cutting, P=1 kW



fs spectroscopy, P=8 W



peak intensity 10^{15} W/cm^2



IISERB
B. Ram Lab

Inertial confinement fusion,
P=1 PW (1×10^{15} W)