Week 3

PHY 106 Quantum Physics
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2) Waves and Particles

revision of movie:

Quantum physics is essentially all about
"things that ought to be particles are **also** waves"
and
"things that ought to be waves are **also** particles".

Thus, let's make sure we are all on the same page regarding waves…..

2.1) Introduction to wave mechanics

What **is** a wave?

Definition of **wave:**

A perturbation of some **property** is transported through a **medium**, without transport of the medium itself

Book: A.P. French, “Vibrations and waves”

Waves

Examples:

fill in lecture

fill in lecture

fill in lecture

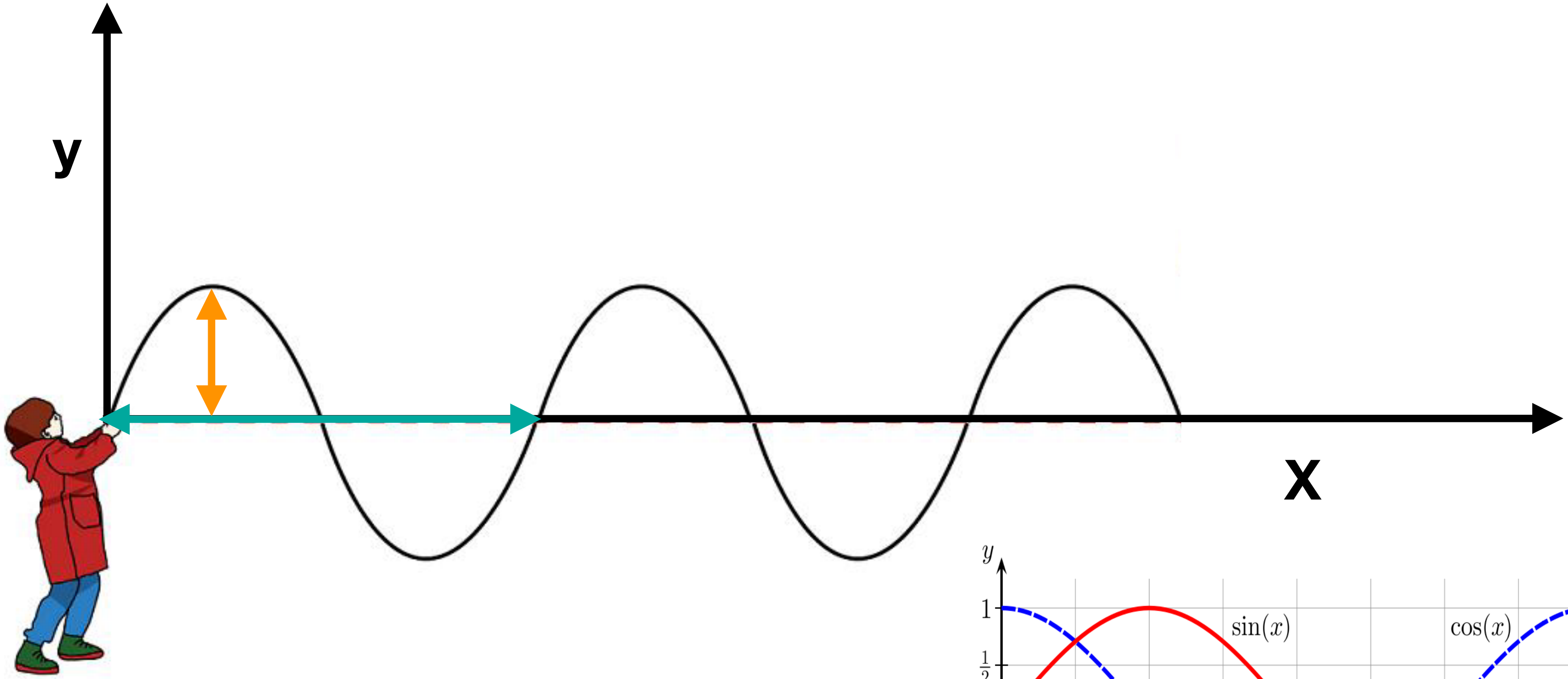
Waves

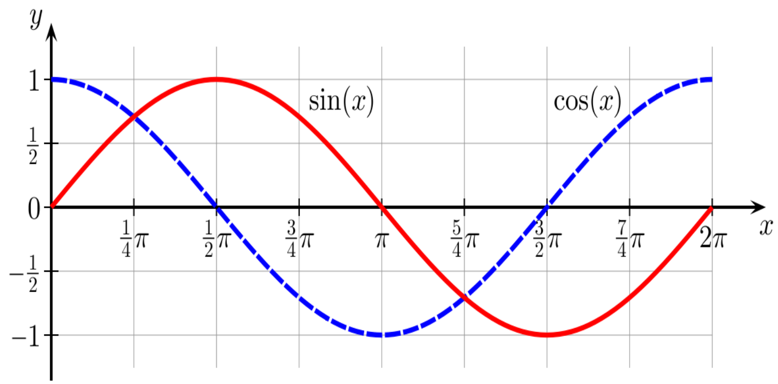
fill in lecture

fill in lecture

fill in lecture

2.1.1) Waves, frequencies, wavelengths



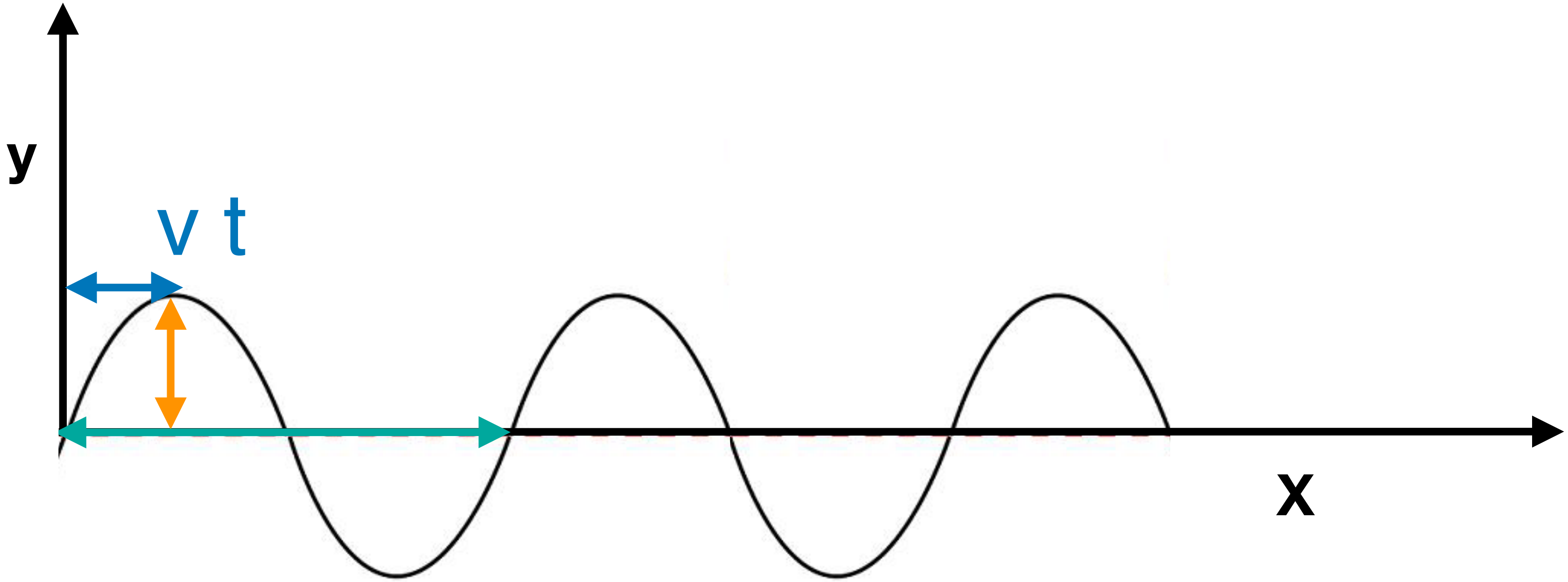


$$y(x) = A \sin\left(\frac{2\pi}{\lambda}x\right)$$

amplitude

wavelength

Waves



$$y(x,t) = \boxed{A} \sin\left(\frac{2\pi}{\boxed{\lambda}}(x - \boxed{V}t)\right) \qquad (5)$$

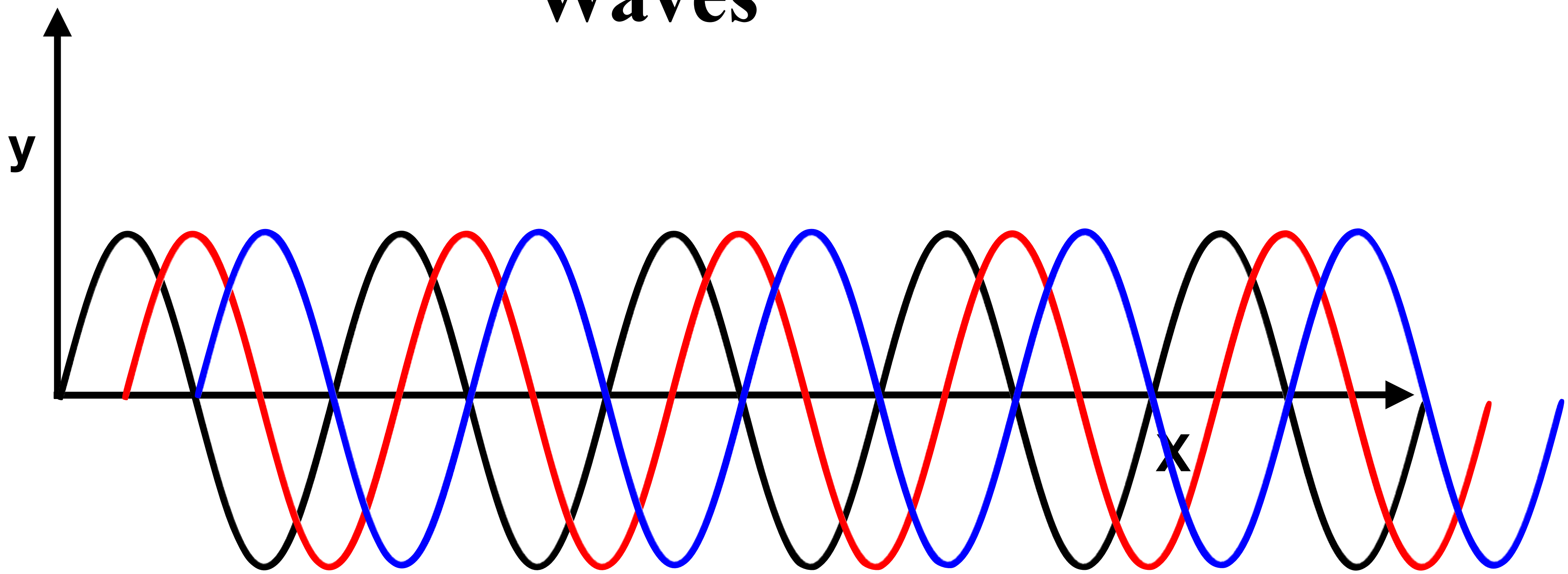
wave velocity

amplitude

wavelength

Form of **progressive/travelling wave**

Waves



$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - Vt)\right)$$

$t = 0$

$t = \frac{\lambda}{4}\frac{1}{V}$

$t = \frac{\lambda}{2}\frac{1}{V}$

Argument of **sin** is called **phase**

V here is the **phase velocity**

Waves

Rewrite wave form:

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - Vt)\right)$$

$$y(x,t) = A\sin\left(2\pi(\frac{x}{\lambda} - \nu t)\right)$$

$$y(x,t) = A\sin\left(kx - \omega t\right) \quad (6)$$

$$k = \frac{2\pi}{\lambda} \quad (7)$$

$$\frac{\omega}{k} = V \quad (8)$$

k is the **wave number** (unit 1/m)

ω is the **angular frequency**

ν is the **frequency** (unit Hz = 1/s)

$$\omega = 2\pi\nu \quad (9)$$

Waves

$$y(x,t) = A\sin\left(kx - \omega t\right) \quad (6)$$

$$k = \frac{2\pi}{\lambda} \quad (7)$$

Relation between **frequency**, **wave length** or **wave number** and **phase velocity** of any wave (unit m/s)

$$\frac{\omega}{k} = V \quad (8)$$

$$\nu\ \lambda = V \quad (10)$$

k is the **wave number**

ω is the **angular frequency**

$$\omega = 2\pi\nu \quad (9)$$

ν is the **frequency**

Wave velocities

$$\frac{\omega}{k} = V \quad \textbf{(8)} \qquad\qquad \nu\ \lambda = V \quad \textbf{(10)}$$

Examples:

sound in solid

$$V = \sqrt{\frac{Y}{\rho}} \approx 5000 \ \textbf{m/s}$$

$$\nu = 440 \ \textbf{Hz} \qquad \lambda = 11.4 \ \textbf{m}$$

gravitational waves

$$V = c = 299792458 \ \textbf{m/s}$$

$$\nu = 440 \ \textbf{Hz} \qquad \lambda = 681 \ \textbf{km}$$

Wave velocities

$$\frac{\omega}{k} = V \quad \textbf{(8)}$$

$$\nu \ \lambda = V \quad \textbf{(10)}$$

Examples II:

water wave (tsunami)

$V = 500$ **km/h**

$\nu = 3.3$ **/h**

$\lambda = 151$ **km**

light wave (elm)

$V = c = 299792458$ **m/s**

$\lambda = 700$ **nm**

$\nu = 4.2 \times 10^{14}$ **Hz**

2.1.2) The wave equation

Is there a general equation that governs wave behavior?

$$y(x,t) = A\sin\left(kx - \omega t\right)$$

We see:

$$\frac{\partial^2}{\partial x^2} y(x,t) = -\,k^2 A \sin\left(kx - \omega t\right) = -\,k^2\, y(x,t) \tag{11}$$

$$\frac{\partial^2}{\partial t^2} y(x,t) = -\,(-\omega)^2 A \sin\left(kx - \omega t\right) = -\,\omega^2\, y(x,t) \tag{12}$$

The wave equation

$$\frac{\partial^2}{\partial x^2} y(x,t) = -k^2\, y(x,t) \quad \textbf{(11)}$$

$$\frac{\partial^2}{\partial t^2} y(x,t) = -\omega^2\, y(x,t) \quad \textbf{(12)}$$

$$y(x,t) = -\frac{1}{\omega^2}\frac{\partial^2}{\partial t^2} y(x,t)$$

With: $\frac{\omega}{k} = V$ **(8)** $\qquad \frac{k}{\omega} = \frac{1}{V}$

General **wave equation**

$$\frac{\partial^2}{\partial x^2} y(x,t) = \frac{1}{V^2}\frac{\partial^2}{\partial t^2} y(x,t) \quad \textbf{(13)}$$

- Change in time **causes** change in space **and vice versa**.

Wave equation

Any function:

$$y(x, t) = f(x - Vt)$$

Chain rule:

$$\frac{\partial^2}{\partial x^2} y(x, t) = (1)^2 \, f''(x - Vt)$$

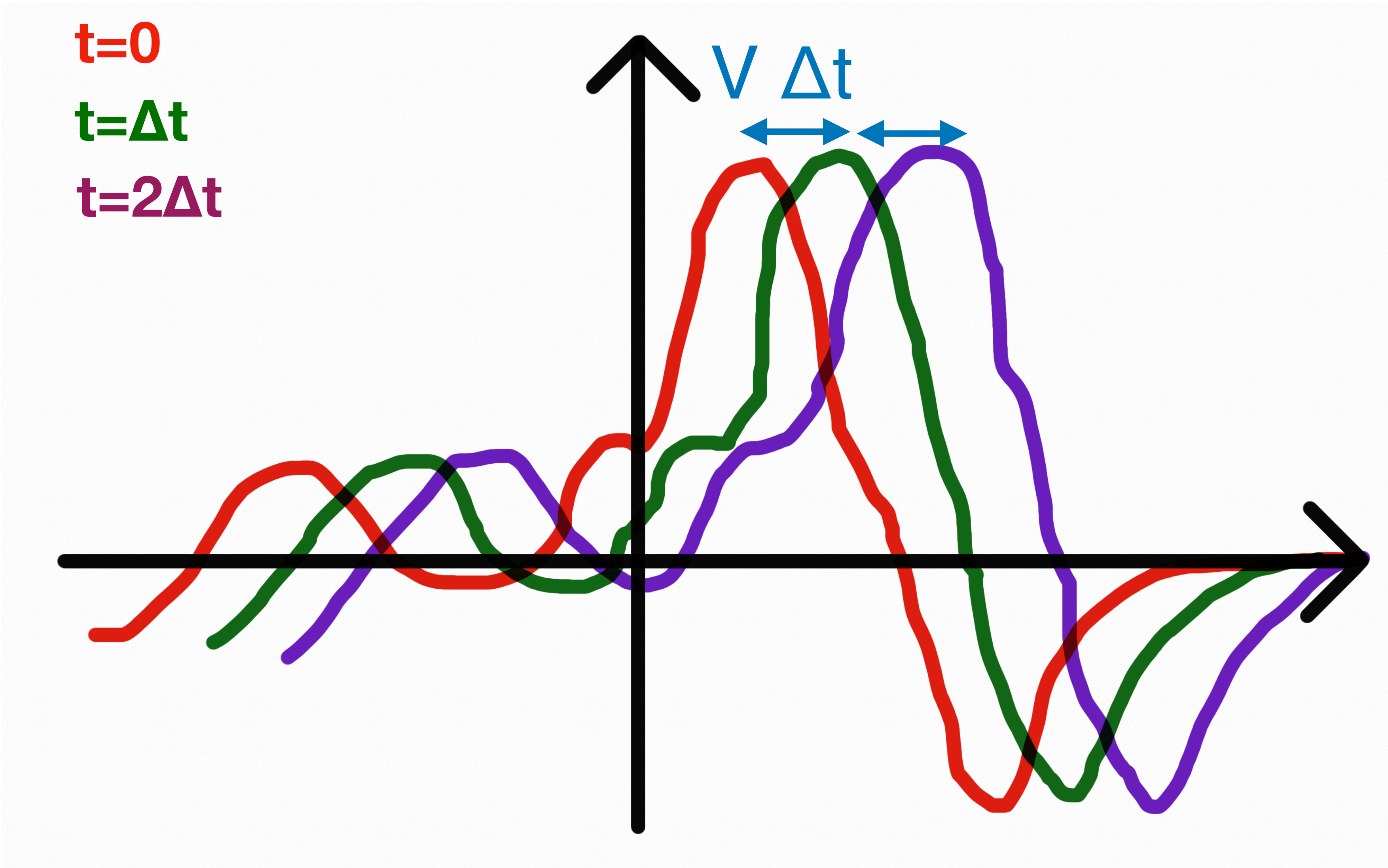
$$\frac{\partial^2}{\partial t^2} y(x, t) = (-V)^2 \, f''(x - Vt)$$

Fulfills wave-equation: **(13)**

$$\frac{\partial^2}{\partial x^2} y(x, t) = \frac{1}{V^2} \frac{\partial^2}{\partial t^2} y(x, t)$$

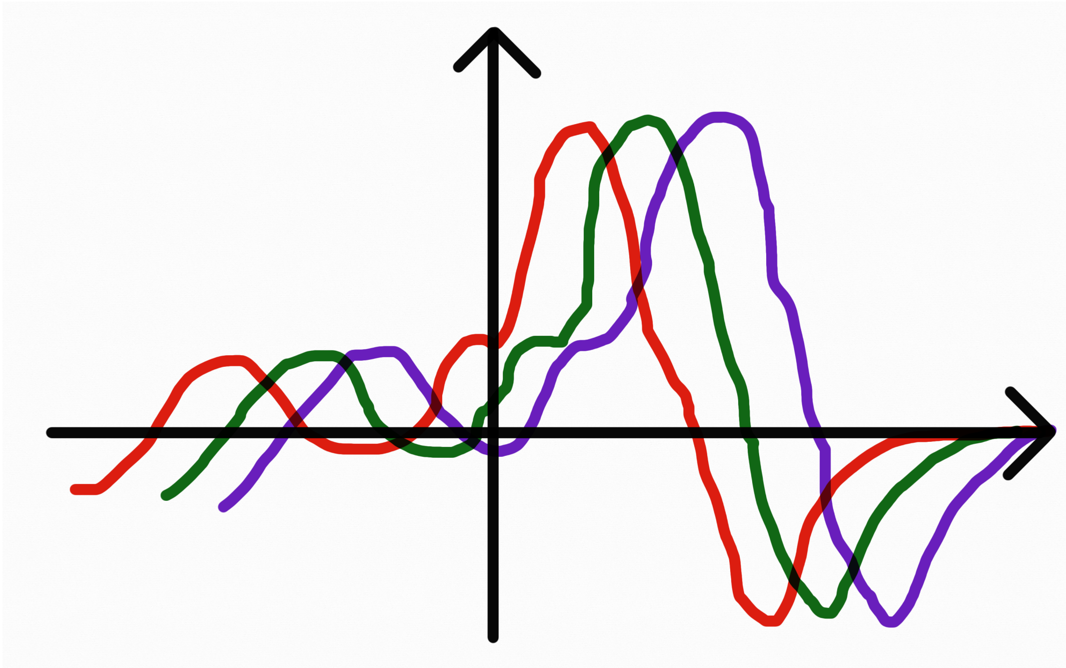
Wave equation

$y(x,t) = f(x - Vt)$ moves to the right with velocity V!!



Wave equation

$y(x,t) = f(x - Vt)$ moves to the right with velocity V!!



- $y(x,t) = f(x + vt)$ Moves to the left with velocity V, also fulfills wave equation

- Can be generalized to 2D, 3D

- There are many wave-equation**s**, one for each medium.

Superposition principle

The wave equation is linear. That means any **combination of waves** is also a **solution**

let:

$$\frac{\partial^2}{\partial x^2} y(x,t) = \frac{1}{V^2}\frac{\partial^2}{\partial t^2} y(x,t)$$

$$\frac{\partial^2}{\partial x^2} w(x,t) = \frac{1}{V^2}\frac{\partial^2}{\partial t^2} w(x,t)$$

Then:

$$\frac{\partial^2}{\partial x^2}\left[y(x,t) + w(x,t)\right] = \frac{1}{V^2}\frac{\partial^2}{\partial t^2}\left[y(x,t) + w(x,t)\right]$$

2.1.3) Standing waves

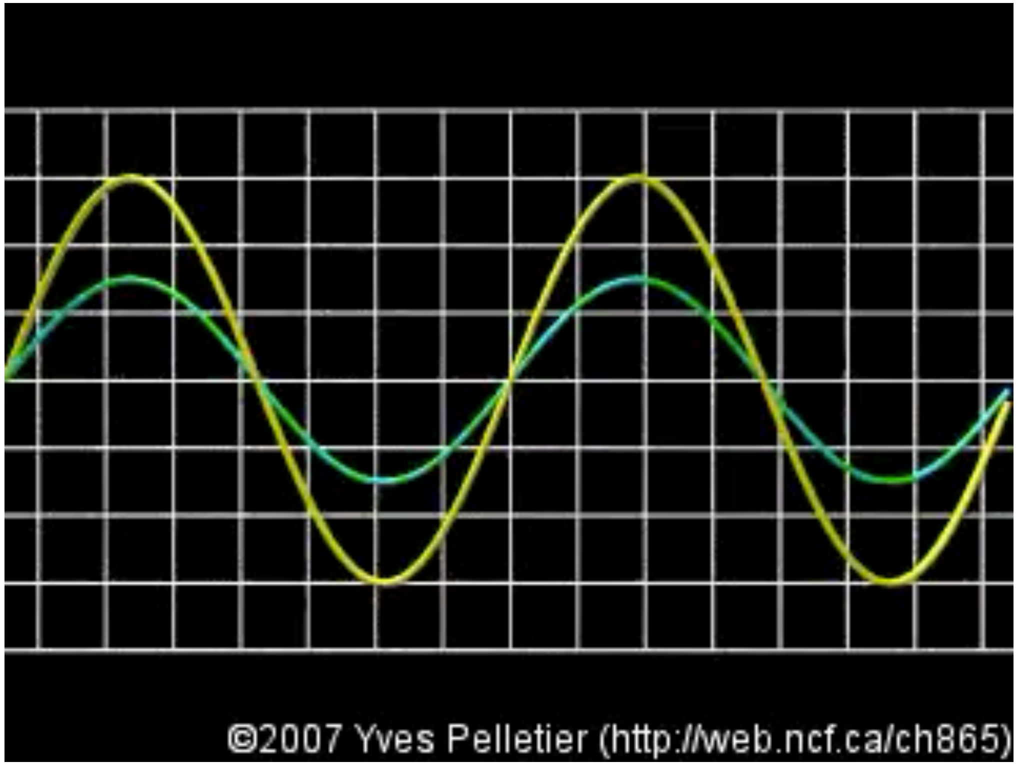
What happens if we combine two identical waves travelling in opposite directions?

https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html

https://www.youtube.com/watch?v=LgJStYrk2fc

2.1.3) Standing waves

What happens if we combine two identical waves travelling in opposite directions?



Animation from: https://www.youtube.com/watch?v=ic73oZoqr70

Using (6), we can write this as:

$$y(x, t) = A \sin\left(kx - \omega t\right) + A \sin\left(-kx - \omega t\right)$$

Standing waves

$$y(x,t) = A\sin\left(kx - \omega t\right) + A\sin\left(-kx - \omega t\right)$$

Trigonometric identity

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \qquad \textbf{(14)}$$

$$y(x,t) = A\Big[\sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t) + \underbrace{\sin(-kx)}_{-\sin(kx)}\cos(\omega t) - \underbrace{\cos(-kx)}_{\cos(kx)}\sin(\omega t)\Big]$$

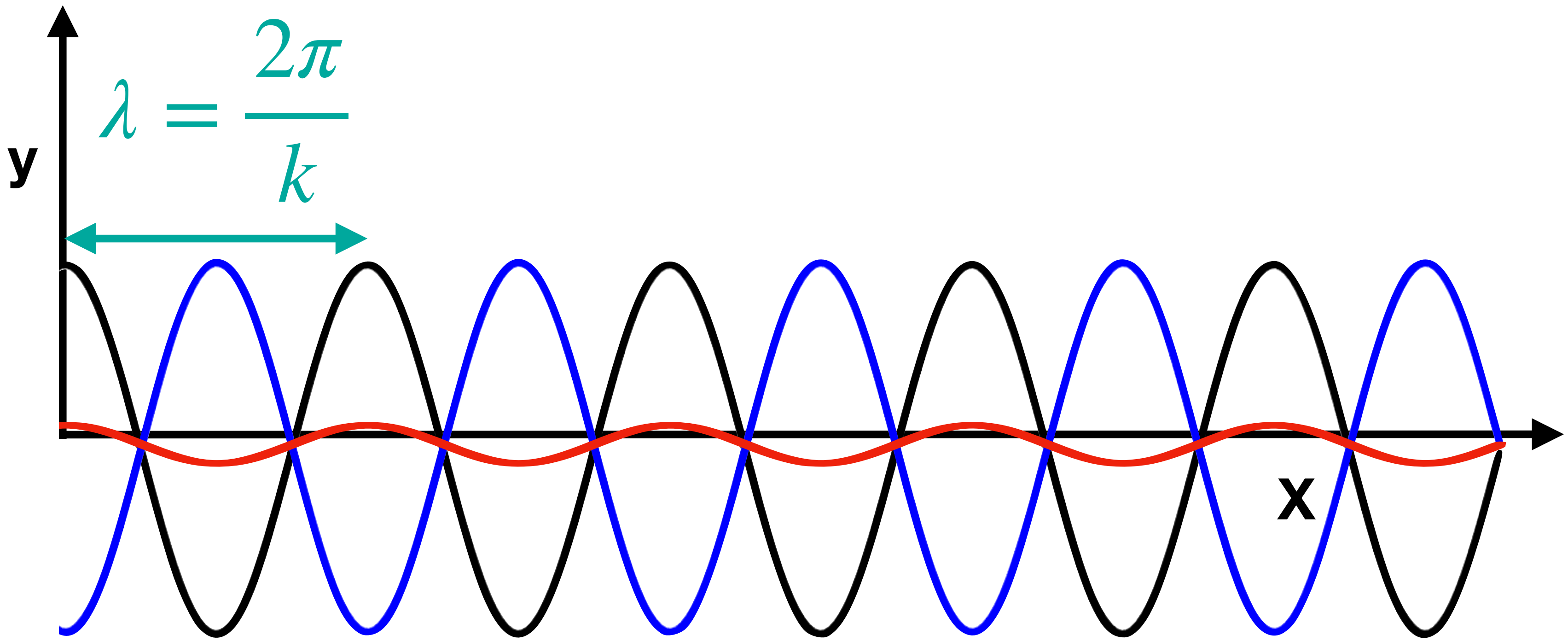
(The terms $\sin(kx)\cos(\omega t)$ and $\sin(-kx)\cos(\omega t)$ are crossed out.)

$$y(x,t) = -\,2A\cos(kx)\ \sin(\omega t)$$

Standing waves

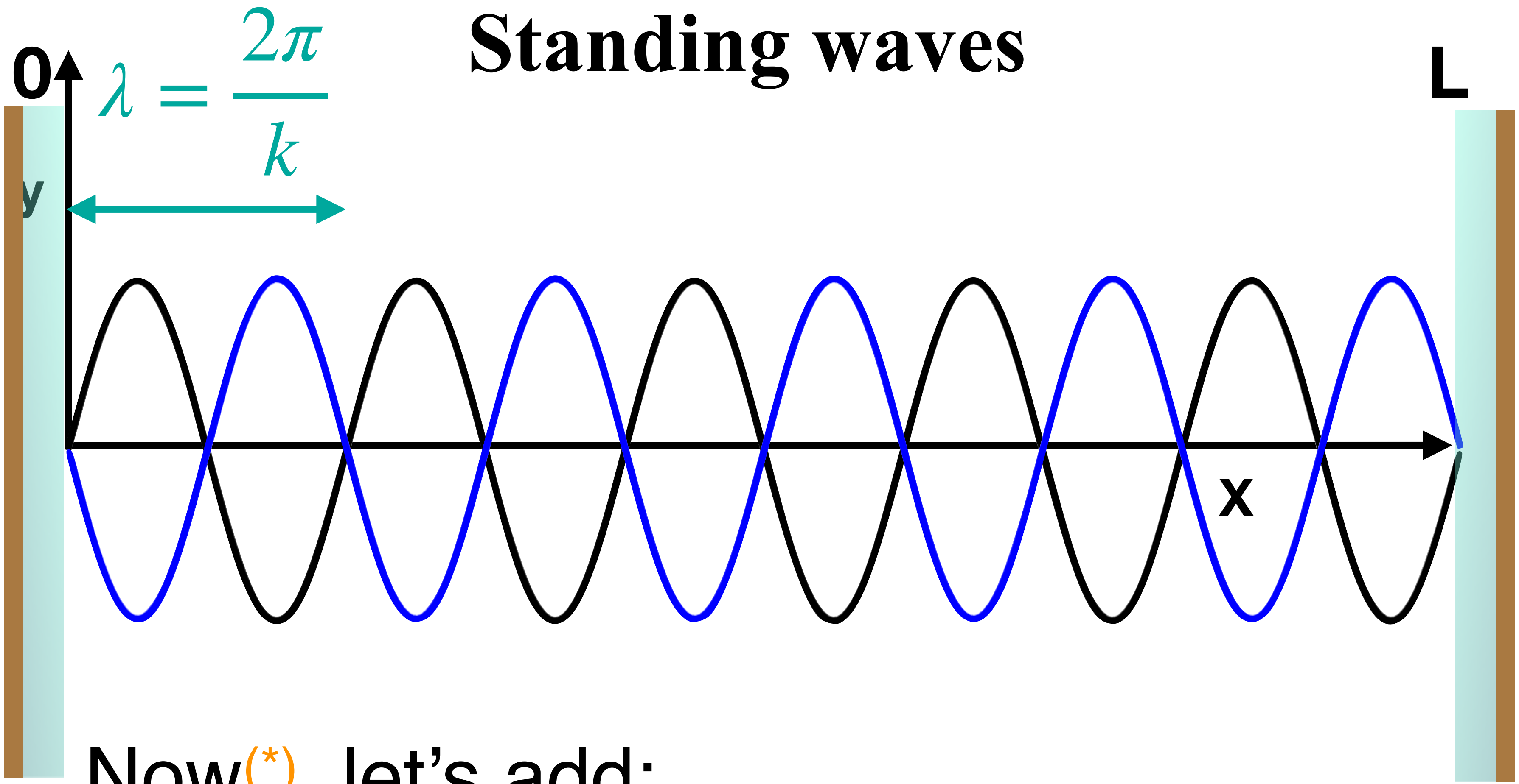
Formula for some **standing wave**

$$y(x,t) = \tilde{A}\cos(kx)\ \sin(\omega t) \tag{15}$$



$t = \frac{\pi}{2\omega}$ just before $t = \frac{\pi}{\omega}$ $t = \frac{3\pi}{2\omega}$

Standing waves



Now(*) let's add:

Boundary condition: $y(0,t) = y(L,t) = 0$ **(16b)**

Resonance condition for **standing wave**

$$L = n\frac{\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad n = 1,2,3\ldots \qquad \textbf{(16)}$$

* Q: Eq. (15) is an example that does not fulfill Eq. (16b). Find another example that does.

Standing waves

Examples:

fill in lecture

fill in lecture

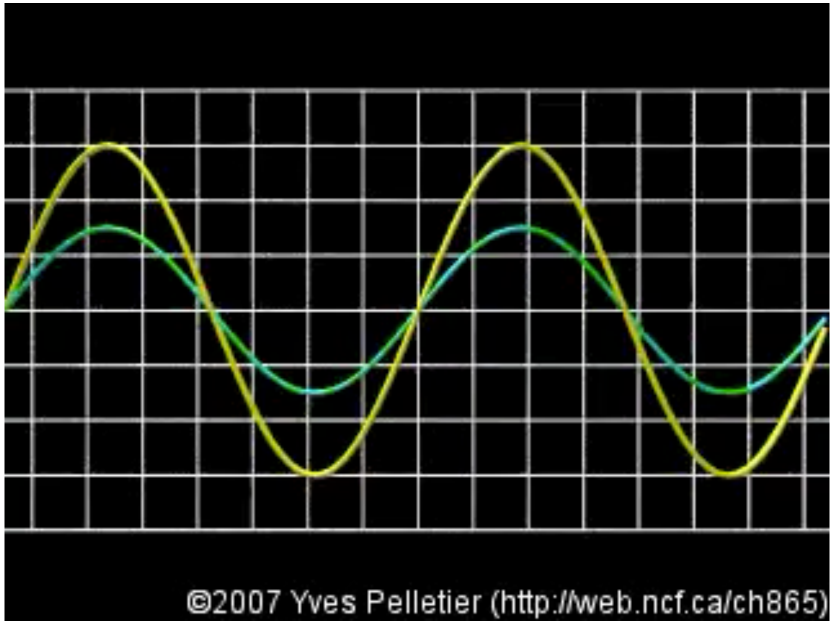
fill in lecture

2.1.4) Phenomena characteristic for waves

Interference

Superposition principle: Waves taking different paths get added.

Standing wave: example where superimposed waves always cancel at anti-node:



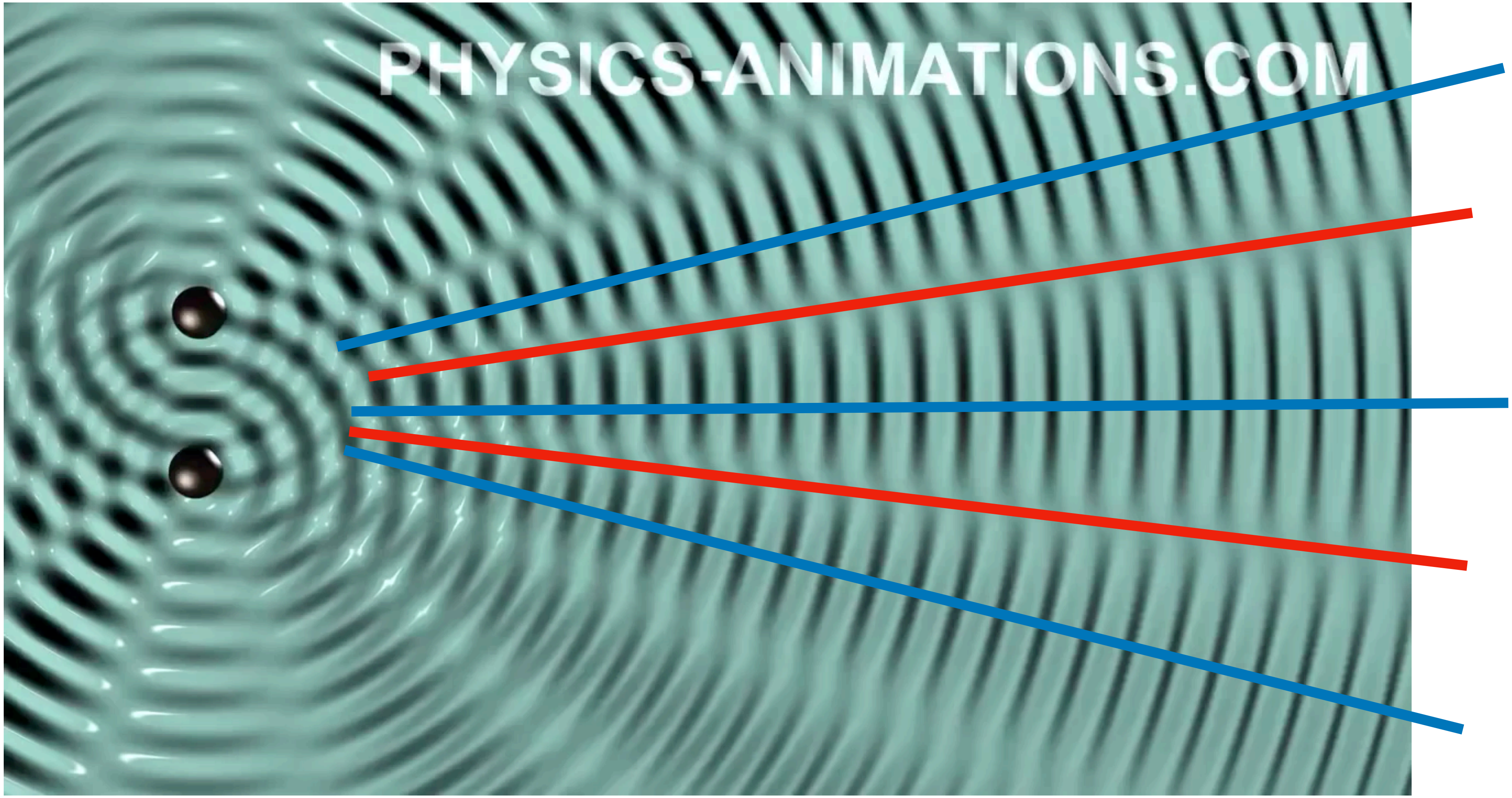
Interference

Usually (2D, 3D) more options:

Circular waves on a water surface

Interference

Usually (2D, 3D) more options:



https://www.youtube.com/watch?v=ovZkFMuxZNc

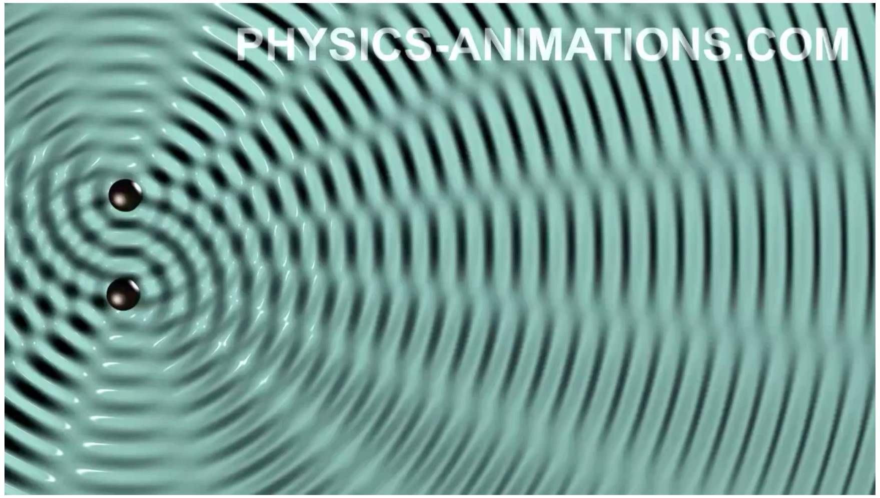
Two circular waves: —— strengthen —— cancel

Interference

Waves can show **interference**

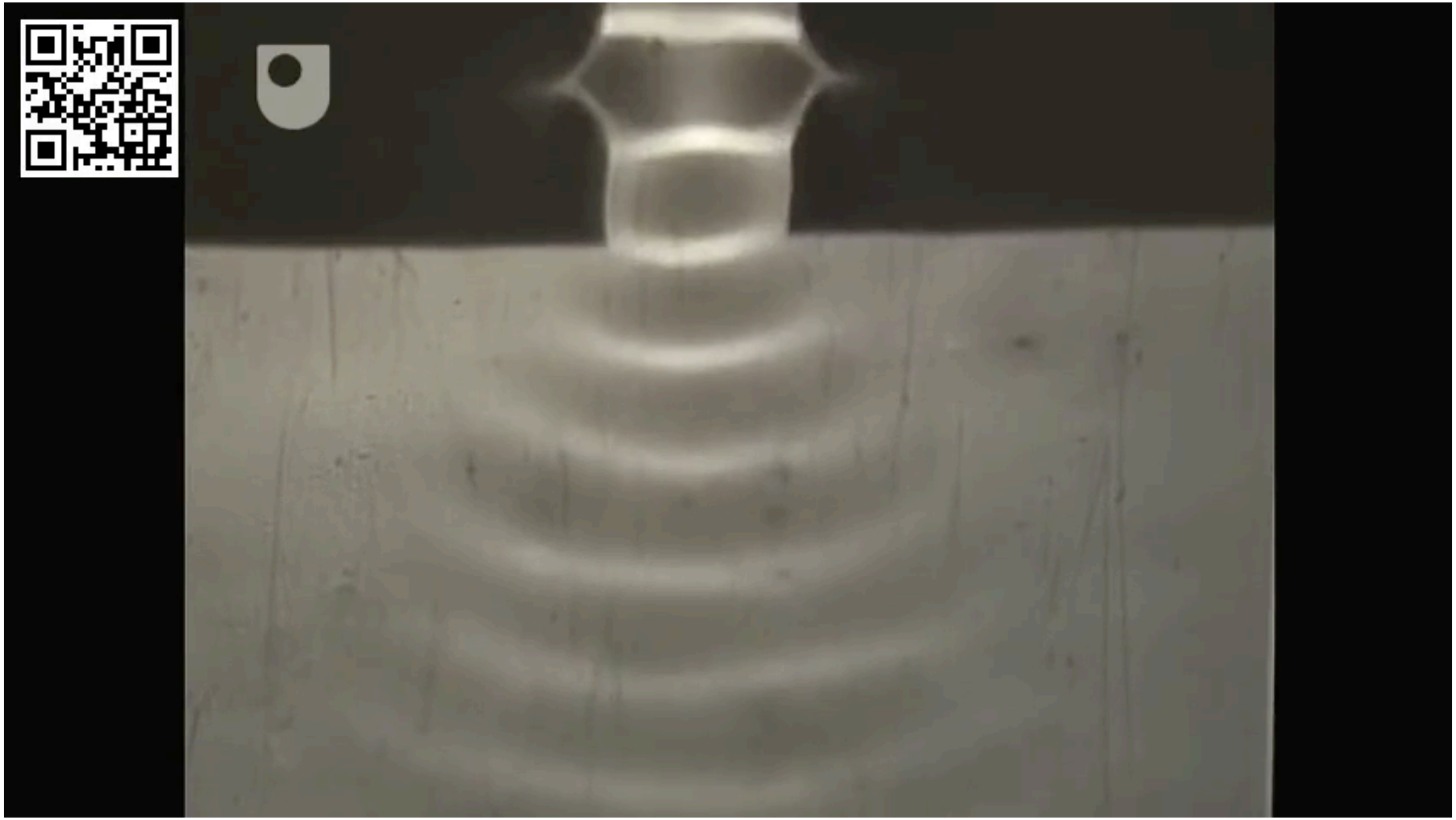
-strengthening in certain directions/ at certain times: **constructive** interference

-weakening in certain directions/ at certain times: **destructive** interference



Diffraction

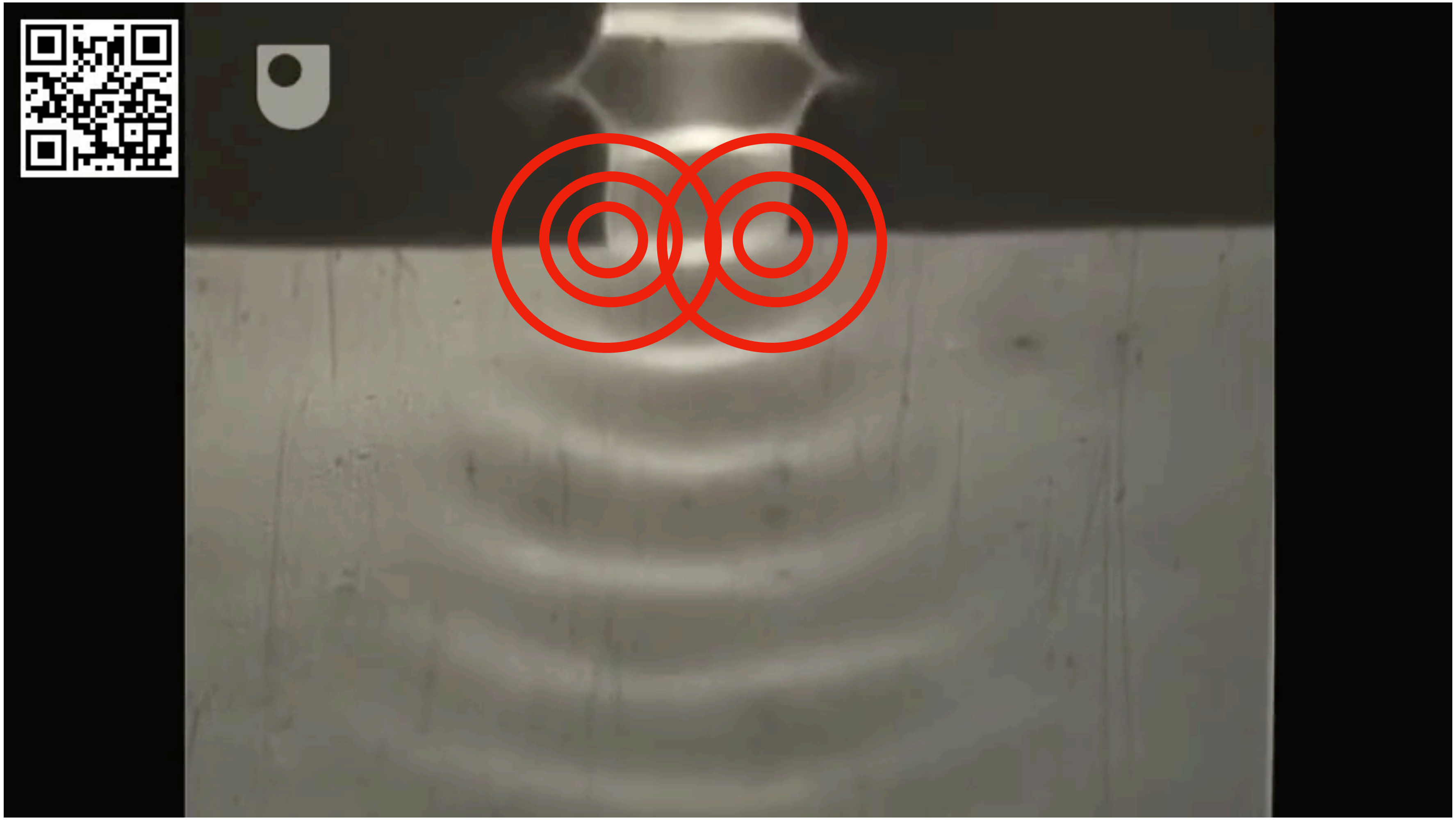
Waves can turn around corners:



https://www.youtube.com/watch?v=BH0NfVUTWG4

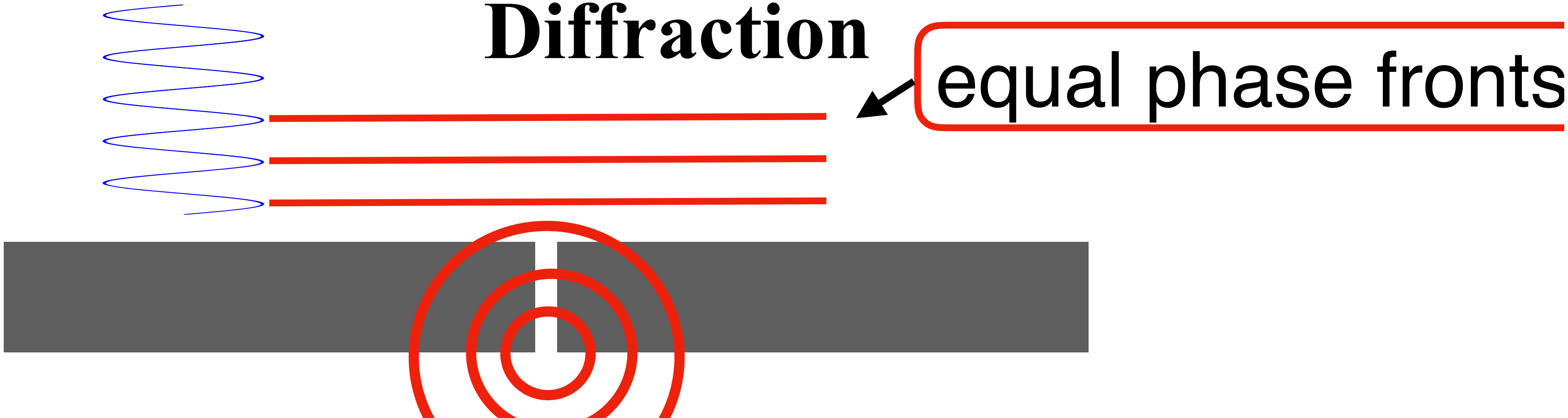
Diffraction

Decompose wave into lots of spherical waves:

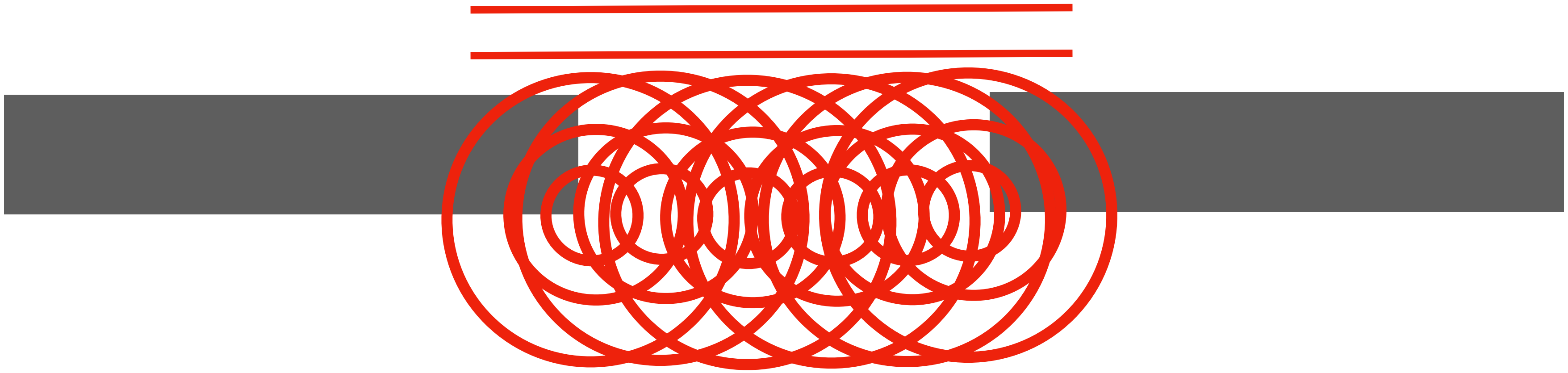


Could see this from 2D wave equation

Diffraction



Slit smaller than wavelength: emits circular waves going in **ALL** directions

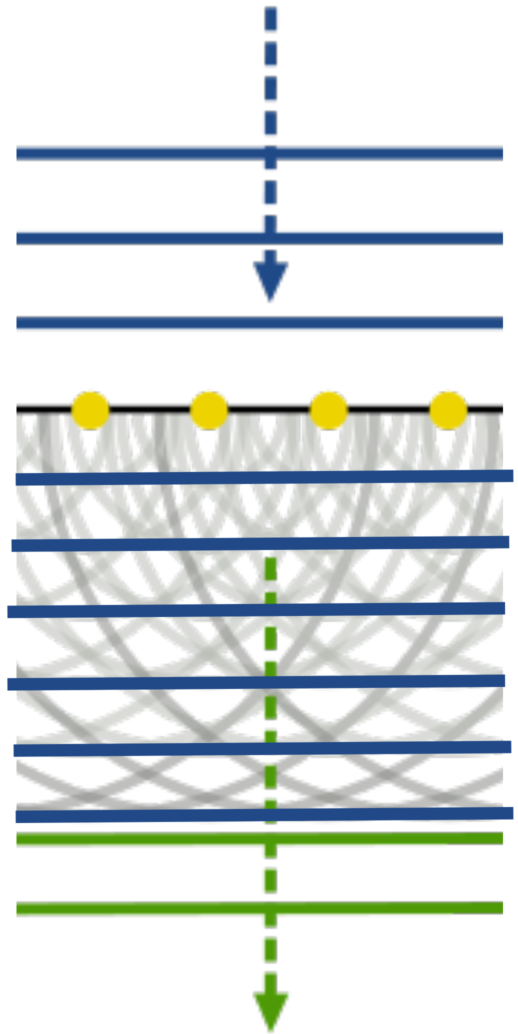


Slit larger than wavelength: waves destructively interfere if direction not almost forward *(tutorial, waves and optics course)*

Excursion: Huygens–Fresnel principle

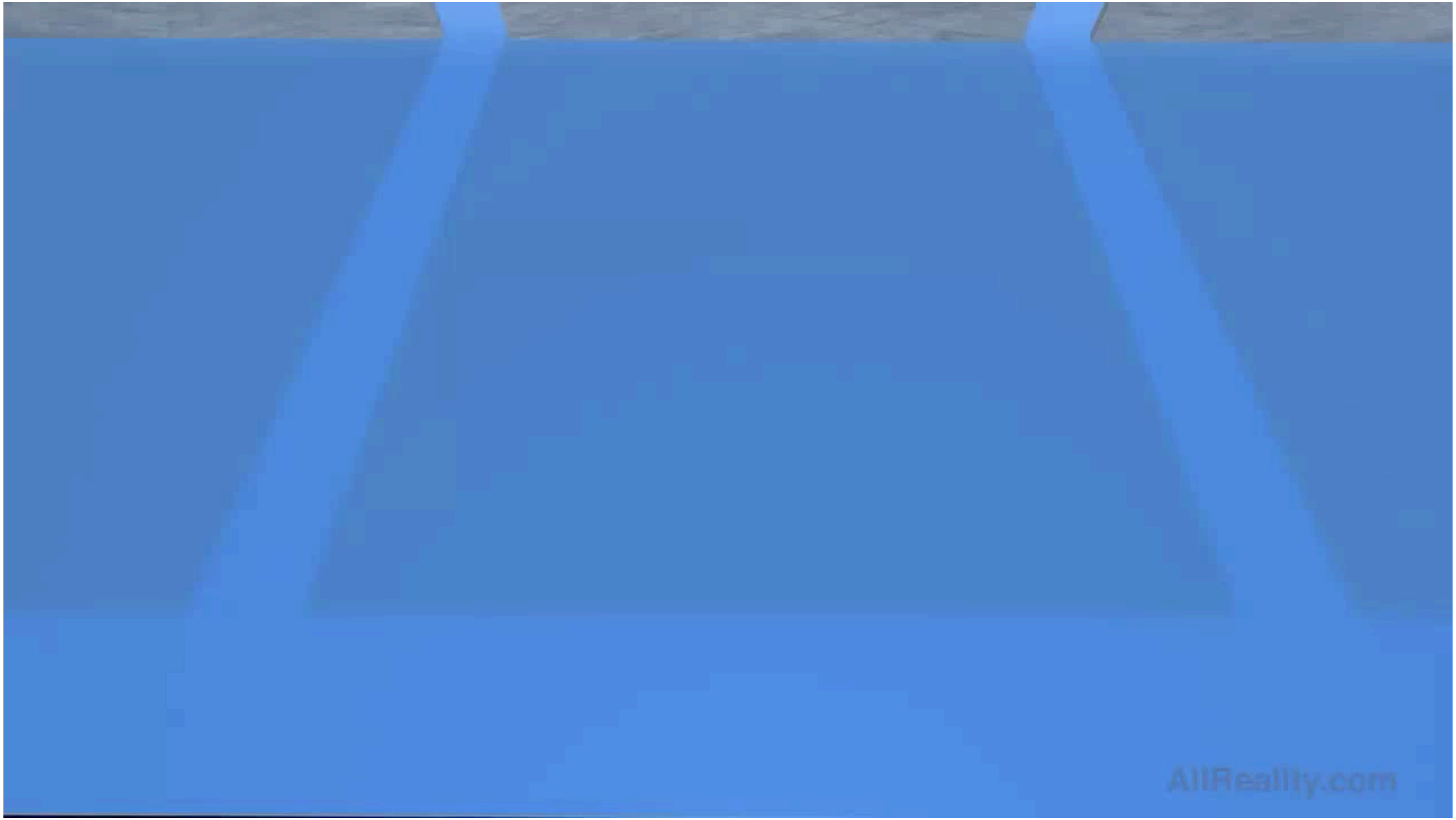
Decompose wave into lots of spherical waves:

Interference of these gives next wave-crest



When plane wave is obstructed: diffraction

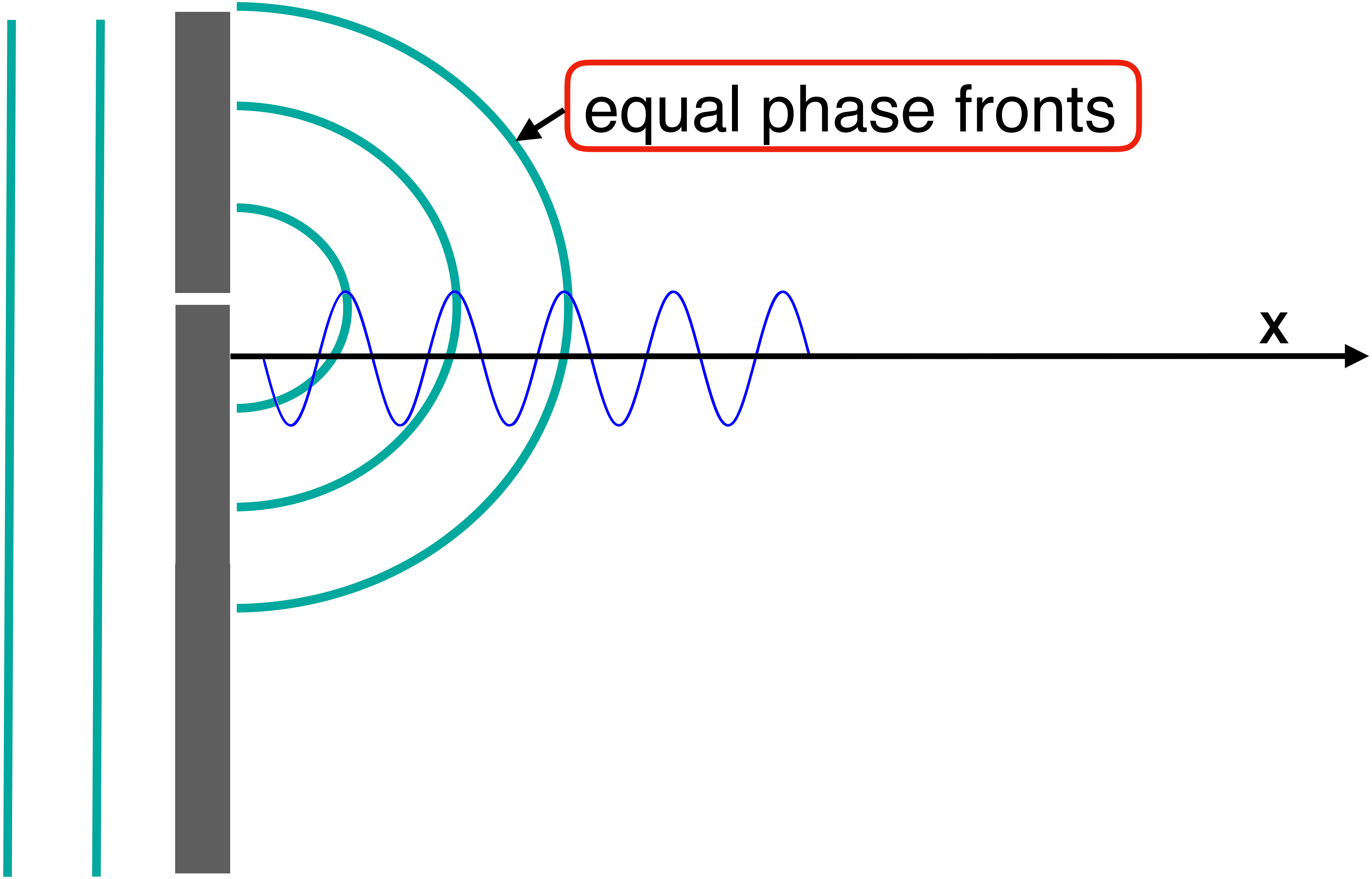
Diffraction and Interference



Double slit interference

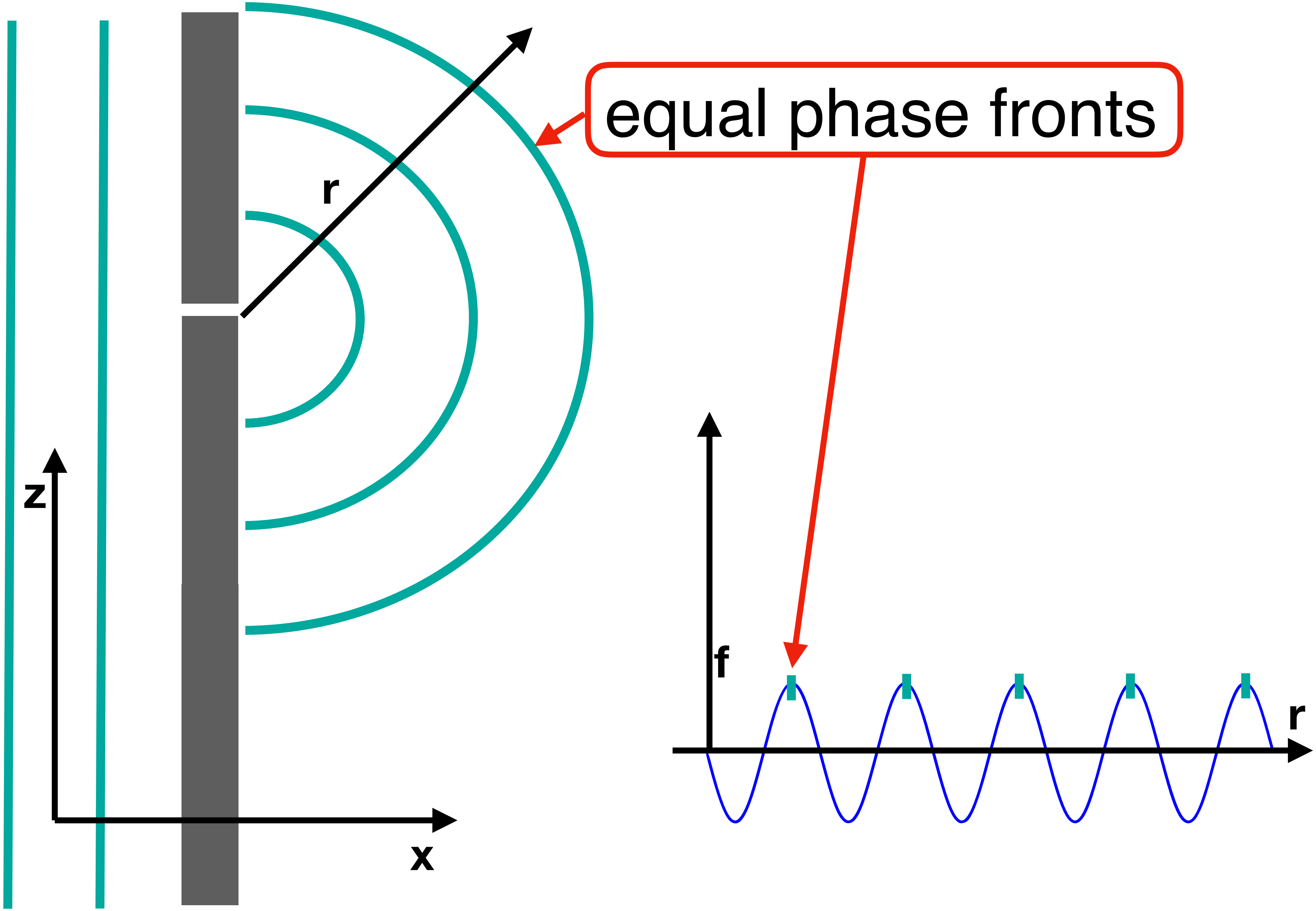
Diffraction and Interference

Double slit interference



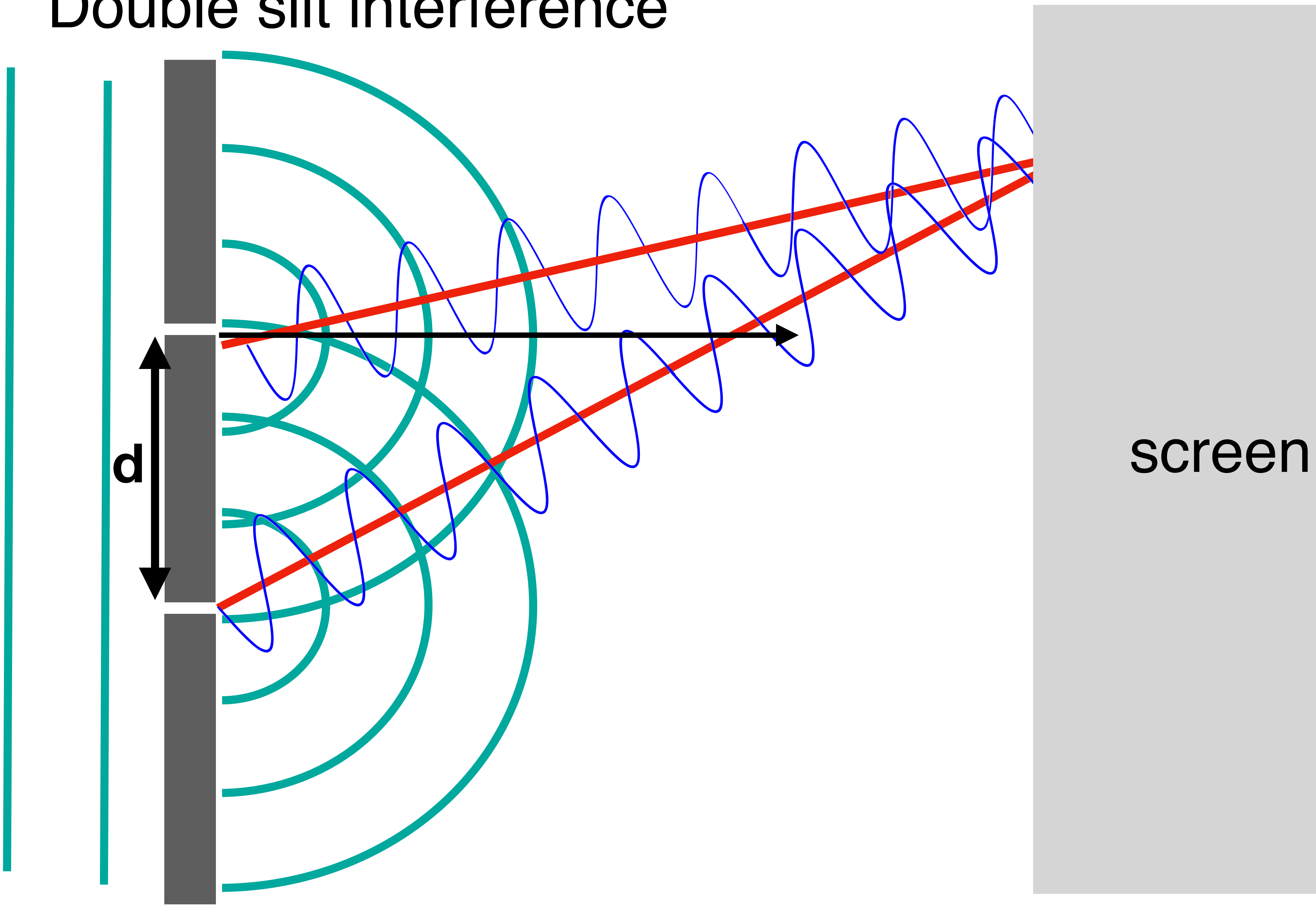
Diffraction and Interference

More tidy picture



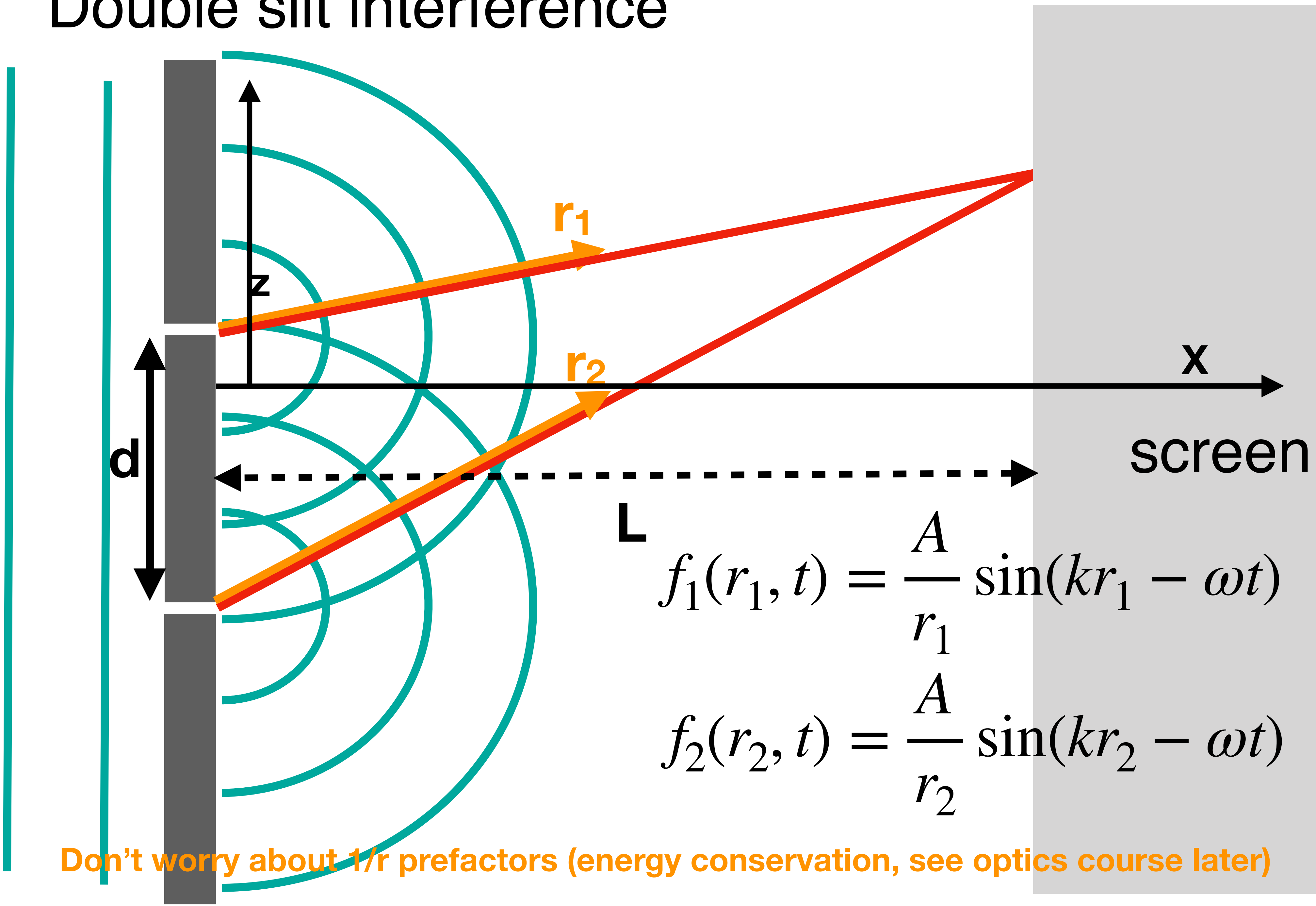
Diffraction and Interference

Double slit interference



Diffraction and Interference

Double slit interference



$$f_1(r_1, t) = \frac{A}{r_1}\sin(kr_1 - \omega t)$$

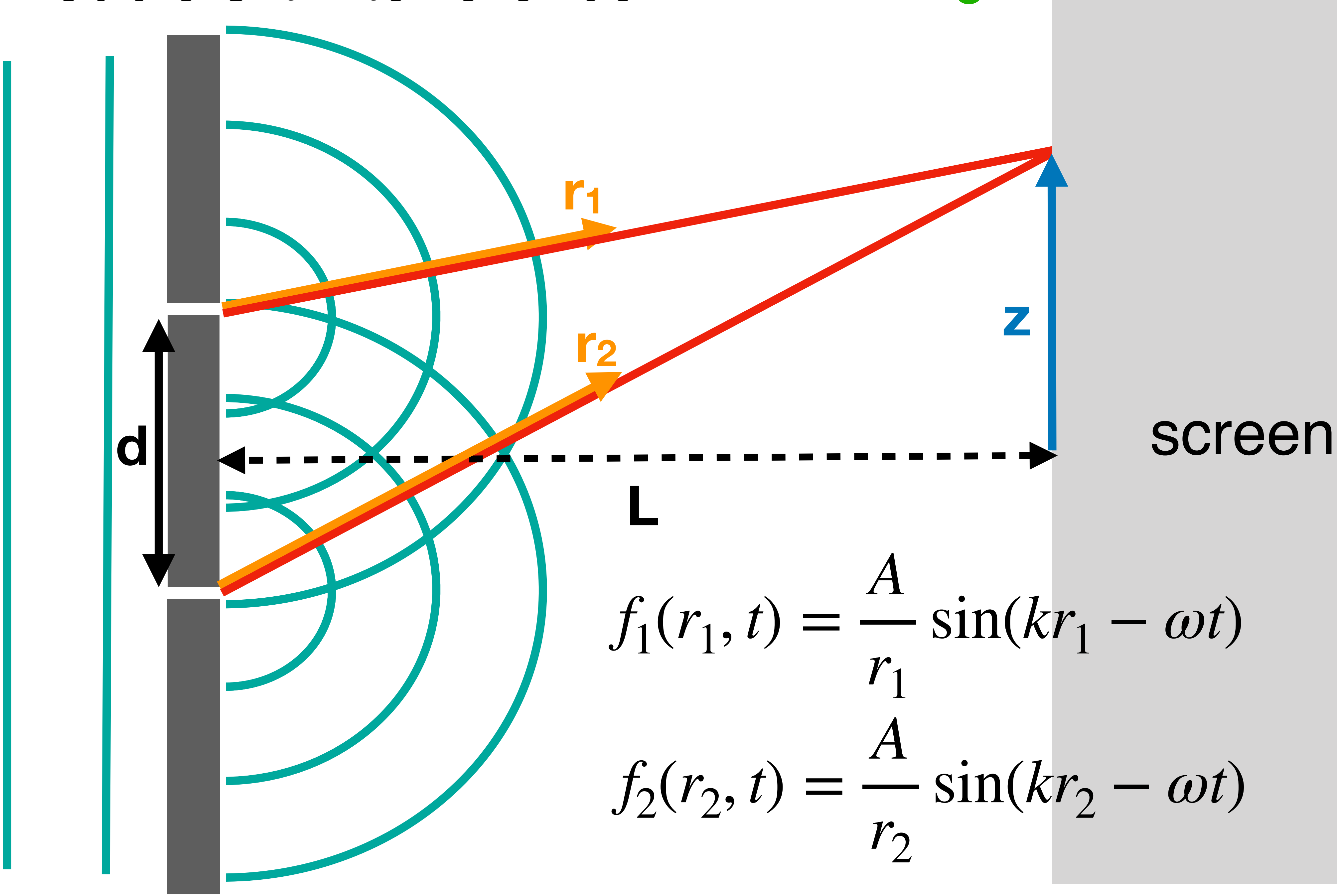
$$f_2(r_2, t) = \frac{A}{r_2}\sin(kr_2 - \omega t)$$

Don't worry about 1/r prefactors (energy conservation, see optics course later)

Diffraction and Interference

Double slit interference

Fig. 2



$$f_1(r_1, t) = \frac{A}{r_1} \sin(kr_1 - \omega t)$$

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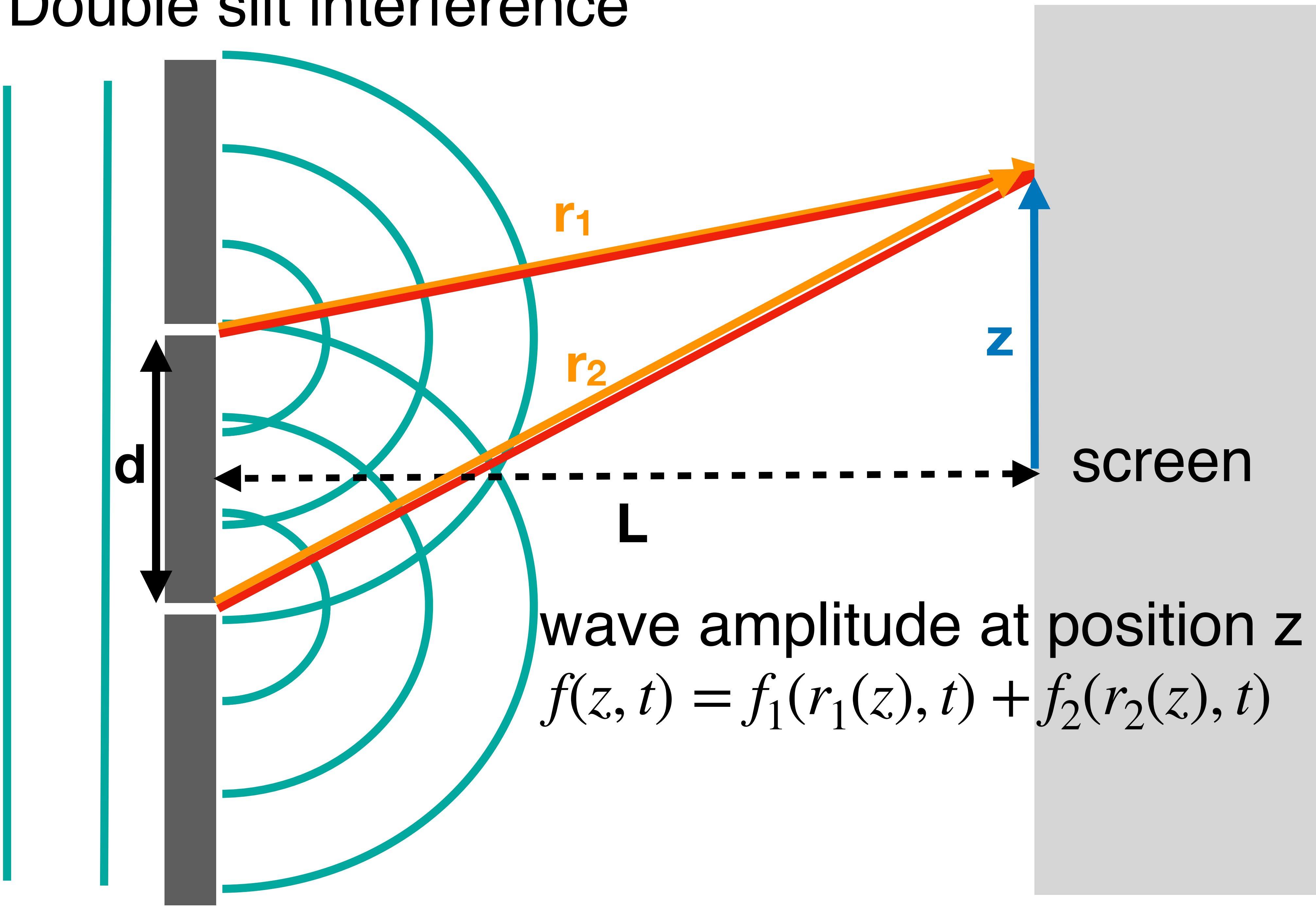
Diffraction and Interference

Double slit interference

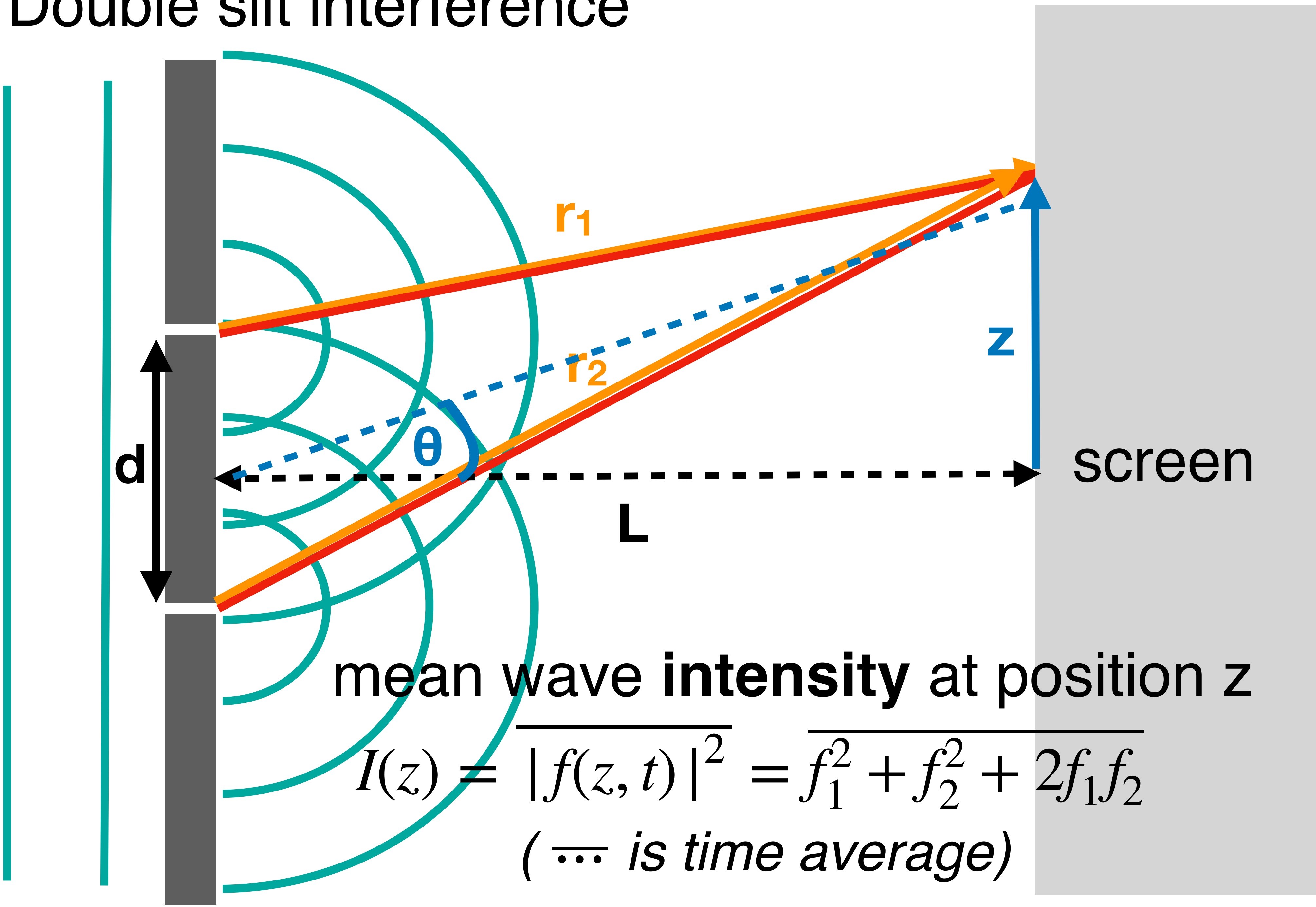


wave amplitude at position z

$$f(z,t) = f_1(r_1(z),t) + f_2(r_2(z),t)$$

Diffraction and Interference

Double slit interference

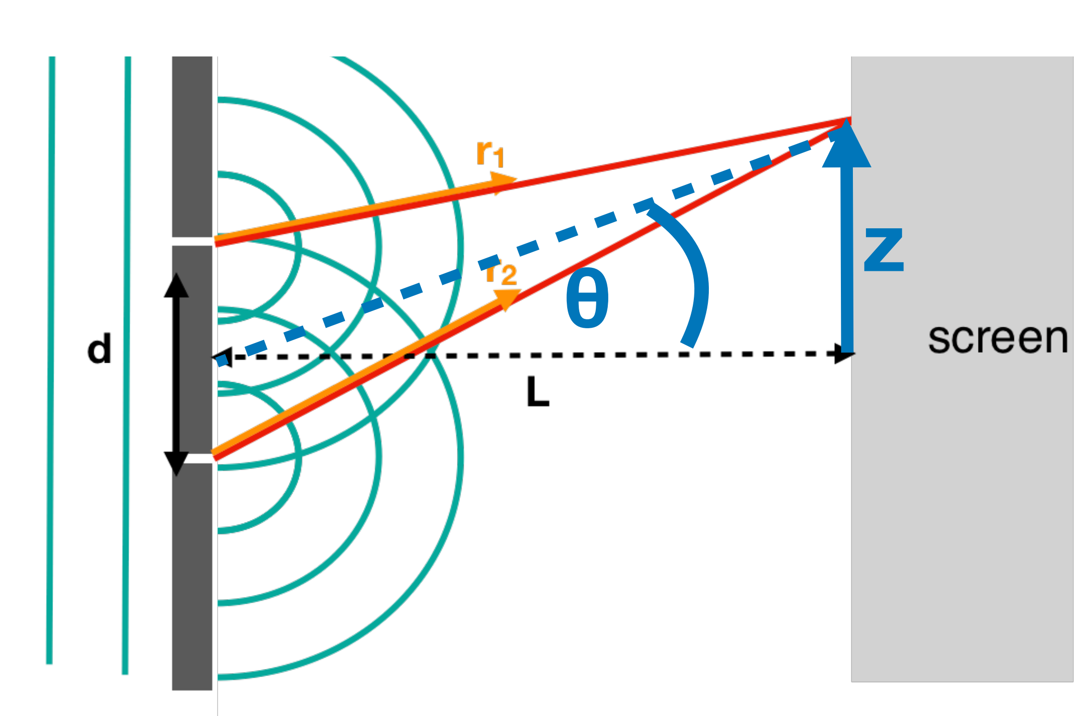


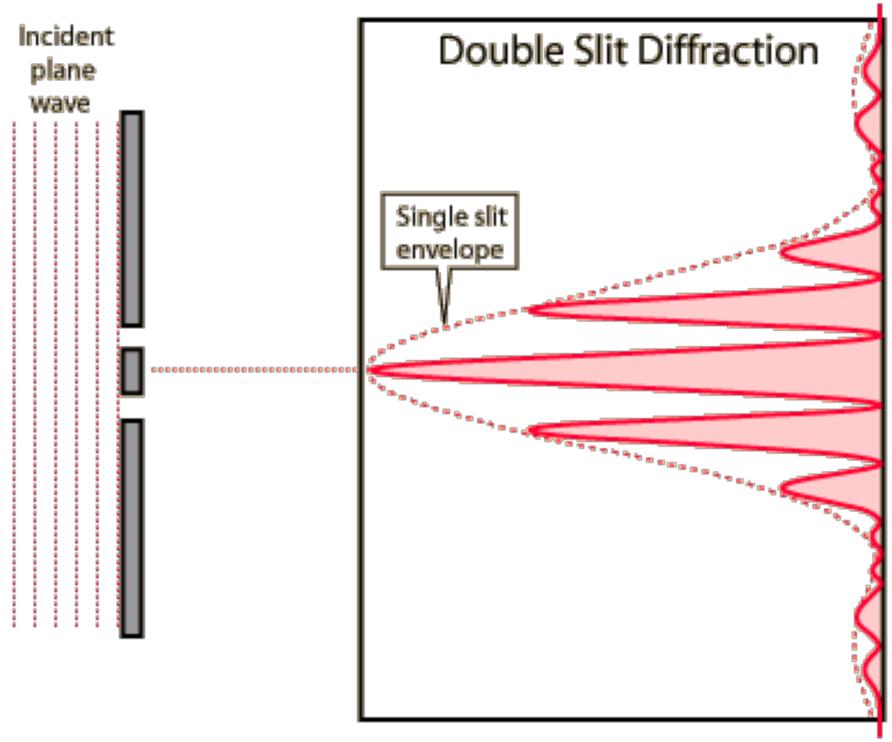
mean wave **intensity** at position z

$$I(z) = \overline{|f(z,t)|^2} = \overline{f_1^2 + f_2^2 + 2f_1 f_2}$$

($\overline{\cdots}$ is time average)

Diffraction and Interference

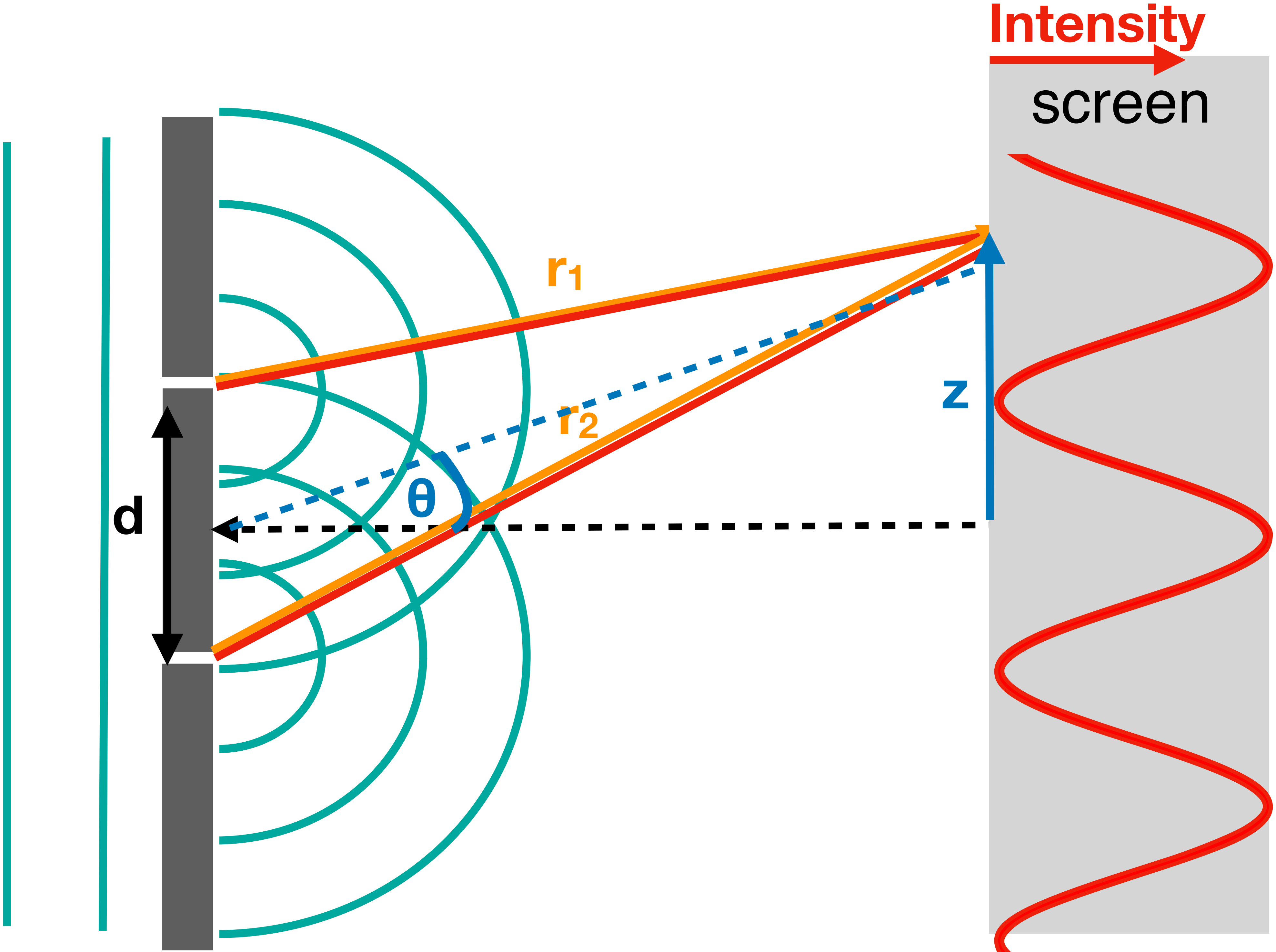




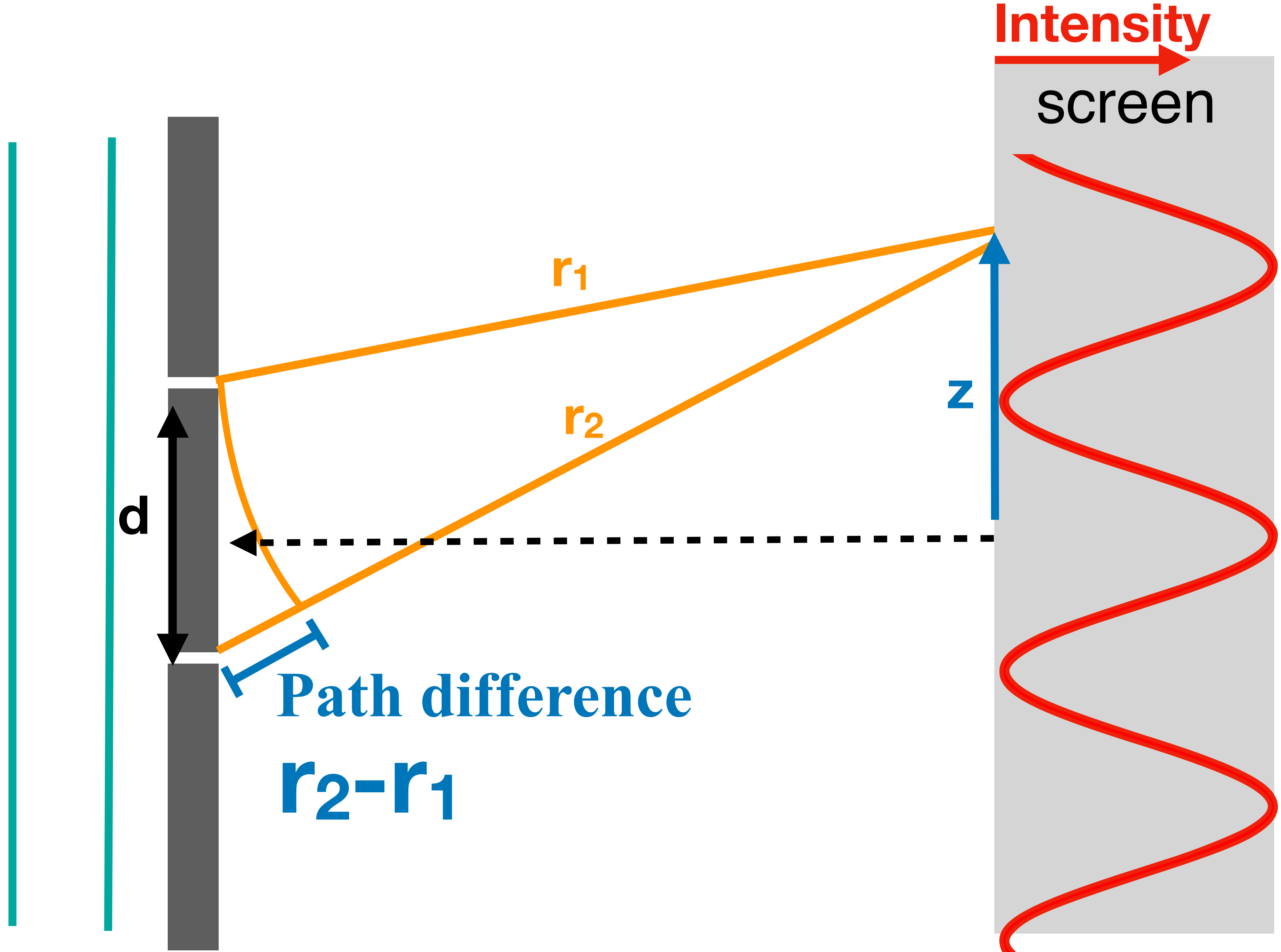
double slit **interference pattern**

$$I(\theta) \approx I_0 \cos^2(\pi d \frac{\sin\theta}{\lambda}) \qquad (17)$$

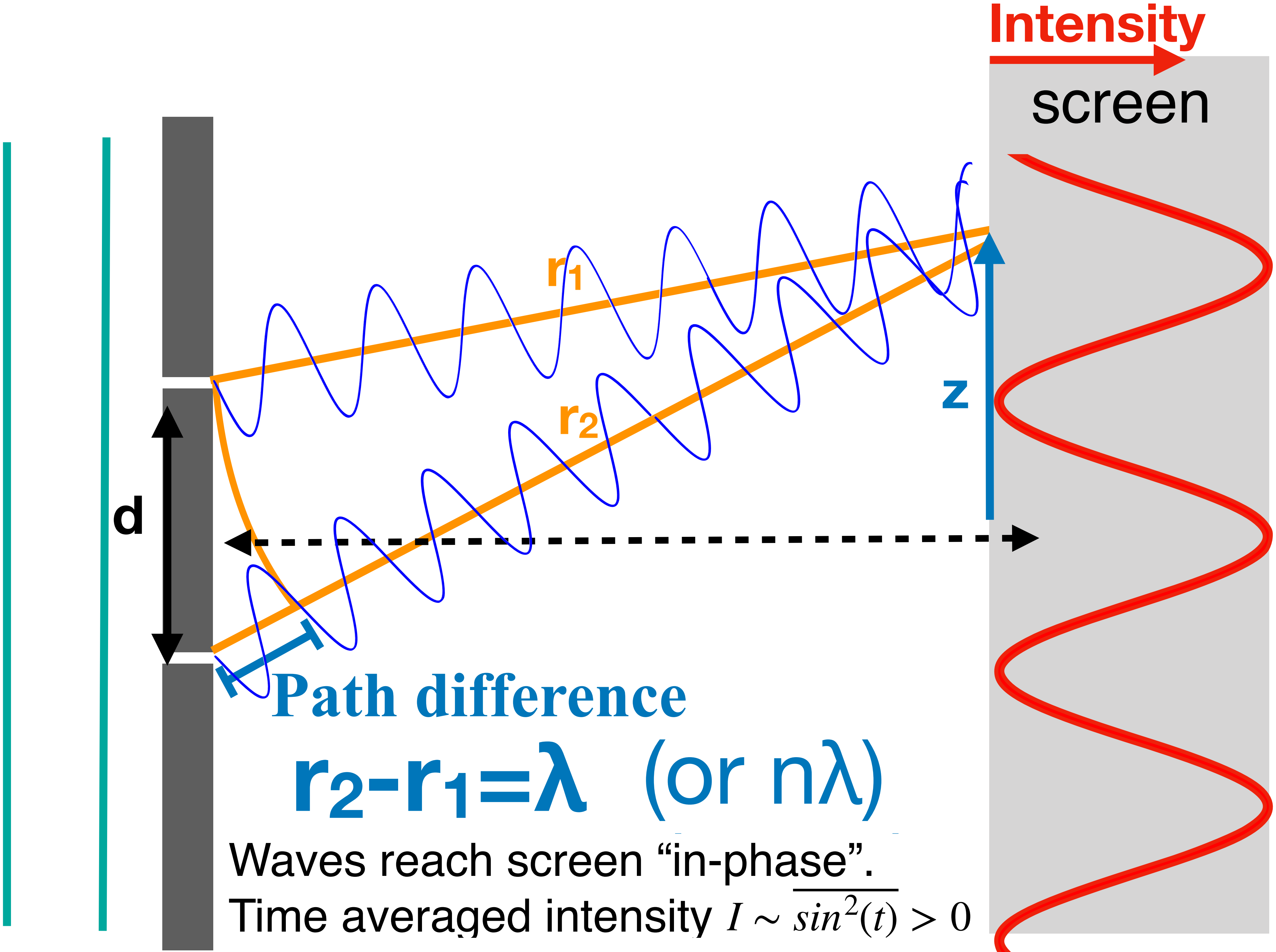
Diffraction and Interference



Diffraction and Interference



Diffraction and Interference

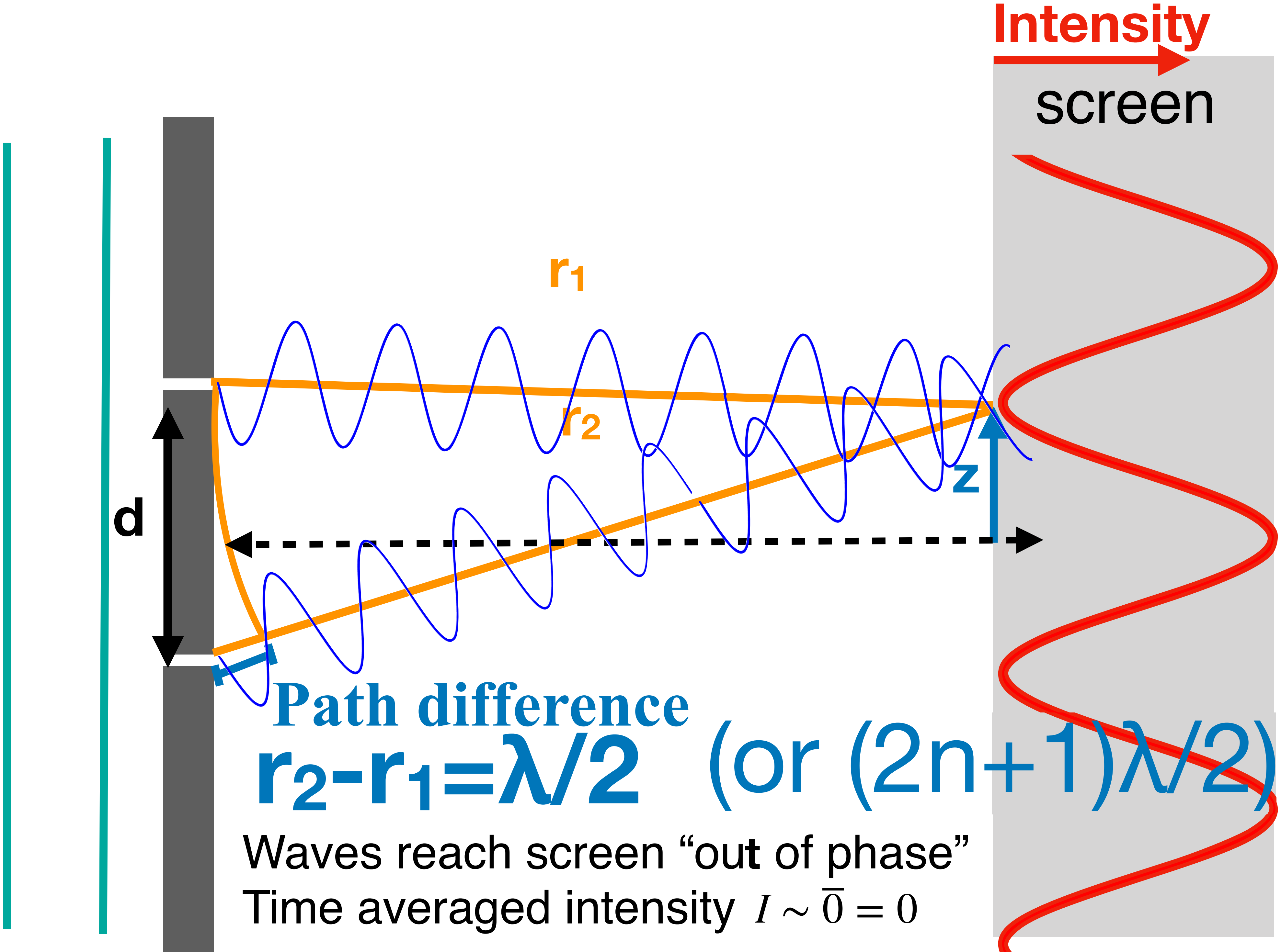


$r_2 - r_1 = \lambda$ (or $n\lambda$)

Waves reach screen "in-phase".

Time averaged intensity $I \sim \overline{sin^2(t)} > 0$

Diffraction and Interference



Path difference

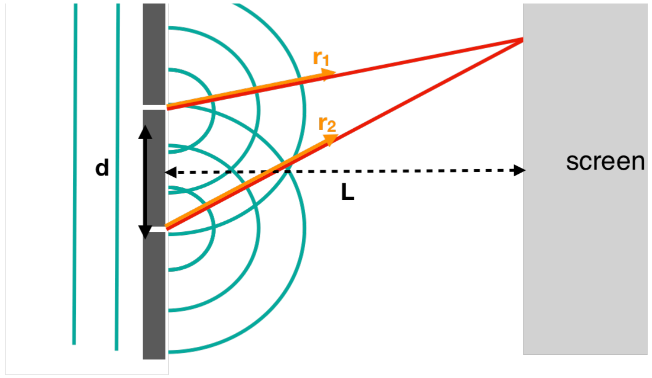
$r_2 - r_1 = \lambda/2$ (or $(2n+1)\lambda/2$)

Waves reach screen “out of phase”

Time averaged intensity $I \sim \overline{0} = 0$

Diffraction and Interference

Summary for two-path / double -slot interference



- path difference integer multiple of wavelength

$$|r_2 - r_1| = n\lambda, \qquad n \in \mathbb{Z}$$

$\Rightarrow$ constructive interference, same phase, waves add up.

Bright fringe on screen

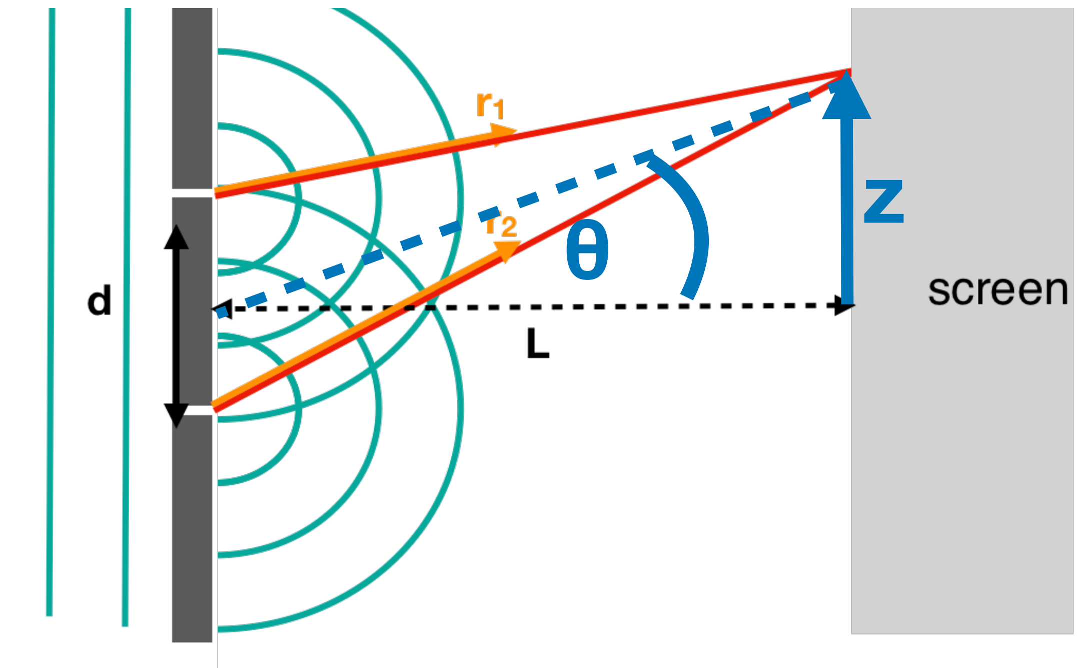
- path difference odd integer multiple of half wavelength

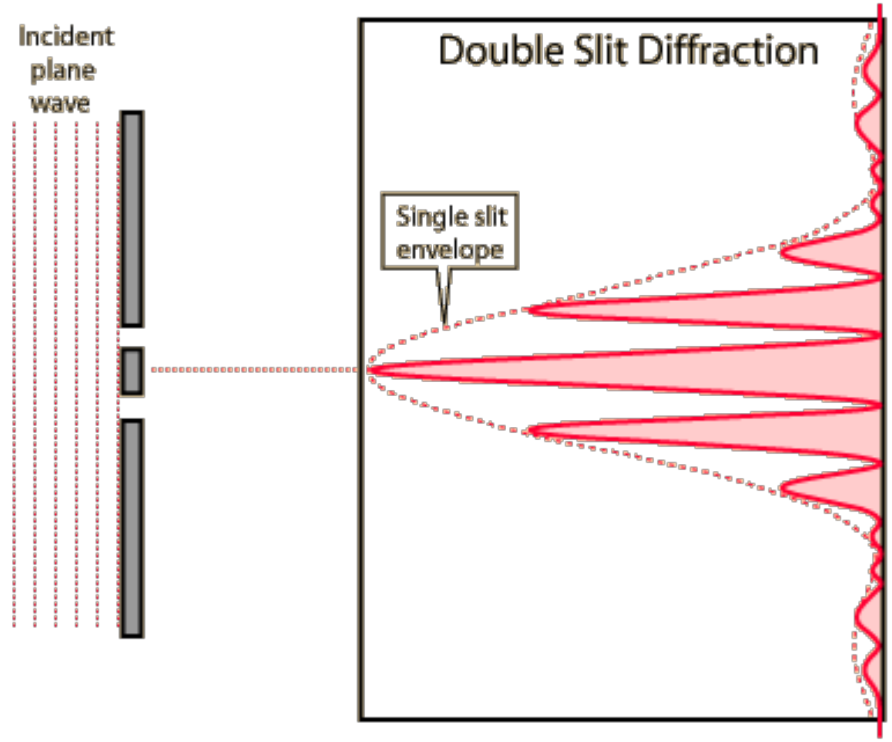
$$|r_2 - r_1| = (2n+1)\frac{\lambda}{2}, \qquad n \in \mathbb{Z}$$

$\Rightarrow$ destructive interference, opposite phase, waves cancel.

Dark on screen

Diffraction and Interference





double slit **interference pattern**

$$I(\theta) \approx I_0 \cos^2(\pi d \frac{\sin\theta}{\lambda}) \quad (17)$$

Diffraction and Interference

Examples:

fill in lecture

fill in lecture

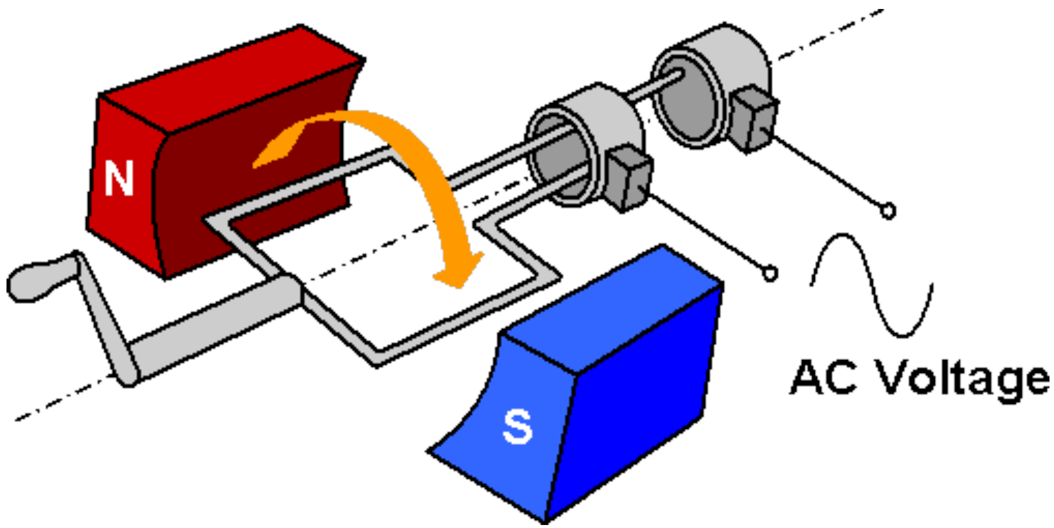
fill in lecture

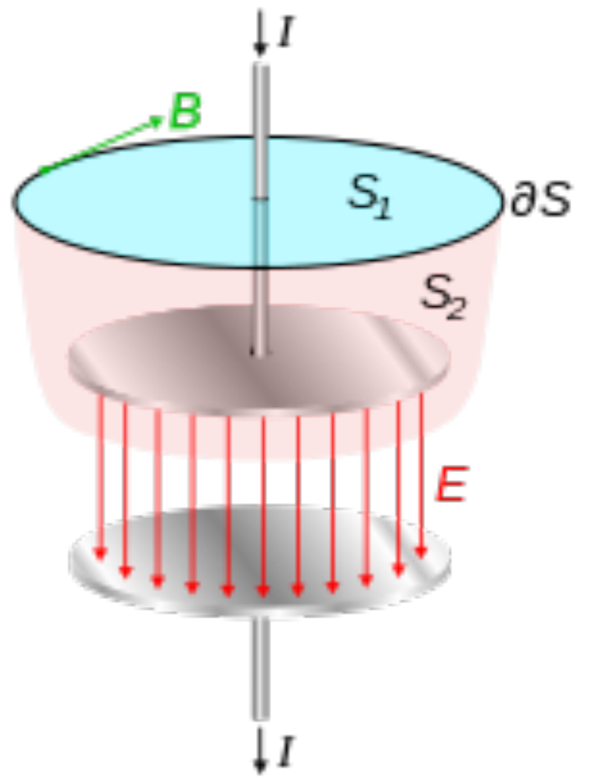
2.1.5) Electromagnetic waves

You will learn in Electro-magnetism lecture:

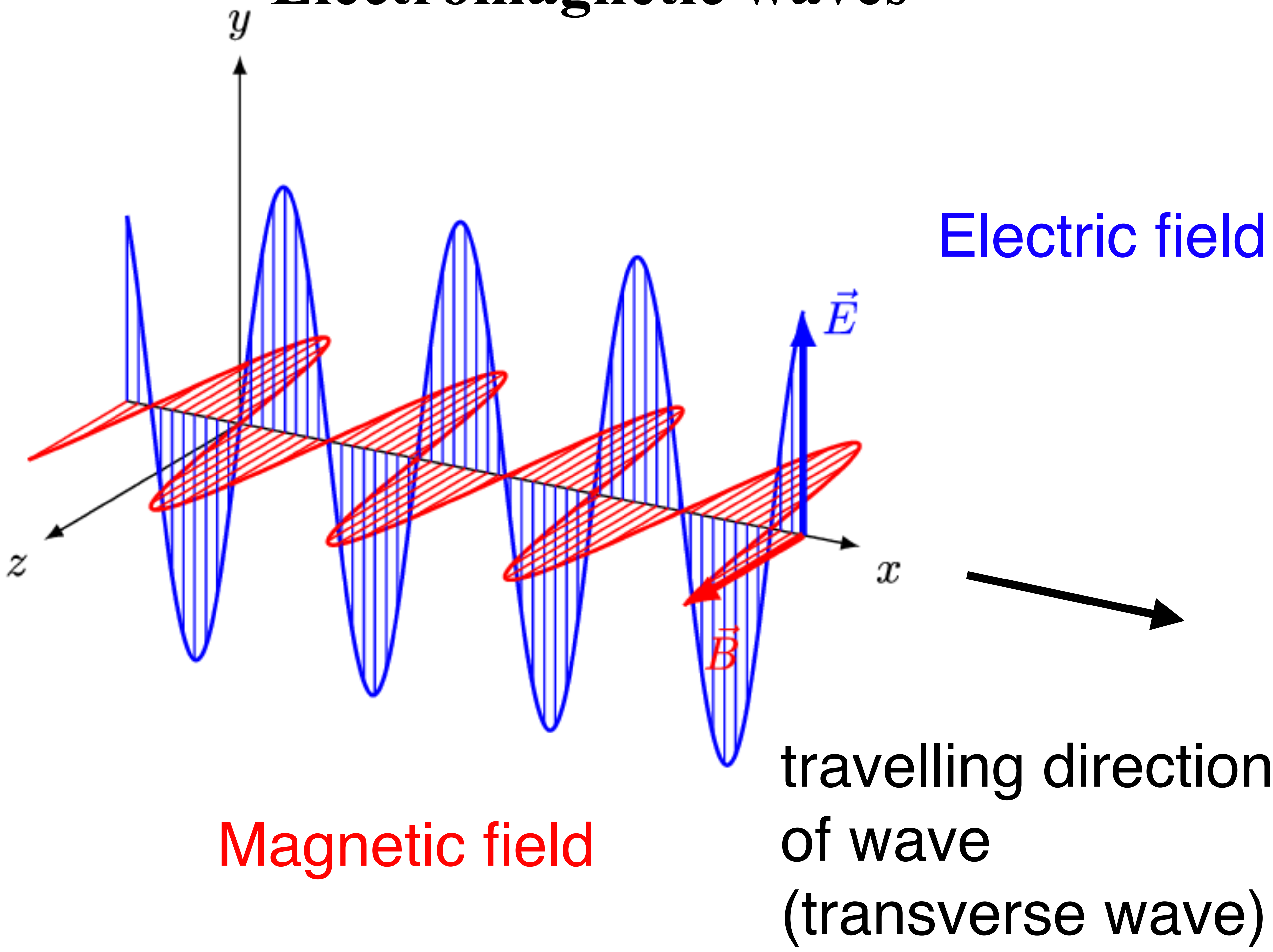
Changing **magnetic** field causes **electric** field (**induction**)

Changing **electric** field causes **magnetic** field

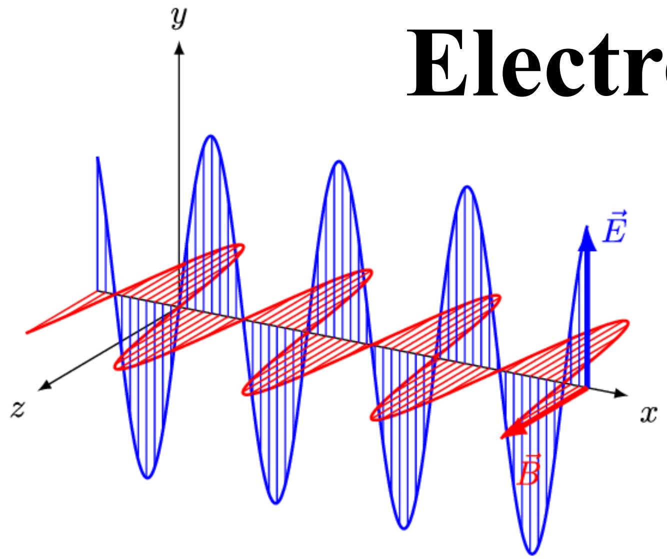




Electromagnetic waves



Electromagnetic waves



Electric field

Magnetic field

Electromagnetic wave equation:

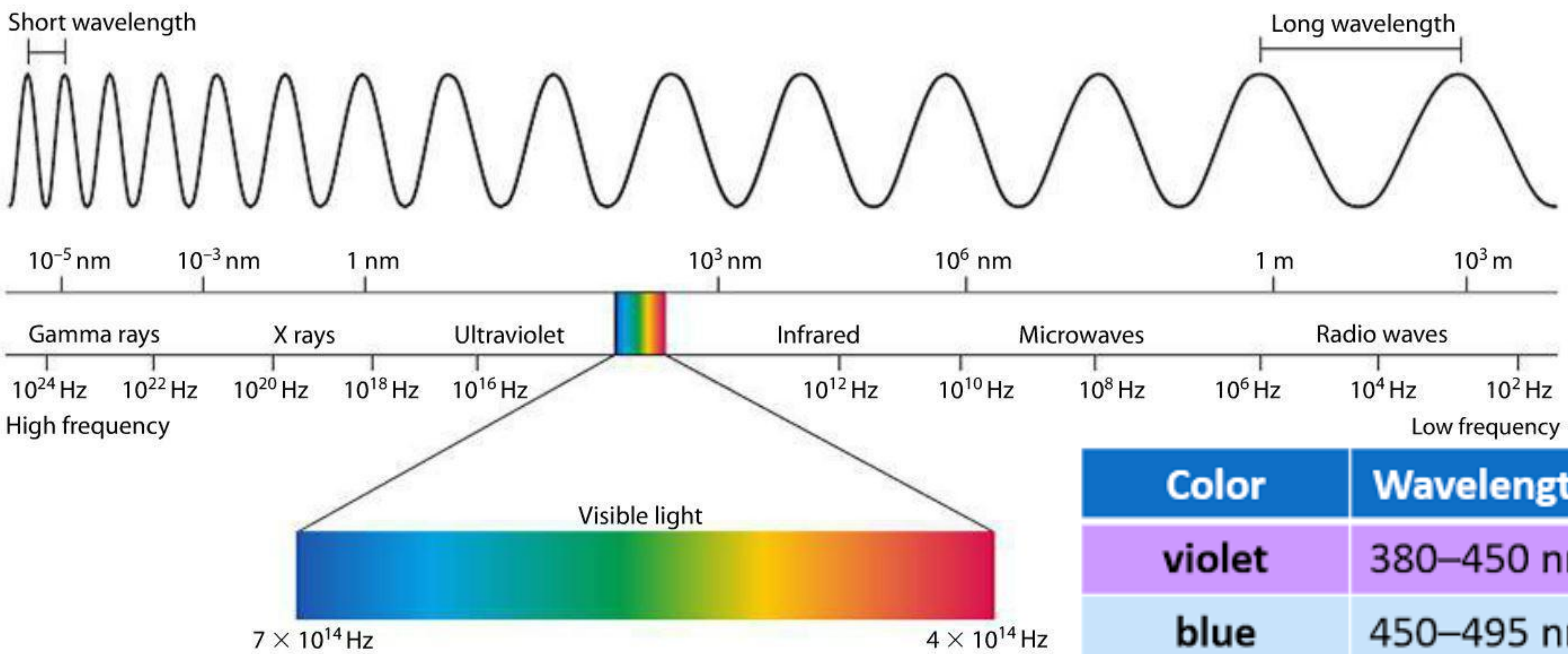
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\mathbf{E}(\mathbf{r}, t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r}, t) \qquad (18)$$

Speed of light (vacuum) $$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \qquad (19)$$

$$c = 29\,97\,92\,458\ m/s$$

Electromagnetic waves

$$\nu \ \lambda = c \qquad \textbf{(10)}$$



Color	Wavelength
violet	380–450 nm
blue	450–495 nm
green	495–570 nm
yellow	570–590 nm
orange	590–620 nm
red	620–750 nm

Electromagnetic waves

Energy density in a (classical) electromagnetic wave:

$$\rho_E = \frac{1}{2}\epsilon_0 \mathbf{E}^2 + \frac{1}{2\mu_0}\mathbf{B}^2 \qquad \textbf{(19b)}$$

Can have **any** value, more energy = stronger electric and magnetic fields.