**IISER-Bhopal, Department of Physics**

**PHYS 106, Quantum Physics** **Name:** ____________________

**Spring 2020, mid-term** **Roll no:** ____________________

**Seat no:** ____________________

**Time Limit: 2h; Permitted aids: Only calculator without plotting function. See equation collection at the back.**

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**README FIRST:**
*(i) Please write all solutions tidily and with concise/ short text or comments accompanying any calculations. Else marks may be subtracted.*
*(ii) For all questions, there is a* **strict** *size limit of whatever space is available on this question sheet for the answer. Any unreasonably small handwriting will be ignored by the marker.*
*(iii) We provided you with some separate sheets of paper for rough work, don't hand that in at the end. Copy your answers neatly onto* **this answer sheet only**, *after you obtained them.*
*(iv) In exceptional cases you may ask us for extra paper, in that case* **clearly mark on this answer sheet** *that some question continues on extra paper.*
*(v) At the end of the exam, please assemble your material in the following order: This question-answer sheet on top, extra sheets below, then tie together. The A4 page with only your seat number can be discarded after inserting seat number on this question sheet.*
*(vi) For all questions with numerical values, if you have a calculator it is sufficient to use 2 significant digits (two largest) digits that are non-zero, if you don't use just 1 significant digit.*

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Points: | 14 | 15 | 13 | 10 | 10 | 10 | 72 |
| Score: | | | | | | | |

1. **Black body radiation:**

   (a) (4 points) Explain what black-body radiation (BBR) is: Discuss which properties of an emitting object affect BBR and in which way. Which wavelengths does BBR contain?

(b) (4 points) Planck's radiation formula is $u(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{h\nu/(k_B T)}-1}$. What does this equation tell you? Draw a sketch of the function $u(\nu)d\nu$ versus $\nu$, either from memory or from that formula, for two different temperatures $T_2 > T_1$. *Make sure your sketch has axes labels. You can also draw $u(\lambda)d\lambda$ as a function of wavelength if you like.*

(c) (2 points) We find that the frequency $\nu_{max}$ where $u(\nu)d\nu$ takes the largest value fulfills $h\nu_{max} = 2.82 k_B T$. A very hot star has a surface temperature of $T = 20000$ K. Assuming it is a black-body, find the peak of its radiation spectrum in frequency and from that the associated wave-length. Which spectral range does this belong to?

(d) (2 points) In your drawing of Planck's formula for part (b) above, now also sketch the result from classical physics called Rayleigh Jeans law using a dashed line. Discuss what is problematic with it.

(e) (2 points) Discuss what has to be changed in the classical calculation in order to get the correct quantum result (Planck's formula). [*Use a text explanation, no need for any calculation*]

2. **X-rays:**

   (a) (2 points) Draw a diagram of an X-ray tube. Label all important elements in it. Next to it, sketch a typical spectrum of the emitted X-rays, make sure the sketch has axis labels.

   (b) (2 points) The X-ray spectrum you sketched in (a) should have two different components. Name these components and describe their different physical origin and whether they depend on the target material.

   (c) (4 points) Now in your X-ray tube, electrons are accelerated to a kinetic energy $E_{\text{kin}}$. Derive the shortest wavelength of X-ray that can then be generated, commenting the steps. Mark this wavelength in your sketch in (a).

   (d) (3 points) Suppose one of these shortest wavelength X-rays (later called "that X-ray") now hits a free electron at rest and scatters off it, changing direction by an angle $\beta$. What will be the wavelength of that X-ray after the scattering and why?

   (e) (4 points) If we specify $E_{\text{kin}} = 30$ keV and $\beta = 180^o$ in (c) and (d), what will be the wavelengths of that X-ray before and after the scattering?

3. **Photo-electric effect:**

   (a) (4 points) The work function (what is that?) for Potassium is $\Phi = 2.2$ eV. If you shine green light with wave length $\lambda = 500$ nm and intensity $I = 20$ W/m$^2$ onto it, can you see photo-electrons? If yes, what is their maximum kinetic energy?

   (b) (2 points) Light intensity is proportional to the number of photons in the light per unit time. What would you expect to happen to the photo electron current in part (a) if you increase the light intensity by a factor of of 10? Explain your answer.

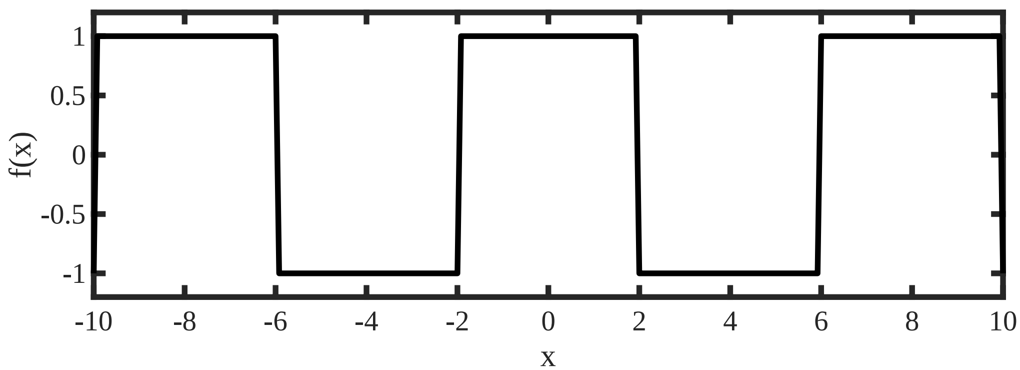
   (c) (4 points) What happens to the conclusions in (a) if we change the light wavelength to $\lambda' = 800$ nm? What happens if we change the light wavelength to $\lambda' = 800$ nm and increase the intensity to $I = 600$ W/m$^2$? Explain both answers.

   (d) (3 points) Discuss in words which three main features of the photo-electric-effect cannot be explained by classical physics, and what classical physics would predict instead.

4. **Wave packets**

   (a) (2 points) What is meant by "Fourier decomposition"? *Let's stick to the case of even periodic functions*

   (b) (2 points) In the figure of a square wave $f(x)$ below, from your intuition, sketch the two largest contributing cosine waves in the Fourier decomposition.




   (c) (6 points) Describe what is meant by "wave packet" and illustrate this with a drawing. What is the difference between "phase velocity" and "group velocity"? When are they the same and when do they differ?

5. **Electromagnetic waves**

   (a) (4 points) Write the seven named parts of the electro-magnetic spectrum as given in the lecture in the order from small photon energy to large photon energy

(b) (4 points) Draw a 3D sketch of the field structure (vectors) and propagation direction of a mono-chromatic electro-magnetic wave travelling in the positive x direction. Comment your drawing.

(c) (2 points) Which physical quantities decides the energy carried by a classical electro-magnetic wave? Which values can this energy take? What changes when considering quantum physics?

6. **Waves** A rope wave $y(x, t)$ travelling to the right has an amplitude 0.1 m, a wavelength 0.5 m, and an angular frequency $4\pi$/s.

(a) (2 points) For this wave, infer also the wave number, non-angular frequency and phase velocity. Don't forget units.

(b) (4 points) Draw at least two wavelengths of this wave into the same $(x, y)$ diagram at times $t = 0$ and $t = 1/8$s. You may pick any phase of the wave. Make sure the axes in the diagram are labelled, including units and axes ticks.

(c) (4 points) Consider that the two waves $f_1(x,t) = A\sin(k_1 x - \omega_1 t)$ and $f_2(x,t) = A\sin(k_2 x - \omega_2 t)$ are interfering. Calculate the sum of the two waves $f_{tot}(x,t) = f_1(x,t) + f_2(x,t)$ and write the result as a product of two trigonometric functions (instead of a sum). For the resultant wavepacket of two waves, determine the phase velocity $c$ and the group velocity $v_g$, assuming similar but not quite equal wave-numbers and frequencies, and justify the answer.

**The end. You may use the remaining page(s) for rough work. It will not be graded.**

## Equation collection

Physical constants:
Planck's constant $h = 6.63 \times 10^{-34}$ J s,
Reduced Planck's constant $\hbar = 1.05 \times 10^{-34}$ J s,
Electron charge $e = 1.60 \times 10^{-19}$ C,
Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/K,
Electron mass $m_e = 9.1 \times 10^{-31}$ kg
Neutron mass $m_n = 1.7 \times 10^{-27}$ kg
Bohr radius $a_0 = 5.29 \times 10^{-11}$ m
Vacuum speed of light $c = 3 \times 10^8$ m/s.

- Harmonic oscillator:
  After transient: $x(t) = C \sin(\omega t + \phi)$, with

$$C = \frac{|F_0|}{\sqrt{\gamma^2\omega^2 + (\omega^2 - \omega_0^2)^2}} \tag{1}$$

$$\varphi = \arctan\left(\frac{\gamma\omega}{\omega^2 - \omega_0^2}\right) \tag{2}$$

- Special Relativity

$$E = \gamma m c^2 \tag{3}$$

$$p = \gamma m v \tag{4}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{5}$$

- Compton scattering formula

$$\lambda' - \lambda = \left(\frac{\hbar}{mc}\right)[1 - \cos(\phi)] \tag{6}$$

- Trigonometric functions:
  $\sin(30^o) = 0.5$, $\sin(45^o) = 0.71$, $\sin(60^o) = 0.87$, $\sin(90^o) = 1$.
  $\cos(30^o) = 0.87$, $\cos(45^o) = 0.71$, $\cos(60^o) = 0.5$, $\cos(90^o) = 0$.

- Trigonometric identities:
  $\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$.
  $\sin(a) \pm \sin(b) = 2\sin\left(\frac{a\pm b}{2}\right)\cos\left(\frac{a\mp b}{2}\right)$