PHY 304, II-Semester 2023/24, Tutorial 7

Wed 10th April 2024

Work in the same teams as for assignments. Do "Stages" in the order below. Discuss on your table in AIR. When all on your table finished a stage, make sure all students at your table understand the solution and agree on one by using the board.

Stage 1 Time dependent perturbation theory

(i) Discuss what is the mission objective of time-dependent perturbation theory. When do we need it? What are alternatives to it?

(ii) Revise the general formula for the transition from state $|\, i\,\rangle$ to state $|\, f\,\rangle$ in first order time-dependent perturbation theory, and describe the key terms in it and which physical properties of the system they relate to.

(iii) Discuss the special case for a periodic perturbation and what physical requirements on the quantum state transition the different mathematical terms encode.

Stage 2 Rabi oscillations: Discuss Rabi oscillations in section 9.3.5 on your table for the resonant case ($\Delta = 0$).

(i) Then run the app http://www.falstad.com/qmatomrad/ , which handles transitions between electronic states of a Hydrogen atom coupled with a periodically oscillating electric field in various choosable directions. The Hamiltonian thus is:

$$\hat{H} = \hat{H}_{hyd} + e\mathbf{E}_0 \cdot \hat{\mathbf{r}} \cos(\omega t) \tag{1}$$

with $\mathbf{E}_0 = E_0 \mathbf{e}_{x,y,z}$ with the direction of $\mathbf{E}_0$ indicated in the app. The app only considers resonant perturbations, such that $\hbar\omega = E_n - E_m$ for two Hydrogen states n and m. Discuss how features you see in the animation relate to what you learnt about Rabi-oscillations, what information decides which two Hydrogen states are involved, and why only two become relevant in this app.

(ii) **Bonus:** Also discuss what behavior of the electron dipole $\langle\, e\hat{\mathbf{r}}\,\rangle$ corresponds to the electron wavefunctions shown, and how this relates to the emission of absorption of radiation (thinking about the latter in terms of classical electromagnetism for now).

(iii) **Bonus:** You can also check out http://www.falstad.com/qm1drad/ , which handles dynamics of the particle in the box subject to a periodically oscillating force along the x-direction, just as we had seen in QM-1 assignment5Q2 and are presently seeing in assignment 6 Q4 (see those for the Hamiltonian). In contrast to the first app, it allows you to chose an arbitrary frequency ω and strength F_0 for the perturbation. Play with those and discuss what happens and understand why.

Stage 3 **Interaction of electro-magnetic waves with matter:** Consider an arbitrary quantum system for which we assume we know the unperturbed spectrum

$$\hat{H}^{(0)}|\,\phi_n\,\rangle = E_n|\,\phi_n\,\rangle. \tag{2}$$

The system contains particles that are affected by electro-magnetic fields, such that the system is subject to a perturbing Hamiltonian

$$\hat{H}'(t) = \hat{V}\cos(\omega t) \tag{3}$$

if a monochromatic electro-magnetic wave of frequency ω hits it. Consider $\hbar\omega$ to be of the order of the $E_n - E_m$. Note that this a very general setting throughout physics, the states could describe eigen-energies of (nuclei/atoms/molecules) or quasi-particle energies in materials (excitons/phonons/polarons etc.) and waves can span the entire spectrum from gamma rays to radio. We assume the system is in the ground-state $|\,\phi_0\,\rangle$ initially.

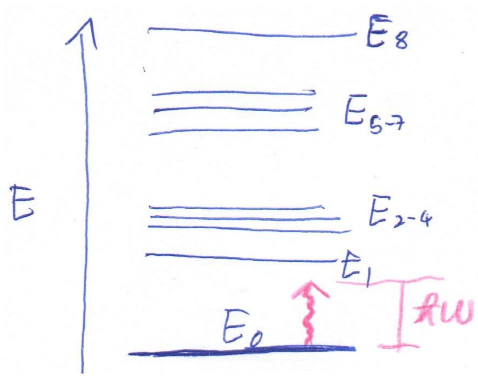

Figure 1: (**stage 3**) Cartoon of a quantum system with discrete energies, interrogated by monochromatic radiation.

(i) Initially assume all $\langle\,\phi_m\,|\hat{V}|\,\phi_n\,\rangle = V_0 \neq 0$. Consider the cases where (a) ω is closer to $E_k - E_0$ than to any other energy differences but $|\hbar\omega - (E_k - E_0)| \gg V_0$, (b) $\hbar\omega = E_k - E_0$ for some k, and $|\hbar\omega - (E_n - E_0)| \gg V_0$ for all other $n \neq k$ and (c) $V_0 \gg |\hbar\omega - (E_n - E_0)|$ for a large number of n. In each case discuss qualitatively (or quantitatively if possible) what you expect to happen once the elm. wave hits the system and how you can know this.

(ii) Typically the $\langle\,\phi_m\,|\hat{V}|\,\phi_n\,\rangle$ will not be all equal. Discuss what information they contain and through which principles you can often infer which ones are non-zero.

(iii) **Bonus:** While the discussion above forms the basis of interactions of electro-magnetic waves with matter, there is lots and lots of complications, details and effects that are not yet considered and you will learn later. Brainstorm some.

(iv) What changes in the picture if we do not start in the ground-state, but an excited state $|\,\phi_e\,\rangle$?