PHY 304, II-Semester 2023/2024, Tutorial 6

27. March 2024

Work in the same teams as for assignments. Do "Stages" in the order below. Discuss on your table. When all teams finished a stage, make sure all students at your table understand the solution and agree on one by using the board.

Stage 1 Wave scattering:

(i) Make drawing of 1D, 2D and 3D quantum scattering scenarios, and qualitatively discuss what degrees of freedom are there in the scattering wavefunction in each case, and which information is fixed by conservation laws. Does it make sense to talk about scattering in ND with $N > 3$?

(ii) Discuss in your team the physical meaning (as opposed to mathematical definition), of "scattering angle", "scattering amplitude", "differential cross section", "total cross section" and "partial wave amplitude".

(iii) Follow this link: https://physics.weber.edu/schroeder/software/ and then start the app "Quantum Scattering in Two Dimensions". Read the description, switch 'Barrier type" to "Circle" (or "Square"), and then do numerical experiments with sliders "Packet energy", "Strength", "Size", "Softness", to make contact with as many concepts from the lecture as possible. *[Important note: STOP the simulation once any wave hits the outer edge of the box, it becomes nonsense afterwards]* Discuss whether and how you can see

- The structure of the scattering state (8.8) discussed in the lecture.
- Interference
- Momentum dependence of scattering
- Others?

You may get back to this applet after doing assignment 5Q3, which provides you with a very similar code.

Stage 2 Partial wave expansion

(i) For the scenarios in Fig. 1, qualitatively discuss which partial waves ℓ you think might be significant and which angular dependence of the scattering amplitude f you would expect. Assume dimensionless units with $\hbar = m = 1$ and the following incoming wavenumber (momentum): (a) $k = 0.5$, (b) $k = 0.5$, (c) $k = 2$, (d) $k = 0.25$ and $k = 4$.

(ii) Consider scattering at low energy E off a spherically symmetric square well potential $V(\mathbf{r}) = -V_0\theta(a - r)$ with $r = |\mathbf{r}|$, such that the radial TISE [Eq. (8.19)] in the $\ell = 0$ channel reads

$$\left[\frac{d^2}{dr^2} - V(r)\right] u_0(r) = -k^2 u_0(r), \tag{1}$$

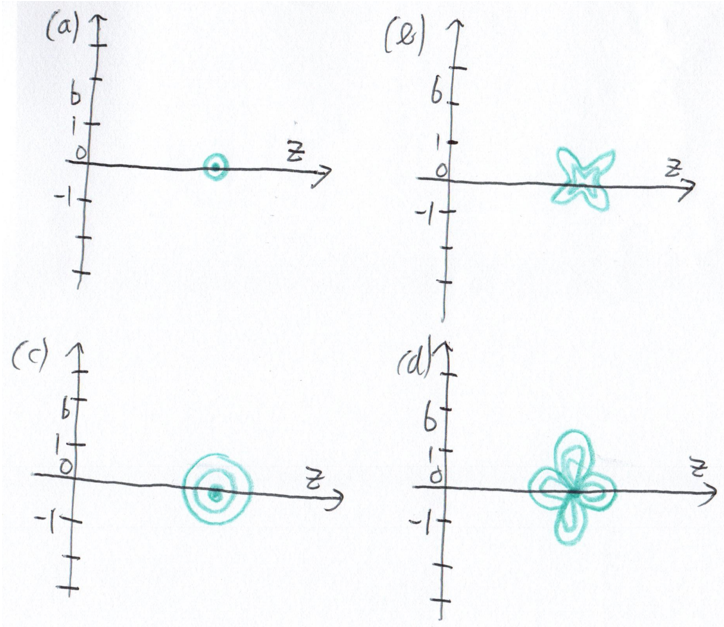

Figure 1: (**stage 1**) Scattering potentials $V(\mathbf{r})$ in the (b, z) plane as contour plots (green). Assume the potential vanishes outside the outermost contour. The precise details within the contours are not important.

and the solution can be written as

$$u_0(r) = \begin{cases} C\sin(Kr), & r \leq a \\ \sin(kr + \delta_0), & r > a. \end{cases} \tag{2}$$

with $k = \sqrt{2mE}$, $K^2 = k^2 + V_0^2$ and real δ_0. Discuss in your team first why/when looking at $\ell = 0$ only is justified, with which steps you can find the total scattering cross section and s-wave scattering length or s-wave scattering phase shift, all the way to the end. If there is time, then also perform those steps (in that case justify (2) first).

Stage 3 Born approximation: Find the total scattering cross section for very low energy scattering from the potential

$$V(\mathbf{r}) = \begin{cases} V_0, & r \leq a \\ 0, & r > a. \end{cases} \tag{3}$$

in the first Born-approximation.

Stage 4 Born series: Discuss in your team your intuitive understanding of the Born-series (or lack thereof, in which case try to get some).