PHY 304, II-Semester 2023/24, Tutorial 1

11. Jan. 2024

Discuss on your table in AIR. When all teams finished a stage, make sure all students at your table understand the solution and agree on one by using the board.

Stage 1 Checklist Let's play a revision game for QM-I on your table: Everyone in turn quickly answer a question from below. Everyone then can agree, disagree or discuss. If you don't know the answer when it is your turn, pass or pick another question, put take note of those points for revision later.

(a) **Basic structure of quantum mechanics** Instead of a well defined position and momentum as in classical mechanics, the quantum mechanical particle is described by a wave function $\Psi(x, t) \in \mathbb{C}$. Answer the following questions as a self-check: What is the interpretation/meaning of this wavefunction?______________ Why is it complex and what does the complex part of the numbers tell us?______________. What is the value of $\int dx |\Psi(x, t)|^2$ and why?______________. What is the importance of a global complex phase $\Psi(x, t) \rightarrow e^{i\varphi}\Psi(x, t)$ with $\varphi \in \mathbb{R}$?______________. Which equation governs how the wavefunction changes in time ?______________. What is the meaning of stationary state and how do we find those?______________.

(b) **Measurements in quantum mechanics** How do we find the probability of a measurement of any observable?______________. Why do we need operators in quantum mechanics?______________. What happens to the quantum state while we find a certain measurement result?______________. How to we find the mean of a large number of measurements?______________. How do we find the uncertainty or standard deviation of a large number of measurements?______________.

(c) **Solutions of the TISE and why we need them** For what do we need the TISE?
______________. In which cases and how can we also find time-dependent information (time-evolution) from the TISE?______________. Which nice properties do solutions of the TISE have?______________. List a few practical aspects needed to solve the TISE, how many different methods do you know?______________. What is the meaning of degeneracy? ______________ List some classifications of solutions of the TISE ______________ List typical properties of bound state solutions of the TISE ______________

(d) **Solutions of the TDSE and why we need them** Why do we need the TDSE in addition to the TISE?______________. How many methods to solve the TDSE do you know?______________. List a few physical phenomena for which knowing the TISE is not enough______________.

(e) **Uncertainty relations** What is the basic mathematical origin of uncertainty relations in quantum mechanics? List an intuitive and a formal reason. ______________. Which uncertainty relations do you know?______________ What is the relation between uncertainty relations and operators sharing eigenfunctions? ______________ What is special with the energy-time uncertainty relation, and why do we have to be careful using it?______________

(f) **Quantum effects** List at least six quantum mechanical phenomena that are in an essential way different from behaviour in classical mechanics. ______________.

(g) **Algebraic solutions to quantum problems** With which trick can you often avoid finding all the eigenstates and energies from the TISE, and instead use the action of operators onto states directly? ______________

(h) **Angular momentum** How is angular momentum dealt with in quantum mechanics?
______________ Which different types of angular momentum do you know?
______________ What rules to angular momentum quantum numbers fulfill?
______________ Which links between angular momentum and particle motion do you know from classical mechanics and how are they preserved in quantum mechanics?
______________ How do we add angular momenta of two particles ?
____

(i) **Many-Partices or dimensions** What changes in the math when you move from a single particle to many particles? or from one dimension to many dimensions? ______________ How can we often tackle those complications to resort back to our easier 1D solutions? ______________ Explain the concept of indistinguishable particles in quantum mechanics?______________
What is the fundamental consequence of this principle? ______________
Which classification of particles does it lead to?______________
What is entanglement and why is it interesting?______________

Stage 2 **Questions and answers:** A first step at having mastered some material is to be able to ask questions about it, and a second step is to be able to answer such questions. Perhaps guided by stage 1, I would like to ask each of you to make two large lists:

(i) Topics from the QM-I course that you feel you understood well.

(ii) topics from the QM-I course that you feel you understood less well.

Share these lists within your group. Then I would like to ask all those which comfortable with a topic [i.e. it was on their list (i)] to explain it to those group-members that are not [i.e. have it on their list (ii)]. This will be beneficial for both sides: only when you can explain something have you fully understood it, and often you only understand it when you attempt to explain it. Afterwards, please make a group-wise collection again with (i) and (ii), in particular listing all topics that none of you felt happy to explain or all/most of you felt comfortable with. Sent this final list with your group ID and all student names to your TA.

Stage 3 Entanglement: *(Only do this if you have done the revision part earlier, or are confident you need none).* For each of the following states of two spin-1/2 particles: (i) find the probability to measure the first spin to be "up", (ii) find the state after measuring the second one in "up", (iii) find the probability to measure first one in "up" after we have measured the second in "up". Discuss.

(i) $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

(ii) $|\Psi\rangle = \frac{1}{2}(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle)$

Hint: The outcome of a measurement can be found using projection operators $\hat{P}$, compare also postulates in section 3.6. If you measure the first particle to be in state $|\varphi_k\rangle$, you project the wavefunction before measurement $|\Psi\rangle$ according to $\hat{P}|\Psi\rangle$, and then re-adjust normalisation to one. In that expression

$$\hat{P} = |\varphi_k\rangle\langle\varphi_k| \otimes \mathbb{1} \otimes \mathbb{1}, \tag{1}$$

which acts as identity $\mathbb{1}$ onto all particles that are not the first.

Stage 4 Indistinguishable particles: *(Only do this if you have done the revision part earlier, or are confident you need none).* Which of the following is a valid quantum state for the two particles listed ($\mathcal{N}$ normalises the wavefunction).

(i) Two ^{87} Rb atoms: $\Psi(x_1, x_2) = \mathcal{N}e^{-\frac{x_1^2+x_2^2}{2\sigma^2}}$.

(ii) Two ^{40} K atoms: $\Psi(x_1, x_2) = \mathcal{N}e^{-\frac{x_1^2+x_2^2}{2\sigma^2}}$.

(iii) A Helium atom and a Hydrogen atom: $\Psi(x_1, x_2) = \mathcal{N}e^{-\frac{x_1^2}{2\sigma^2}}e^{ikx_2}$.