PHY 304, II-Semester 2023/24, Assignment 6

Instructor: Sebastian Wüster
Due-date: 14. April. 2024

(1) Kicked quantum dot [**10 pts**] Consider the particle in an infinite square well potential, as discussed in QM-I, section 2.2.1, which is initially in the ground-state $n = 1$ and then subject to a briefly pulsed perturbation

$$\hat{H}'(t) = \kappa \left(\hat{x} - \frac{a}{2}\right)^3 \sin^2\left(\pi t/T\right) \text{ for } 0 < t < T, \tag{1}$$

and again $\hat{H}'(t) = 0$ afterwards.

(a) To first order in κ, find the transition probabilities from the ground state $n = 1$ initially, to any other state n'. [6 pts]

(b) Checkout these probabilities in the limits $T \to 0$ and $T \to \infty$ and justify those based on physical arguments. [2pts]

(c) Use `assignment6_question1_draft.nb`, which solves the above scenario numerically, to verify your calculation from part (a) by implementing your solution in the indicated spot. Discuss where you see agreement, what differs, and why that might be. [2 pts]

(2) Shift of a harmonic oscillator: [**5 pts**] Assume a particle of mass m is initially for $t < 0$ in the ground-state of a harmonic oscillator potential with frequency ω:

$$V(x) = \frac{1}{2} m\omega x^2. \tag{2}$$

Now at $t = 0$, we suddenly shift the potential by a displacement x_0:

$$V(x, t > 0) = \frac{1}{2} m\omega \left(x - x_0\right)^2. \tag{3}$$

What is the state of the oscillator immediately after this shift and why? Also find the probability distribution of the particle position at all $t > 0$. What is the fastest velocity with which we could shift the potential slowly from being centered at 0 to being centered at x_0, without causing significant excitations of the oscillator?

(3) Charged particle in 3D within quantum dot [**5 pts**] A particle of mass m with electric charge q is confined to a three-dimensional cubical box of side length L and in the ground-state until $t = 0$. It now feels an electric field $\mathbf{E} = E_0 e^{-\alpha t}\mathbf{e}_x$ for $t > 0$ only, where α is a constant, and $\mathbf{e}_x$ is the unit vector in the x-direction. Calculate the probability that the charged particle is excited to the first excited state by the time $t = \infty$. Discuss the dependence of your result on α. How can you separately understand the limits $\alpha = 0, \infty$?

(4) Driven quantum dot [10 pts] Let us change the perturbation of the (otherwise unchanged) quantum dot in Q1 to

$$\hat{H}'(t) = F_0 \left(\hat{x} - \frac{a}{2} \right)^3 \sin(\omega t) \tag{4}$$

at all t. Let us try to recreate evolution similar to example 46 page 144, from the initial state $|\,\Psi(0)\,\rangle = |\,\phi_2\,\rangle$ [See online code, we use a Hamiltonian in matrix form].

(a) Insert the missing pieces into the code `assignment6_question4_draft_v1.nb`, most of which you can take from Q1.

(b) Using the parameters provided in the code, *quantitatively explain* all features of the dynamics (amplitude and period of main oscillation, choice of states, why is there which oscillation). We are using dimensionless units with $\hbar = m = 1$. [4pts]

(c) Moderately vary the perturbation amplitude F_0 (but keep it $F_0 < 0.05$ here) and discuss what happens. Change the frequency to $\omega = (E_3 - E_2)/\hbar$ and $\omega = (E_2 - E_1)/\hbar$ instead of the default ($\omega = (E_5 - E_2)/\hbar$). Discuss what happens and why. [4pts]

(d) Now go to large $1 < F_0 < 10$. Discuss what you see and why? [2pts]