PHY 304, II-Semester 2023/24, Assignment 2

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Due-date: 27. Jan. 2024

(1) Symmetries: [8 pts]

(a) First, supply some of the unproven arguments in our section on symmetries. (i) Show that the eigenvalue z of a unitary operator must have unit modulus $|z| = 1$. (ii) Show that the parity operator is Hermitian. (iii) Show that the rotation operator around the z axis is:

$$\hat{R}_z(\alpha) = e^{-i\frac{\alpha}{\hbar}\hat{L}_z}. \tag{1}$$

using similar arguments as those that led to Eq. (6.3). [4 pts]

(b) Consider a particle of mass m in the potential

$$V(x, z) = \kappa(\hat{x}^2 + \hat{y}^2) + \bar{\eta}(\hat{x}^4 + \hat{y}^6), \tag{2}$$

for $\kappa > \bar{\eta} > 0$. Make a meaningful 2D plot of this potential with a computer (or by hand).

Find all symmetries of the Hamiltonian. Based on that discuss whether or not you expect the spectrum to show degeneracies. [4 pts]

(2) Perturbed square well potential: [8 pts] A particle is subject to the infinite square well potential (confined within $x = 0$ and $x = a$, see e.g. Eq. (2.10) of QM-1), with an additional potential of

$$W(x) = W_0 x/a. \tag{3}$$

(a) Make a drawing of the complete potential, and discuss under which conditions we can handle it in perturbation theory. [2 pts].

(b) Then find the energy corrections to the lowest two states of the original square well potential to first order perturbation theory. [6 pts]

(PTO)

(3) Charged oscillator: [**14pts**] Consider a particle of mass m and charge q in a harmonic potential with frequency ω subject to a constant electric field of strength $\mathcal{E}$. The Hamiltonian is hence

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 - q\mathcal{E}\hat{x}, \tag{4}$$

(a) Define a suitable splitting of the Hamiltonian for the use of perturbation theory, if the field is sufficiently weak. What constitutes "sufficiently weak"? [2 pts]

(b) Find all the matrix elements $\mathcal{M}_{nm} = \langle\, n\,|\hat{H}'|\, m\,\rangle$ of whatever you choose as perturbation above. Then use the special case $\langle\, n\,|\hat{H}'|\, n\,\rangle$ to evaluate the first order correction $E_n^{(1)}$ to the energies. *Hint: Use the ladder operators from Eq. (2.43, QM-I)* [2 pts]

(c) Use the $\mathcal{M}_{nm}$ to also find an expression for the second order correction $E_n^{(2)}$ to the energy and the first order correction to the states $|\,\psi_n^{(1)}\,\rangle$. [4 pts]

(d) Trying to analytically evaluate $E_n^{(2)}$ and $|\,\psi_n^{(1)}\,\rangle$ is not going to be very illuminating, instead let us explore their use on a computer. `Assignment2_program_draft_v1.nb` is set up to help you compare all your results above with those provided by a numerical solution of the complete problem. For that adjust the script at the places with XXXXX: (i) Complete the definition of `TISELHSpert`, which should contain the left hand side of the TISE <u>with</u> the perturbation. (ii) Execute the solution of the unperturbed oscillator numerically in the line below, and verify eigenvalues are as expected. (iii) In the definition of `Energyfirstorder`, insert your first order result from (b). In the definition of `Matrixelement` insert $\mathcal{M}_{nm}$ from (c). (iv) Also use this and `Energy[n]` to complete the definitions of second order energies, first order states and normalisation of the perturbed state. Then run all the commands for comparison of energy eigenvalues and wavefunctions. Discuss and compare with your expectations from (c). [4 pts]

(e) Why could we have guessed the change under this perturbation of all energy eigenvalues directly from the start, without doing perturbation theory? [2 pts]