PHY 304, II-Semester 2023/24, Assignment 1

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Due-date: 13th Jan. 2024

(1) Perturbed Hydrogen atoms [10pts]: Through interaction with light, a Hydrogen atom has ended up in an equal superposition of the states $|\,3s0\,\rangle$ (with $n = 3$, $\ell = 0$, $m = 0$) and $|\,3p0\,\rangle$ (with $n = 3$, $\ell = 1$, $m = 0$) at $t = 0$:

$$\Psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{2}} \left(\phi_{300}(\mathbf{r}) + \phi_{310}(\mathbf{r})\right). \tag{1}$$

(a) Find the explicit probability distribution of the position of the electron as a function of time. Try to visualize this as best you can and discuss. [2pts]

(b) What are the possible outcomes of measuring the magnitude of angular momentum of the electron in the state (1), what are their probabilities, and how do those probabilities change in time? [2pt]

(c) What is the probability of finding the electron at $z > 0$ in the state (1), and how does this probability change in time? [2pts]

(d) What is the expectation value of the electronic dipole operator $\hat{\mathbf{d}} = -e\hat{\mathbf{r}}$ as function of time? [2pts]

(e) From your knowledge of waves, optics and electro-magnetism, discuss the expected interplay between an atom in this state and electro-magnetic waves. [2pts]

(2) Entanglement [10pts]:

(a) For the following two-particle states, discuss whether they are entangled or not [3pts]

(i) $\Psi(x, y) = \frac{1}{\sqrt{2}}[\phi_0(x - x_0)\phi_3(y - y_0) + \phi_2(x - x_0)\phi_1(y - y_0)]$ (position of particle 1 is x, position of particle 2 is y, $\phi_n(x)$ are normalised eigenstates of the simple harmonic oscillator)

(ii) $\Psi(x, y) = \phi_0(x - x_0)\phi_3(y - y_0)$

(iii) $\Psi(x, y) = \frac{1}{2}[\phi_0(x)\phi_0(y) - \phi_0(x)\phi_1(y) - \phi_1(x)\phi_0(y) + \phi_1(x)\phi_1(y)]$

(iv) $\Psi(x, y) = \frac{1}{2}[\phi_0(x)\phi_0(y) - \phi_0(x)\phi_1(y) + \phi_1(x)\phi_0(y) + \phi_1(x)\phi_1(y)]$

(v) $|\,\Psi\,\rangle = |\,s = 1; m_s = -1\,\rangle \otimes |\,s = 1; m_s = +1\,\rangle$ (two different spin-1 particles)

(vi)

$$\begin{aligned} |\,\Psi\,\rangle = \frac{1}{2}\Big[& |\,s = 1; m_s = -1\,\rangle \otimes |\,s = 1; m_s = -1\,\rangle \\ & + |\,s = 1; m_s = +1\,\rangle \otimes |\,s = 1; m_s = -1\,\rangle \\ & + |\,s = 1; m_s = -1\,\rangle \otimes |\,s = 1; m_s = 0\,\rangle \\ & + |\,s = 1; m_s = +1\,\rangle \otimes |\,s = 1; m_s = +1\,\rangle \Big] \end{aligned} \tag{2}$$

(b) Give another 2 examples of entangled and 2 examples of separable states involving three particles and discuss [2 pts].

(c) Einstein-Podolsky-Rosen correlations: Consider the entangled state for two spin-1/2 particles: $|\Psi\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$. Show that the correlation of the spin projection onto axis $\mathbf{a}$ for particle one, with that onto axis $\mathbf{b}$ for particle two is:

$$\langle(\mathbf{a}\cdot\hat{\mathbf{S}}^{(1)})(\mathbf{b}\cdot\hat{\mathbf{S}}^{(2)})\rangle = -\frac{\hbar^2}{4}\mathbf{a}\cdot\mathbf{b}, \tag{3}$$

where $\hat{\mathbf{S}}^{(k)}$ is the spin-operator for particle k. [5pts]

(d) (Bonus) Read and understand the proof of Bell's theorem in Griffith (page 446), that shows that Eq. (3) cannot be explained by a classical local hidden variable theory.

(3) (Pseudo) Spin-1/2 particle [10pts]: Let the Hamiltonian of a spin-1/2 particle (or any other two-level system that we can map onto a pseudo spin-1/2) be:

$$\hat{H} = \kappa\sigma_y + E(\sigma_z + \mathbb{1}), \tag{4}$$

where σ_k are Pauli matrices.

(a) Write the most general time-dependent state and then the TDSE in terms of its coefficients. [2pts]

(b) Solve the TDSE for the initial state $|\Psi(0)\rangle = |\uparrow\rangle$. *You may use mathematica.* [5 pts]

(c) Make drawings or plots of the probability to be in state $|\downarrow\rangle$ as a function of time, for $E = 0$, $E \approx \kappa$ and $E \gg \kappa$, discuss your results. [3pts]