

PHY 304, II-Semester 2023/24, Assignment 5

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Due-date: 31. March. 2024

(1) Optical theorem: [10 pts] Use the partial wave expansion of the scattering amplitude $f(\theta)$ to show that the total scattering cross section σ is given by

$$\sigma = \frac{4\pi}{k} \text{Im}[f(0)], \quad (1)$$

where k is the scattering wavenumber and $f(0)$ the forward scattering amplitude. This is known as the “optical theorem”.

(2) Rutherford scattering: [10 pts] Consider the scattering of two particles through the potential

$$V(\mathbf{r}) = -\frac{ZZ'e^2}{r} e^{-\alpha r} \quad (2)$$

in the first Born approximation.

- (a) Find the scattering amplitude, and express it in terms of the momentum transfer \mathbf{q} .
- (b) In the limit $\alpha \rightarrow 0$, find the corresponding scattering cross-section. Why can we interpret this now as scattering through the Coulomb potential? Compare your result with the Rutherford scattering cross-section (e.g. PHY 106) and discuss.

(3): Two-dimensional scattering of wavepackets: [10pts] The code `Assignment5_2Dscattering_draft_v1.m` solves the TDSE in two dimensions

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \left[-\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + V(x, y) \right] \Psi(x, y), \quad (3)$$

for the initial state

$$\Psi(x, y) = \frac{1}{(\pi^2 \sigma_x^2 \sigma_y^2)^{1/4}} e^{-\frac{(x-x_0)^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}} e^{ik_{\text{in}} x}, \quad (4)$$

and scattering potential $V(r) = A_{\text{pot}} e^{-r^2/\sigma_{\text{pot}}^2}$ with $r = \sqrt{x^2 + y^2}$. The wavepacket is set up broad compared to the range of the scattering potential, so this is the closest we can get to the incoming plane wave discussed in the lecture, without having to worry about boundary conditions at the edge of the simulated domain in x and y .

- (a) Run `Assignment5_2Dscattering_draft_v1.m` without changing parameters, and using the script `Assignment5_scattering_slideshow_v1.m` inspect the time evolution of the probability density $|\Psi(x, y, t)|^2$. Discuss what you see [3pts]. *Hint: Since this 2D simulation might take a little long to run on weaker computers, we also provide snapshots in Fig. 1 based on which you can answer Q3. But if you can run it, please see the real data, since that is more fun.*

- (b) Now, for various values of k_{in} in the range $k_{in} = 0.5..2$, run first the script `Assignment5_2Dscattering_draft_v1.m` to generate the evolution of the wavepacket without scattering potential $\Psi_{nsc}(x, y, t)$, and then `Assignment5_2Dscattering_draft_v1.m` with the same value of k_{in} , to generate the evolution with scattering potential $\Psi(x, y, t)$. Finally use `Assignment5_comparison_wfct_slideshow_v1.m`, which first calculates only the scattered part of the wavefunction $\Psi_{out}(x, y, t) = \Psi(x, y, t) - \Psi_{nsc}(x, y, t)$ and then plots the density of that $\rho_{out} = |\Psi_{out}(x, y, t)|^2$, to analyse the results (see figures (2) and (3)). Comment on what you see and why you see it. [5pts]
- (c) (Bonus) Feel free to do and report further numerical experiments, changing the potential parameters, inventing new potentials and moderately changing k_{in} .

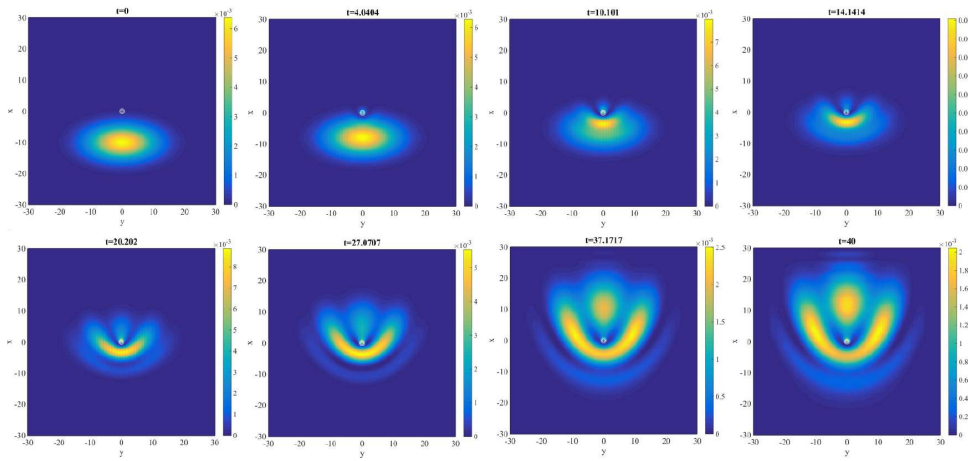


Figure 1: **For Q3(a)** Time evolution of probability density (colorshade) at the times t indicated above. For parameters used see `Assignment5_2Dscattering_draft_v1.m`.

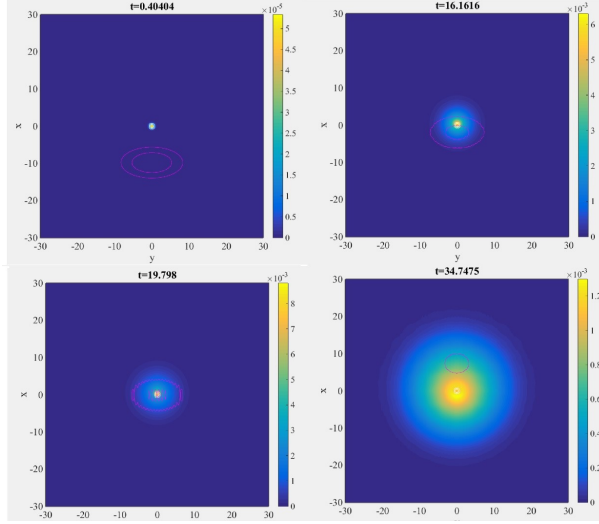


Figure 2: **For Q3(b)** Time evolution of only the scattered part of the density $|\Psi_{out}(x, y, t)|^2$ for $k_{in} = 0.5$. Violet contours indicate where the incoming wavepacket would be in the absence of a scattering potential.

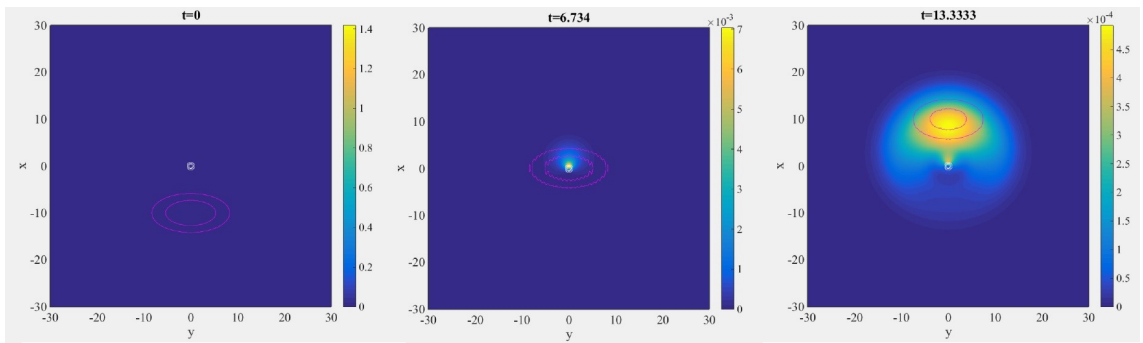


Figure 3: **For Q3(b)** Time evolution of only the scattered part of the density $|\Psi_{out}(x, y, t)|^2$ for $k_{in} = 1.5$. Violet contours indicate where the incoming wavepacket would be in the absence of a scattering potential.